

Problem Set 1, Due January 31, 2008

Each pair of students only needs to submit one set of answers.

1. In this problem, we will assume that three advertisers have made bids on the same keyword in a search engine. The search engine (which acts as the auctioneer) assigns  $(\alpha, \beta)$  priors to each advertiser, and uses their Gittins' index to compute the winner. The advertisers are:
  - Advertiser (a) has  $\alpha = 2$ ,  $\beta = 5$ , has bid \$1 per click and no has budget constraint.
  - Advertiser (b) has  $\alpha = 1$ ,  $\beta = 4$ , pays \$0.2 per impression and additionally \$1 if his ad is clicked. He has no budget constraint.
  - Advertiser (c) has  $\alpha = 1$ ,  $\beta = 2$ , has bid \$1.5 per click and his ad can only be shown 5 times (including this one).

There is a single slot, the discount factor is  $\theta = 0.95$  and a first price auction is used. Which ad would the auctioneer allocate the slot to? Briefly comment on what a second price auction could charge to the winner.

2. Consider a first price auction for a single good, where there are  $n$  bidders. Assume that bidder  $i$  has valuation  $v_i$  and bid  $b_i$ . Bids are said to be in Nash equilibrium if no bidder can immediately benefit by changing his bid. Give an example where bidding the true values is not a Nash equilibrium. Argue that there exists a set of bids in the first price auction where the bids are in Nash equilibrium and the auctioneer generates the same revenue as would be generated if everyone bids truthfully in a second price auction. This phenomenon is known as revenue equivalence. Assume that bidding is done in increments of cents; assume that no two valuations are the same; also ignore the difference of 1 cent when comparing the revenue of different auctions.
3. Consider three merchants A, B, and C having a valuation-per-click of 50, 48, and 10 cents respectively bidding for two slots on a keyword. Assume that each merchant has a CTR of 0.2 in position 1 and a CTR of 0.15 in position 2. Compute the total revenue earned by the next price auction if each merchant bids truthfully. Identify the merchant which can immediately profit by changing her bid. Identify two different Nash equilibria in this setting. What does this imply for the stability of the system?
4. Consider an online stock broker and a soft drink company. Which of the two would find it more beneficial to buy online banner advertisements as opposed to a TV advertisement? Why?
5. Write a 500 word essay on current state and trends in social network monetization and any insights about future directions.