Introduction to Optimization
MS&E 111/ENGR 62, Autumn 2007-2008, Stanford University
Instructor: Ashish Goel
Handout 11: Homework 5. Given 11/16/07. Due 11/28/07 in class.

Collaboration policy: You can solve Problems 1 and 2 with a partner. If you choose to do so, both of you should turn in a copy of your Answer Reports as requested and clearly indicate who you worked with. Additionally, on any problem you can discuss general strategies with other students in this class but cannot collaborate on the actual final answer. You cannot discuss the HW with anyone not in the class.

Problem 1 Suppose you are in charge of inventory management for a company that manufactures a single product. At the beginning of day i, starting from day $i = 1$, a set of customers shows up and (in total) orders d_i items. They pay p for each item. However, due to market fluctuations, the cost of manufacturing varies over time. Each day it costs c_i per item to manufacture. There are also inventory costs. Storing each item has a holding cost of h per day. Finally, if you cannot satisfy all customers on a given day, it costs you b for every day you make a customer wait to get the item (due to a promotional agreement.)

a) As the experienced inventory manager suppose you know what all of the demands will be up to day T. Model the problem as a min-cost flow to find the most profitable inventory planning up to day T where $T = 3$. Assume that you can produce items instantly (meaning you can sell them the same day they are produced) and there is no limit on the number of items you can manufacture per day. In your solution you should clearly indicate what your nodes and edges are, as well as all other parameters that are required for a min-cost flow problem. Also clearly specify how you would retrieve from the solution to your model the number of items you would manufacture and sell every day. You can use a graph to represent your answer as long as it is clearly labeled.

b) Using Excel, find the optimal plan when $T = 5, p = 8, h = 1$, and $b = 2$. The demands and costs are given in the table below.

i	1	2	3	4	5
d_i	4	5	3	6	8
c_i	6	4	3	6	8

Write up your solution clearly indicating what your decision variables represent and the number of items you would manufacture and sell every day. Also print out and submit your Answer Report.

Problem 2 Consider the following production problem. A furniture company builds tables and chairs using only wood and nails. Each table requires 4 planks of wood and 2 boxes of nails. Each chair requires 1 plank of wood and 2 boxes of nails. The company currently has 2000 planks of wood and 1200 boxes of nails. Each table you build can be sold for $100 and each chair can be sold for $40.

a) Formulate an LP that will solve this problem. (Assume that you can build fractional tables and chairs.)

b) Now formulate the dual for this problem.

c) Using Excel, solve the dual LP. Write up your solution. Also print out and attach your Answer Report.

d) Without solving the primal LP, what is the maximum revenue the company can make? How do you know?

e) In one concise sentence, explain what the dual variables represent with respect to the original problem.

f) The company realizes that it actually has one more plank of wood. Without solving another LP, what is the new maximum revenue?

g) If the company instead had to pay \$15 for an additional plank of wood, should it do so? What if it had to pay \$15 more for an additional box of nails?

Problem 3 (From VRM 4.2)

a) Convert the following optimization problem

$$\begin{array}{rl} \text{maximize} & -x_1 - 2x_2 - x_3 \\ \text{subject to} & x_1 + x_2 + x_3 = 1 \\ & |x_1| \leq 4 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

into a linear program of the form

$$\begin{array}{rl} \text{maximize} & c^T x \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array}$$

Note: $|x|$ denotes the absolute value of x.

b) Find the dual of this LP.

Problem 4 a) Create a shortest path instance where the primal (the original minimization problem we set up in class to solve this problem) is infeasible and the dual is unbounded. Clearly draw and label the network with all necessary information for a shortest path problem.

b) Find a shortest path instance where the primal is unbounded and the dual is infeasible. Again, clearly draw and label the network with all necessary information for a shortest path problem.

c) Find a shortest path instance where both the primal and dual are infeasible. Again, clearly draw and label the network with all necessary information for a shortest path problem.

Problem 5 There are N teams in a college football conference, and the j-th team in this conference has won $t(j)$ games so far. The only remaining games are between teams in the same conference. Let $k(i,j)$ denote the number of games remaining between teams i and j in this conference. Our goal is to determine whether team Q can end the season with the most victories (or at least tied for the most victories), assuming ties are not allowed in games. Model this as a max-flow problem. Clearly indicate how the nodes and edges would be created as well as indicating all other information about nodes and edges required in a max-flow problem.