MS&E 235: Internet Commerce

Problem Set 3. Due February 28 by 5:00 pm.

Each pair of students only needs to submit one set of answers.

1. Consider an information network which is trying to predict an event whose intrinsic probability is q. Suppose that the only shares in the system are YES shares. If the event happens, everyone who holds a share gets paid a Dollar. Suppose there exists a risk-neutral insider who knows this probability q.

   (a) (5pts) Let us define the clearing price to be a price such that the insider will neither buy nor sell shares at this price. Find the clearing price of the above market.

   (b) (15pts) Now imagine that this market is used to predict an event whose probability changes with popular perception. If the price of the YES share is p, the probability of the event changes to $(p+q)/2$. Find the clearing price. Use this to comment on the resilience of information markets.

2. (20 pts) In this problem we consider a personalized version of HITS for product recommendation. Given a set of products and a set of consumers, we set $L_{uv} = 1$ if consumer u has bought product v and $L_{uv} = 0$ otherwise. Let h_u be the hub score for consumer u and a_v be the authority score for product v. Note that we do not associate hub scores with products or authority scores with consumers. For $\epsilon \in [0, 1]$, the personalized HITS equations for consumer i are:

$$h_u = (1 - \epsilon) \sum_v L_{uv} a_v + b_u,$$

   where $b_u = \epsilon$ if $u = i$ and 0 otherwise; and

$$a_v = \sum_u L_{uv} h_u.$$

   We have the following data for consumers $\{c_1, c_2, c_3, c_4, c_5, c_6\}$ and products $\{p_1, p_2, p_3, p_4, p_5, p_6\}$:

   - Consumer c_1 has bought products p_1 and p_2.
   - Consumer c_2 has bought products p_1, p_2 and p_3.
   - Consumer c_3 has bought products p_5 and p_6.
   - Consumer c_4 has bought products p_4 and p_6.
   - Consumer c_5 has bought products p_3, p_5 and p_6.
   - Consumer c_6 has bought product p_6.

   Compute product recommendations for consumer c_1 for $\epsilon = 0.1$ and $\epsilon = 0.6$. In particular, compute the hub scores of consumers and the authority scores of products. Which product would be recommended to c_1 in each case? Explain.

   Hint: Use matlab, Excel or any other program to compute the solution iteratively. Remember to normalize after each step.

3. (10 pts) In class it was shown that by iteratively computing successive values of $\pi^{(i)}$ according to $\pi^{(i)} = (1-\epsilon)\pi^{(i-1)}P + b$ we converge to the true PageRank. In particular, it was shown that the error $e^{(i)} \equiv \sum_v |\Delta_v^{(i)}|$ converges to zero. Compute an upper bound on the number of steps required so that $e^{(i)} \leq \delta$ for some $\delta > 0$.

4. (10 pts) We have seen in class how PageRank can be gamed and now consider how HITS can be gamed. Suppose HITS is being used by a collaborative recommendation web-site. Products play the role of authorities, and users of the system play the role of hubs. There is a link from a user to a product if the user recommends that product. One of the users is actually a pseudonym for the manufacturer of one of the products. If this user is allowed to place exactly k product recommendations, intuitively explain which products he should recommend in order to increase the authority score of his product. Assume that he can not post more than one recommendation for a particular product.

5. (10pts) Suppose every real individual were to have a unique reviewer id which is used across all the reputation systems on the Internet, i.e. while rating books on Amazon, hotels on tripadvisor, corporations on rapleaf, click-based ranking on search engines, products on epinions, etc. Which major problem associated with online reputation systems will this solve? Is this going to automatically make all these reputation systems robust? Describe one subsidiary business that might emerge that would help merchants/publishers/hotels game the system.

6. (10 pts) eBay has recently announced upcoming changes in its reputation system: http://pages.ebay.com/services/forum/new.html. One of the changes is that it will ban sellers from leaving negative comments about buyers. Briefly discuss the implications of this change.