Introduction to Optimization
MS&E 111/ENGR 62, Autumn 2008-2009, Stanford University
Instructor: Ashish Goel
Homework 3. Given 10/13/08. Due 10/20/08 in class.

Collaboration policy: There are two Excel problems: 1 and 5. You must do problem 1 on your own. You can solve Problem 5 with a partner. If you choose to do so, both of you should turn in a copy of your Answer Reports and clearly indicate who you worked with. Additionally, on any problem you can discuss general strategies with other students in this class but cannot collaborate on the actual final answer. You cannot discuss the HW with anyone not in the class.

Problem 1 Consider the problem of separation of private and public schools:

a) Set up an LP that will minimize the maximum violation (i.e. the max of the positive and negative violation) subject to the constraint that that the sum of all the positive and negative violations is less than equal to some constant B.

b) Consider the following data:

Private Schools

College	cost of living	tuition
MIT	116.3	34986
Yale	123.1	34530
Harvard	116.3	31456
Princeton	130	34202
Stanford	138.9	34800
Valparaiso	92.9	23200

Public Schools

College	cost of living	tuition
Wisonsin Madison	94.4	21438
Washington-Seattle	104.6	22131
Berkeley	138.9	28004
Minnesota Twin cities	99.1	19580
Texas Austin	89.6	27468

Solve this LP in Excel for both $B = 4$ and $B = 6$ and attach your Answer Reports for both problems.

c) What trivial solution do you obtain if you solve this LP without the constraint that the sum of all errors must be less than B? (Hint: you should not need to actually solve the LP.)

d) What problem would you solve to determine the minimum B value for which this problem has a solution?

e) For what range of values of B could we be confident that we would always get a solution, and the solution would not be the one you discussed in part c?

Problem 2 Suppose we have 4 jobs and 4 agents. The table below indicates how many hours it takes each consultant to perform each task.

Agent	hrs/task 1	hrs/task 2	hrs/task 3	hrs/task 4
A	12	6	9	23
B	11	7	10	21
C	13	6	8	18
D	17	5	8	19

Each job can be split among different agents, and an agent can perform multiple jobs. Also, multiple agents can work on their piece of the job at the same time. The completion time of the last job to be completed is known as the "makespan" of the problem.

a) Formulate an LP to minimize the makespan.

b) Solve the LP in Excel.

Hint: Check out Chapter 4 of Supplementary Notes by Professor Goel.

Problem 3 ARC Consultants needs to know how long it will take to complete a particular job. The job consists of n different tasks. Each task k takes t_k hours to complete (regardless of how many people work on it). Additionally, certain tasks cannot be performed until other tasks have been completed. This information is represented in a nxn matrix P where the (i, j)th element of the matrix is a 1 if task j must be performed before task i and a 0 otherwise. (Note that all elements on the diagonal must be zero as a task cannot be performed before itself.) There is no limit on the number of tasks that can be performed at once.

Set up a linear program that determines how soon ARC can complete the job. As always, clearly specify your decision variables, objective function and constraints.

Problem 4 In class, we considered the following problem: in a world with N assets and M possible future states, a payoff vector $b \in \mathbb{R}^M$ can be super-replicated using a set of securities. More specifically, let $\rho \in \mathbb{R}^N$ be the vector of prices of the securities and the payoff matrix P be defined by

$$P = \begin{pmatrix} a_1 & a_2 & \cdots & a_N \end{pmatrix}.$$

where vector $a_j \in \mathbb{R}^M$ is the possible payoffs of security j, then the minimum cost super-replicating portfolio can be found by solving the LP

$$\begin{array}{rl} \text{minimize} & \rho^T x \\ \text{subject to} & Px \geq b \end{array}$$

Define $z^{(M)}$ as the optimal value for the super-replicating problem when there are M possible future states of the world corresponding to the value r of the currency after one year being $1/M$, $2/M$, $3/M$, ..., 1. For the example from class, there are 100 possible future states, so the optimal value for that problem is $z^{(100)}$.

a) Is it always the case (for all possible desired payoffs b) that $z^{(100)} \geq z^{(50)}$? Explain briefly.

b) Is it always the case (for all possible desired payoffs b) that $z^{(100)} \geq z^{(200)}$? Explain briefly.

Problem 5 Consider the following two sets of sample data, U and V where each row represents a point in $\Re^2$:

$$U = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & .5 \end{bmatrix}.$$

$$V = \begin{bmatrix} 5 & .5 \\ 3 & 3 \\ 0 & .5 \\ 2 & 0 \end{bmatrix}.$$

a) Using the technique from class, in Excel find a linear separator in $\Re^2$ that minimizes the sum of the error terms. Attach your Answer Report.

b) Plot the points and the linear separator. You can do this by hand or with the program of your choice. Would you say that this is a good separator?

c) Create 4x4 matrices U' and V' where each row of U' is of the form $U'_{i*} = [U_{i1}, U_{i2}, U^2_{i1}, U^2_{i2}]$, and likewise for V'.

d) Using Excel find a linear separator that minimizes the sum of error terms for this data. Attach your Answer Report.

e) Can you determine from your answer to d) if this is a good separator? If so, how?

f) Recall that a non-rotated ellipse in Re^2 is of the form $a(x - x_0)^2 + b(y - y_0)^2 = c$. Does your solution from d) correspond to an ellipse in the original 2 dimensional space of the problem? If so, write the equation that specifies the ellipse.