

### Introduction to Optimization

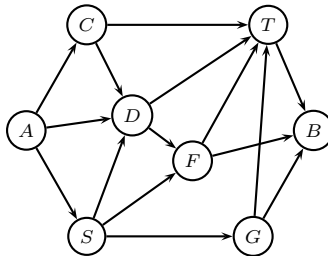
MS&E 111/ENGR 62, Autumn 2007-2008, Stanford University

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Homework 4. Given 11/5/07. Due 11/14/07 in class.

*Collaboration policy:* You can solve Problems 1 and 2 with a partner. If you choose to do so, both of you should turn in a copy of your Answer Reports and clearly indicate who you worked with. For all other problems you can discuss general strategies with other students in this class but cannot collaborate on the actual final answer. You cannot discuss the HW with anyone not in the class.

**Problem 1** [10 points]



Consider the oil pipeline network in the figure above. The table below gives the capacity of each pipeline (in million gallons per hour), the cost per million gallons of sending oil over a pipeline, and the failure probability.

Edge	Capacity	Cost	Failure Prob.
AC	5	\$7000	.4
AD	4	\$4000	.9
AS	2	\$1000	.1
CD	1	\$2000	.2
CT	1	\$4000	0
DT	3	\$2000	.5
DF	1	\$2000	.1
SD	1	\$1000	.3
SF	2	\$3000	.4
SG	2	\$5000	.2
FB	1	\$2000	.2
FT	4	\$6000	.2
GT	5	\$0	1
TB	2	\$1000	.6
GB	2	\$2000	.8

For each of the following, provide an LP whose solution gives the desired information and solve in Excel:

- (a) the cheapest way of sending 1 million gallons per hour from  $A$  to  $B$
- (b) the cheapest way of sending 5 million gallons per hour from  $A$  to  $B$
- (c) the most reliable path from  $A$  to  $B$
- (d) the maximum amount of oil that can be sent from  $A$  to  $B$

**Problem 2** [10 points]

- (a) Suppose that we have  $N$  different currencies (e.g., US Dollars, Euros, etc.). For each pair of currencies,  $(i, j)$ , one unit of currency  $i$  can be exchanged into  $r_{ij}$  units of currency  $j$ .

We wish to find an arbitrage opportunity if it exists, i.e. a sequential series of currency trades that allows us to make a profit with no risk. Formulate this as a min cost flow problem and provide an LP with a bounded feasible polytope which solves it. Assume no transaction costs.

- (b) Can you find an arbitrage opportunity given the currency exchange rates in the spreadsheet `currencies.xls`? If so, describe the corresponding trades. What is the maximum amount of profit that can be made for each \$1.00 invested in this procedure?

- (c) Suppose now that the exchange charges a  $p\%$  transaction fee on each unit of currency traded. In other words, if we wish to trade currency  $i$  into currency  $j$ ,  $p\%$  of our initial amount of  $i$  is “taken away” before doing the exchange.

Modify your LP from (a) to take these fees into account. Assuming  $p = 0.50$ , how does this change your answer for (b)?

**Problem 3** [5 points]

You are given a min-cost flow problem where the total supply is larger than the total demand. Any excess left over supply at any node must be discarded at a cost of  $f$  per unit. Briefly describe, in words, how we can model this as a standard min-cost flow problem where the total supply equals the total demand.

**Problem 4** [5 points]

Suppose every edge has a lower bound as well as an upper bound on the amount of flow that can go through the edge. For example, a pipeline might rust if a minimum amount of oil is not shipped through that pipeline. What additional conditions (beyond those described in class) should the problem satisfy for you to claim that all basic feasible solutions have integer components? Explain why very briefly.

**Problem 5** [10 points]

You are given an  $N \times N$  matrix  $A$  such that each entry of the matrix is a non-negative number. Further assume that the sum of the entries in any row or column is an integer. You are allowed to round each fractional entry in the matrix, i.e. to change each non-integer entry to either the next higher or next lower integer. Prove that there is a way of rounding each entry such that the row and column sums remain unchanged.

*Hint:* Consider a min cost flow problem with  $2N$  nodes, one for each row and one for each column. Then, apply the integrality theorem from class along with the results of problem 4 above.