Introduction to Optimization
MS&E 111/ENGR 62, Autumn 2007-2008, Stanford University
Instructor: Ashish Goel
Homework 6. Given 11/28/07. Due 12/5/07 in class.

Collaboration policy: You can solve Problems 1 and 2 with a partner. For all other problems you can discuss general strategies with other students in this class but cannot collaborate on the actual final answer. You cannot discuss the HW with anyone not in the class.

Problem 1 [10 points]
The file blast.txt (posted on the course web page), contains four paragraphs. The first two are short descriptions of BLAST from the web. The third is a typical kind of plagiarism: some material has been deleted from paragraph 1 and then a couple of sentences have been imported from paragraph 2. The fourth is a review of the movie "Spider-Man."

Using Excel, find the length of the longest common subsequence between paragraphs 1 and 2. Repeat for the paragraph pairs 1,3 and 1,4. For your convenience, a list of words in each paragraph (with very short words eliminated since they are usually too common to be useful) is also provided.

Since this Excel sheet will be too large to easily print, please email your solutions to yolken@stanford.edu by 3:30PM on the due date. If you work on this problem with a partner, only one of you needs to send the solution- just make sure that the other person's name is noted in the email.

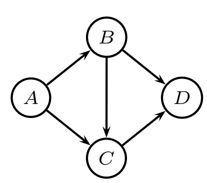
Problem 2 [10 points] The file largemaxflow.xls (posted on the course web site), contains information on the edges of a graph. Using Excel, find a minimum cut between nodes 1 and 20.

Attach a copy of your answer report along with a brief explanation of your formulation and final answer.

Problem 3 [10 points] Consider a *multi-commodity* min cost flow problem, i.e. one for which we are transporting/shipping/sending multiple products (e.g. oil, water, milk) over the same network. Assume that each link has just a single capacity and cost. However, nodes can have separate demands for each of the various commodities. For a complete example, see Lab 4.

Show that a BFS in such a problem is not necessarily integral, even if the demands and capacities are all integers. *Hint:* There is a counterexample with 3 nodes and 3 commodities.

Problem 4 [10 points] Consider the following network:



Assume all costs are 1.

(a) Write down an LP which can be solved to give the shortest path from A to D. Provide two optimal solutions- one that is a BFS and one that is not a BFS.

(b) Formulate the dual of the LP in (a) and solve (either by inspection or in Excel).

(c) Write down the complementary slackness conditions for the given primal/dual pair. Verify that these hold irrespective of which part (a) primal solution you use.

(d) Provide an example primal/dual pair where, in optimality, a primal constraint is tight and the corresponding dual variable is also zero.

Extra Credit Challenge Problem [up to +10 points]
The government has decided to distribute educational funds to N cities, subject to various constraints (e.g., the total amount given is less than some value, all cities in state X receive at least a certain amount, etc.). Let x represent the (non-negative) amounts given to each of the cities with $Ax \leq b$ being the system of feasible allocations.

Given this setup, the government wishes to maximize the total amount given to the K lowest recipients, i.e. the sum of the K lowest values in the vector x; the higher this objective is, the more "fair" the program will appear in the eyes of the press and voters.

Formulate an LP which finds the feasible allocation maximizing the above "fairness" value for some fixed K. Since N may be large, your answer should have as few extra constraints and variables as possible.