Introduction to Optimization
MS&E 111/ENGR 62, Autumn 2007-2008, Stanford University
Instructor: Ashish Goel
Handout 1: Homework 1. Given 10/1/07. Due 10/8/07 in class.

Collaboration policy: You can solve Problems 1 and 2 with a partner. If you choose to do so, both of you should turn in a copy of your Answer Reports and clearly indicate who you worked with. Additionally, on any problem you can discuss general strategies with other students in this class but cannot collaborate on the actual final answer. You cannot discuss the HW with anyone not in the class.

Problem 1 A chicken farmer is trying to decide what to use as feed and is considering using a combination of feeds from local suppliers. The farmer would like to feed the chickens at minimum cost while making sure the chickens get an adequate supply of calories and vitamins. The cost, calorie content, and vitamin supply of each feed is given in the table below:

Contents	Feed Type A	Feed Type B
Calories (per lb)	500	700
Vitamins (units per lb)	50	25
Cost (per lb)	\$1	\$2

Each chicken requires at least 600 calories per day and 40 units of vitamins. As an additional constraint, the farmer would like at least half of the diet to consist of feed type B for public relations purposes.

a) Formulate this problem as a linear program. Clearly indicate your decision variables, your objective function, and your constraints.

b) Solve this in Excel. Attach the Answer Report to your HW. Be sure to indicate on it if you worked with anyone else.

Problem 2 The XYZ Corporation is undergoing a radical restructuring and needs to hire consultants to help in several key areas. XYZ has contracts with four consultants (A, B, C, and D). The table below indicates, for each of the consultants, what they charge per hour and how many hours they have available to work for XYZ.

Consultant	rate/hr	total hours available
A	\$300/hr	60
B	\$250/hr	60
C	\$350/hr	50
D	\$200/hr	40

XYZ has four main tasks that need to be completed. The table below indicates how many hours XYZ estimates that it would take each consultant to perform each task. If no amount of time is indicated for the consultant to complete a task, then the consultant cannot perform the task.

Consultant	hrs/task 1	hrs/task 2	hrs/task 3	hrs/task 4
A	30	20		40
B	30	30	30	
C	25	25	25	25
D			40	40

XYZ want to determine how to allocate tasks to consultants at minimum cost. Note that all tasks must be completed, but not all consultants need to be used to their full availability. You may also allow consultants to split tasks. If they do so, you can assume that each consultant gets a constant fraction of the task completed per unit of time worked. For example, Consultant A would complete 1/2 of task 1 in 15 hours.

a) Formulate this problem as an LP. Clearly indicate your decision variables, your objective function, and your constraints.

b) Solve this in Excel. Attach the Answer Report to your HW. Be sure to indicate on it if you worked with anyone else.

Problem 3 A company produces two kinds of products. A product of the first type requires 1/4 hours of assembly labor, 1/8 hours of testing, and \$1.20 worth of raw materials. A product of the second type requires 1/3 hours of assembly, 1/3 hours of testing, and \$0.90 worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing each day. Products of the first and second type have a market value of \$9 and \$8, respectively.

a) Formulate a linear programming problem that can be used to maximize the daily profit of the company.

b) Suppose that up to 50 hours of overtime assembly labor can be scheduled, at a cost of \$7 per hour. Modify the linear program from a) to incorporate this change. Clearly specify the entire new LP, not just any changes.

Problem 4 A search engine needs to decide which advertisement to show on the search results page each time a user searches for a keyword. Assume there are m keywords that users can search for, and the number of queries for keyword j is estimated to be v_j. There are n advertisers. The i-th advertiser has budget b_i and is willing to pay a price p_{ij} for an advertisement shown on the search

results page for keyword j. Assume that only one advertisement can be shown on each search results page.

The search engine needs to decide, for each (i, j), the fraction (or if you prefer, the number) of search results pages for keyword j that will show an advertisement from advertiser i. The search engine's goal is to maximize its revenue. Formulate this as a linear program.

Problem 5 a) Convert the following LP with the two decision variables z_1, z_2

$$\begin{array}{rl} \text{maximize} & 2z_1 + z_2 \\ \text{subject to} & z_1 \geq z_2 \\ & z_1 + z_2 \leq 3 \\ & z_1 \geq 0 \end{array}$$

to the vector notation form

$$\begin{array}{rl} \text{minimize} & c^T x \\ \text{subject to} & Ax \geq b \\ & x \geq 0 \end{array}$$

Clearly indicate what x, c, A, and b are. If you have added or replaced any decision variables, clearly indicate the relationship between the original decision variables and the new ones. Also, clearly indicate the relationship between the value of your new objective function and the original one.

b) Show how $|x-1/2| \leq 1/2$ can be represented in terms of linear constraints.

c) If $|x-1/2| \geq 1/2$ could be represented in terms of linear constraints, then we could represent binary variables in LPs. In one concise sentence, explain why this is true.

d) Can $|x-1/2| \geq 1/2$ be represented in terms of linear constraints? Explain your answer in one concise sentence.