Introduction to Optimization
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Problem 1

Consider the following problem:

$$\begin{aligned} \text{minimize} \quad & 2x_1 - 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \le 1 \\ & x_1 \ge 0 \end{aligned} \tag{1}$$

(a) Derive the dual and determine its optimal solution by inspection.

(b) How can one use the complementary slackness property and the optimal solution of the dual to identify the optimal solution to the primal?

(c) Suppose that the coefficient of x_1 in the primal objective is c_1 rather than 2. For what values of c_1 does the dual problem have no feasible solutions? For these values, what does duality theory imply about the primal problem?

Problem 2

Recall the linear program for "feeding an army":

$$\begin{aligned} \text{minimize} \quad & x_1 + 0.25x_2 && \text{(cost of diet)} \\ \text{subject to} \quad & 40x_1 + 200x_2 \ge 400 && \text{(carbohydrate requirement)} \\ & 5x_1 + 40x_2 \ge 40 && \text{(dietary fiber requirement)} \\ & 100x_1 + 20x_2 \ge 200 && \text{(protein requirement)} \\ & x \ge 0 \end{aligned} \tag{2}$$

The optimal solution and objective function value are:

$$x^* \approx \begin{bmatrix} 1.67 \\ 1.67 \end{bmatrix} \qquad c^{\mathrm{T}} x^* \approx 2.08 \tag{3}$$

(a) Suppose that the dietary fiber requirement is some value b_2 rather than 40. Plot the optimal objective function value versus b_2.

(b) Consider the dual linear program:

$$\begin{aligned} \text{maximize} \quad & 400y_1 + 40y_2 + 200y_3 \\ \text{subject to} \quad & 40y_1 + 5y_2 + 100y_3 \le 1 \\ & 200y_1 + 40y_2 + 20y_3 \le 0.25 \\ & y \ge 0 \end{aligned} \tag{4}$$

(i) Provide an intuitive interpretation of the constraints and objective.

(ii) What is the optimal objective function value?

(iii) How does the optimal solution y^* relate to the plot from part (a) above?