Introduction to Optimization
MS&E 111/ENGR 62, Autumn 2008-2009, Stanford University
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Problem 1   Consider a stock that can take on prices in $\{\$1, \ldots, \$100\}$.
Assume that the current price of the stock is $50, there is a zero-coupon bond
available at a price of $0.9, and there is a European call option expiring in one
year with strike price of $50 available at a price of $10. There is also a European
put option expiring in one year with strike price of $50 available but we don't
know its price.

A butterfly spread is a contingent claim that pays its holders $\max\left(K - |L - S|, 0\right)$
if the price of the stock is S at some future time T, where K, L, and T are pa-
rameters of the contract. Consider a butterfly spread that pays its holder one
year from now, with $K = 10$ and $L = 50$.

(a) What is a reasonable, arbitrage-free price for the put option described
above? (*hint:* see VRM 2.4.2)

(b) What are the payoff vectors for the available securities?

(c) What is the payoff vector for the given butterfly spread?

(d) Formulate a linear program to find the cheapest super-replicating port-
folio, and solve in Excel.

(e) Suppose now that there are *transaction costs*. In particular, you must
pay $0.50 per stock, $0.10 per bond, $0.20 per call, and $0.15 per put traded
(i.e., bought or sold). Reformulate the super-replication problem to include
these costs in the objective, and solve in Excel.