

Introduction to Optimization

MS&E 111/Engr 62, Fall 2008, Stanford University

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Sample Midterm Problems (The actual midterm will be shorter in length)

1. VRM Chapter 3, Problem 10(a) (formulate, but do not solve). Without solving the problem, show that if an optimal solution exists, then there is an optimal solution that uses at most 3 alloys. In addition, is it possible to have an optimal solution that uses 4 alloys?
2. VRM Chapter 3, Problem 12(a), (c)
3. Four customers, A, B, C, D are in the market to buy five loaves of bread each. The baker has three types of bread, P, Q, and R. There are 8 loaves of type P, 6 loaves of type Q, and 6 of type R. The baker knows how many dollars each customer is willing to pay for each loaf of bread:

	A	B	C	D
P	1	2	2	1
Q	2	3	2	1
R	3	1	2	1

The baker wants to decide how many of each type of loaf to sell to each customer to maximize her profit. Formulate this as an LP. You may assume that loaves can be split between multiple customers.

Prove that there exists an optimum solution with at most 7 non-zero variables.

(Extra credit challenge) Prove that there exists an optimum solution with at most 6 non-zero variables.

4. You are given N tasks, and M agents. Agent i can execute a fraction f_{ij} of task j in one hour. You have to assign tasks to agents to minimize the time t when all the jobs get completed, i.e. when the latest finishing job completes. Assume that jobs can be done simultaneously.

Formulate this as an LP.

Prove that there exists an optimum solution with at most $N + M - 1$ pairs (i, j) such that agent i executes a non-zero amount of task j .

5. There are 12 possible food items that can be part of an emergency food package for dropping from an airplane to areas affected by California wildfires. Each package must provide 200% of the daily recommended value of carbs and fat each. The maximum allowed cost for a package is \$20. Let s_i represent the cost in dollars of 1 lb of food i . Let r_i, f_i denote the percentage of the daily recommended value of carbs and fats, respectively, in 1 lb of food i . The goal is to minimize the total weight of the food in a package.

- (a) You need to formulate this problem as a linear program of the form

$$\begin{aligned} & \text{minimize} && b^T \cdot y \\ & \text{subject to:} && \\ & && Ay \geq c \\ & && y \geq 0. \end{aligned}$$

More specifically, write down the dimensions of b, A, y , and c , and clearly explain what the components of these matrices/vectors correspond to in this problem.

- (b) How many different food items could be present in the optimal package obtained by solving this linear program using a typical commercial linear programming solver (eg. Excel)? Explain briefly. Assume that the LP has a basic feasible solution as well as an optimum solution.
6. Given points x, y, z , suppose a is a convex combination of x, y and b is a convex combination of a, z . Show that b is also a convex combination of x, y, z .