MS&E 235: Internet Commerce, Practice Midterm

1. We assume that N bidders compete for a single item. The valuation of bidder i is v_i and $v_1 > v_2 > ... > v_N$.

   (a) First assume that a first price auction is used and show that the revenue to the auctioneer is at least v_2 at any Nash equilibrium. [5 pts]

   (b) Now assume that a second price aucton is used and show there exist Nash equilibria at which the revenue to the auctioneer is arbitrarily small. In particular, show that for any $\epsilon \in (0, v_N)$ there exists a Nash equilibrium at which the revenue to the auctioneer is ϵ. [5 pts]

2. Consider a search engine with discount factor θ and an advertisement with the following characteristics.

   - When the advertiser pays x per impression (but nothing if the advertisement is clicked), then the Gittins' index is g_1.
   - When the advertiser pays y per click (but nothing per impression), then the Gittins' index is g_2.

   Now assume that the advertiser pays x per impression and additionally pays y if the advertisement is clicked. Is the Gittins' index $g_1 + g_2$? [8 pts]

3. Consider a graph with five nodes and two directed 3-cycles. Node A has an edge to node B, B has an edge to C, and C has an edge to A. Node A has an edge to node D, D has an edge to E, and E has an edge to A. Compute the naive PageRank of each node. [8 pts]

4. Is the following statement true of false? Both search and display online advertisement have been steadily growing at a fast pace. [2/-1 pts]

5. Suppose $m_i(t) = (t/i)^{1.5}$. Which of the following is true? [2/-1 pts]

   (i) The median is infinite and the expected product rank is infinite.

   (ii) The median is bounded and the expected product rank is infinite.

   (iii) The median is bounded and the expected product rank is bounded.

   (iv) The median is infinite and the expected product rank is bounded.