

1. Is eBay's bidding mechanism closer to a second price auction or a first price auction? Explain very briefly.

It is closer to a second price auction.

2. Suppose there are  $m$  web pages of type 1 and  $n$  web pages of type 2. Let the type 1 web pages be  $A_1, A_2, \dots, A_m$  and let the type 2 web pages be  $B_1, B_2, \dots, B_n$ . Each web page of type 1 has a link to a web page of type 2. Also, each web page of type 2 has a link to a web page of type 1. What is the naive PageRank of each page in terms of  $m$  and  $n$ ?

By symmetry, all type 1 pages will have the same PageRank, say  $\pi(A)$ , and all type 2 page will have the same PageRank, say  $\pi(B)$ .

$$\pi(A) = \frac{n}{m}\pi(B)$$

$$m\pi(A) + n\pi(B) = 1$$

Thus,  $\pi(A) = 1/(2m)$  and  $\pi(B) = 1/(2n)$ .

3. Which of the following Bernoulli random networks are connected with high probability as  $N$  goes to infinity?
  - (a) The probability  $p$  is 0.1
  - (b) The average degree is 10
  - (c) The average degree varies as  $\sqrt{N}$ .

We use the fact that the graph is connected with high probability if  $p > \log N/N$  or equivalently  $p \cdot N > \log N$ , where  $p \cdot N$  is the average degree. Thus, (a) and (c) will be connected with high probability, while (b) will not.

4. Consider a line network with  $N$  nodes in which each node knows its two neighbors. Further, there is an edge between the  $i$ -th and  $j$ -th nodes with probability  $1/|j - i|$ . Describe a simple message forwarding protocol on this network such that the expected time for a message to get delivered from node  $a$  to node  $b$  is proportional to  $\log N$  (again, rough order of growth is all that is required). Explain your reasoning. Each node  $k$  that gets the message forwards it to closest node on the left of  $b$  that  $k$  is directly connected to. It was shown in class that the average path length is  $\log N$ .
5. Which of the following are likely to benefit from network neutrality? Which are not? Explain very briefly.

- (a) AT&T
- (b) Comcast
- (c) Amazon
- (d) Netflix
- (e) Skype
- (f) BitTorrent

AT&T and Comcast would not benefit (as Internet Service Providers). Amazon, Netflix, Skype, and BitTorrent would benefit.