MS&E 235: Internet Commerce, Practice Midterm

1. We assume that N bidders compete for a single item. The valuation of bidder i is v_i and $v_1 > v_2 > ... > v_N$.

   (a) First assume that a first price auction is used and show that the revenue to the auctioneer is at least v_2 at any Nash equilibrium. [5 pts]

   Suppose the revenue to the auctioneer is less than v_2. Then it must be that the two highest bids are less than v_2. But then either bidder 1 or bidder 2 will have an incentive to increase their bids.

   (b) Now assume that a second price aucton is used and show there exist Nash equilibria at which the revenue to the auctioneer is arbitrarily small. In particular, show that for any $\epsilon \in (0, v_N)$ there exists a Nash equilibrium at which the revenue to the auctioneer is ϵ. [5 pts]

   Let b_i be the bid of bidder i. Then $b_1 = v_1$, $b_i = \epsilon$, $i \neq 1$ is a Nash equilibrium. Bidder 1 wins the item and pays ϵ and no other bidder can strictly increase his payoff by changing his bid.

2. Consider a search engine with discount factor θ and an advertisement with the following characteristics.

   - When the advertiser pays x per impression (but nothing if the advertisement is clicked), then the Gittins' index is g_1.
   - When the advertiser pays y per click (but nothing per impression), then the Gittins' index is g_2.

   Now assume that the advertiser pays x per impression and additionally pays y if the advertisement is clicked. Is the Gittins' index $g_1 + g_2$? [8 pts]

   Yes. Note that $g_1 = x/(1-\theta)$, since the per impression profits are deterministic. Let $V_2(\alpha, \beta, I)$ be the value function when the advertiser only pays per click. Then $V_2(\alpha, \beta, g_2) = g_2$. Let $V_3(\alpha, \beta, I)$ be the value function when the advertiser pays both per impression and per click and let g_3 be the corresponding index. Then $V_3(\alpha, \beta, g_3) = g_3$. We observe that $V_3(\alpha, \beta) = V_2(\alpha, \beta) + g_1$, since the value that the payments per impression are deterministic. Thus, $g_3 = g_1 + g_2$.

3. Consider a graph with five nodes and two directed 3-cycles. Node A has an edge to node B, B has an edge to C, and C has an edge to A. Node A has an edge to node D, D has an edge to E, and E has an edge to A. Compute the naive PageRank of each node. [8 pts]

   Let π_i be the PageRank of node i. By symmetry, nodes B, C, D and E will have the same PageRank, i.e. $\pi_B = \pi_C = \pi_D = \pi_E$. Also $\pi_A = \pi_C + \pi_E$. Thus, $\pi_A = 1/3$, $\pi_B = \pi_C = \pi_D = \pi_E = 1/6$.

4. Is the following statement true of false? Both search and display online advertisement have been steadily growing at a fast pace. [2/-1 pts]

   It's false. The display online advertisement has slightly decreased. This was pointed out by a panelist.

5. Suppose $m_i(t) = (t/i)^{1.5}$. Which of the following is true? [2/-1 pts]

(i) The median is infinite and the expected product rank is infinite.

(ii) The median is bounded and the expected product rank is infinite.

(iii) The median is bounded and the expected product rank is bounded.

(iv) The median is infinite and the expected product rank is bounded.

(ii)