

## Introduction to Optimization

MS&E 111/ENGR 62, Autumn 2008-2009, Stanford University

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Homework 2. Given 10/6/08. Due 10/13/08 in class.

Collaboration policy: You can solve Problems 4 and 5 with a partner. If you choose to do so, both of you should turn in a copy of your Answer Reports and clearly indicate who you worked with. Additionally, on any problem you can discuss general strategies with other students in this class but cannot collaborate on the actual final answer. You cannot discuss the HW with anyone not in the class.

**Problem 1** List all basic feasible solutions of the following LP:

$$\begin{array}{ll} \text{maximize} & x_1 + 3x_2 \\ \text{subject to} & x_1 + 0.1x_2 \leq 2 \\ & 0.4x_1 + 2x_2 \leq 3 \\ & x_1 + 1.1x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{array}$$

**Problem 2** Recall that the problem of finding an arbitrage opportunity in a market with  $N$  assets whose prices are given as  $\rho \in \mathcal{R}^N$  and whose payoffs are given as  $P \in \mathcal{R}^{M \times N}$  can be posed as the LP:

$$\begin{array}{ll} \text{minimize} & \rho^T x \\ \text{subject to} & Px \geq 0 \\ & \rho^T x = -1 \end{array}$$

Suppose now that there are transaction costs. In particular, for each  $j \in \{1, \dots, N\}$  we must pay a transaction cost of  $q_j > 0$  per unit of the  $j^{\text{th}}$  contingent claim bought or sold short. Provide a linear program that finds an arbitrage opportunity (if it exists) which minimizes the transaction cost incurred for every unit of current profit. Make sure that the optimization problem you provide is written as a linear program; that is the objective function and constraints are linear.

**Problem 3** A policy maker in Sacramento county needs to allocate some water to a group of  $n$  farmers in California from  $m$  reservoirs. Because the media will highlight any farmer who is very unhappy, the policy maker wants to maximize the minimum amount that any of the farmers receives. Reservoir  $i$  has capacity  $u_i$ . If farmer  $j$  is served from reservoir  $i$ , then a fraction  $f_{ij}$  of the water gets lost in evaporation while being channeled to the farmer.

(a) Formulate this as a linear program.

(b) Suppose farmer 2 cannot be served by reservoir 3 because the farmer is at a higher altitude than the the reservoir. How can you take this into account in your model without increasing the number of constraints or variables?

**Problem 4 (VRM 3.13)** MSE Airlines (pronounced messy) needs to hire customer service agents. Research on customer demands has led to the following requirements on the minimum number of customer service agents that need to be on duty at various times in any given day:

Time Period	Staff Required
6am to 8am	68
8am to 10am	90
10am to noon	56
Noon to 2pm	107
2pm to 4pm	80
4pm to 6pm	93
6pm to 8pm	62
8pm to 10pm	56
10pm to midnight	40
Midnight to 6am	15

The head of personnel would like to determine the least expensive way to meet these staffing requirements. Each agent works an 8 hour shift, but not all shifts are available. The following table gives the available shifts and daily wages for agents working various shifts:

Shift	Daily Wages
6am-2pm	\$180
8am-4pm	\$170
10am-6pm	\$160
Noon-8pm	\$190
2pm-10pm	\$200
4pm-Midnight	\$210
10pm-6am	\$225
Midnight-8am	\$210

(a) Write a linear program that determines the least expensive way to meet the staffing requirements.

(b) Solve the linear program using Excel.

**Problem 5** You can find the data for this problem in the Excel file "hw2prob5.xls" on the course web site. In the file, you will find a list of securities, along with their attributes: asset type, price, expected return, and maximum loss per dollar invested (guaranteed by your broker). Assume that you are given an endowment of \$100 to invest in these securities, and that you can invest in any fraction of any asset.

(a) Write a linear program in vector notation to allocate your endowment in the securities to maximize the expected return of your portfolio, and such that

- At least 40% of the portfolio (dollars invested) are in government bonds;
- No more than 10% of the portfolio (dollars invested) be in alternative investments;
- The maximum allowable loss of the portfolio is 30%;
- Due to the recent financial meltdown, you are not allowed to short sell any security (invest in a negative amount).

Solve the linear program using Excel.

(b) How should the maximum expected return change if the maximum allowed loss is increased?