E62 / MS&E 111

Lab 5: Solutions

a.) primal:

$$\min \ 2x_1 - 4x_2$$
$$\text{s.t.} \quad x_1 + x_2 \le 1$$
$$x_1 \ge 0$$

convert to standard form $\Longrightarrow$

$$\max \ -2x_1 + 4(x_2^+ - x_2^-)$$
$$\text{st} \quad x_1 + x_2^+ - x_2^- \le 1$$
$$x_1, x_2^+, x_2^- \ge 0$$

dual:

$$\min \ y$$
$$\text{st} \quad y \ge -2$$
$$y \ge 4$$
$$-y \ge -4$$
$$y \ge 0$$

simplify $\Longrightarrow$

$$\min \ y$$
$$\text{st} \quad y \ge -2$$
$$y = 4$$
$$y \ge 0$$

$$\boxed{y^* = 4}$$

b.) $(b - Ax^*)^T y^* = 0$

$(A^T y^* - c)^T x^* = 0$

$$\left(\begin{bmatrix} 4 \\ 4 \\ -4 \end{bmatrix} - \begin{bmatrix} -2 \\ 4 \\ -4 \end{bmatrix} \right)^T x^* = 0$$

$$\begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} x_1^* \\ x_2^{+*} \\ x_2^{*-} \end{bmatrix} = 0 \Longrightarrow \boxed{x_1^* = 0}$$

$$(1 - (x_1^* + x_2^{+*} - x_2^{-*}))\, y^* = 0$$

$$(1 - [x_2^{+*} - x_2^{-*}])\, 4 = 0$$

$$\Rightarrow x_2^{+*} - x_2^{-*} = 1$$

$$\Rightarrow \boxed{x_2^* = 1}$$

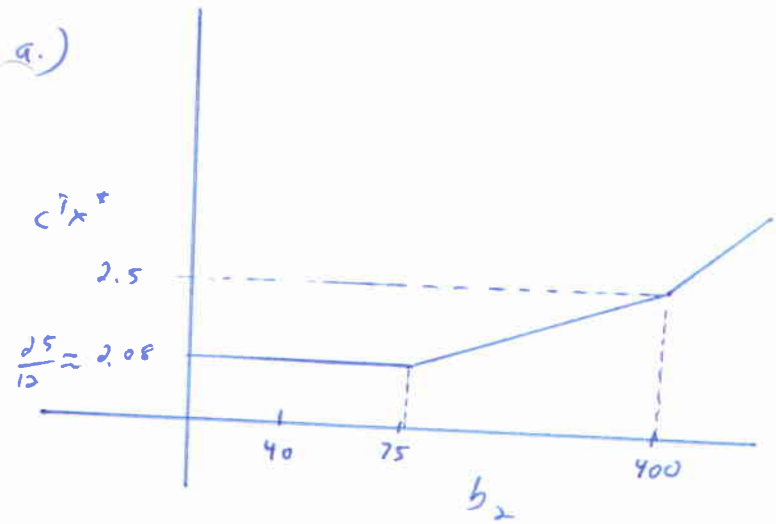
c.) Dual is infeasible when $c_1 < -4$

In this case, primal is unbounded

(although, in general, primal could be infeasible)

Problem 2

a.)



(Not to scale)

b.) i) Imagine company creates carbohydrate, fiber, and protein pills to satisfy army's dietary requirements. Dual seeks to maximize company's profit subject to constraints that pills are no more expensive than food items they are replacing

ii) $b^T y^* = 2.08$ (by strong duality)

iii.) We must have $y_2^* = 0$ because optimal objective function value is insensitive to changes in b_2 around 40 (see plot above).