Credit Networks

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Based on
https://arxiv.org/abs/1007.0515
http://dl.acm.org/citation.cfm?id=2722264

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- Decentralized payment infrastructure introduced by [DeFigueiredo, Barr, 2005] and [Ghosh et. al., 2007]
- Do not need banks, common currency
- Models trust in networked interactions
- A robust "reputation system" for transaction oriented social networks

- Barter: If I need a goat from you, I had better have the blanket that you are looking for. Low liquidity.
- Centralized banks: Issue currencies, which are essentially IOUs from the bank. Very high liquidity; allows strangers to trade freely.
- Credit Networks: Bilateral exchange of IOUs among friends.

Illustration: Centralized Currency

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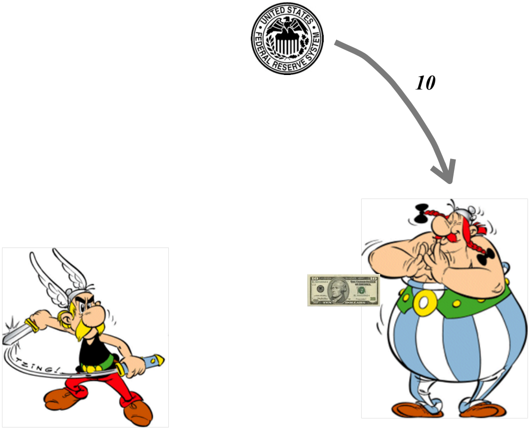
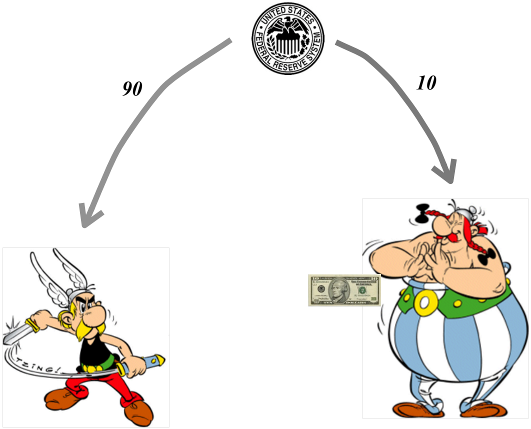


Illustration: Centralized Currency



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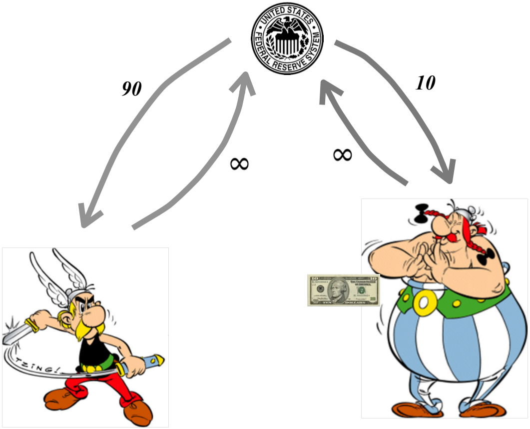

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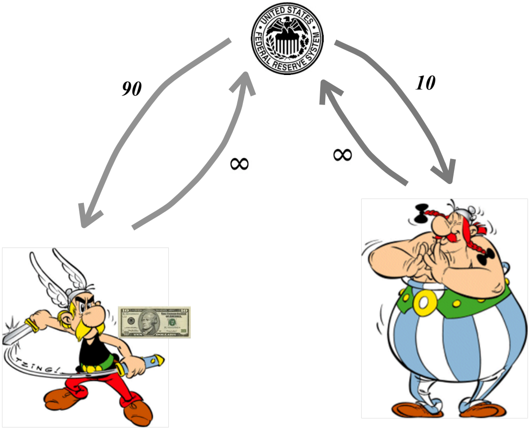

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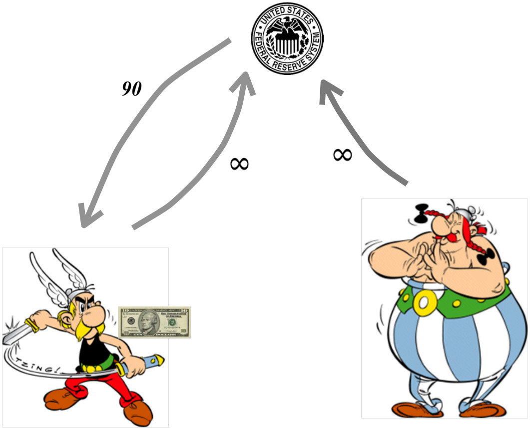

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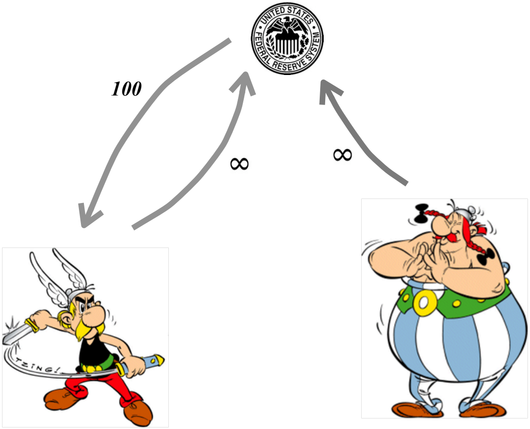

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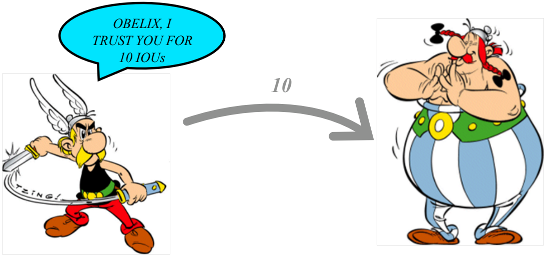

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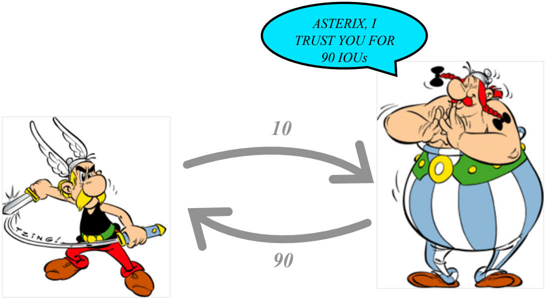

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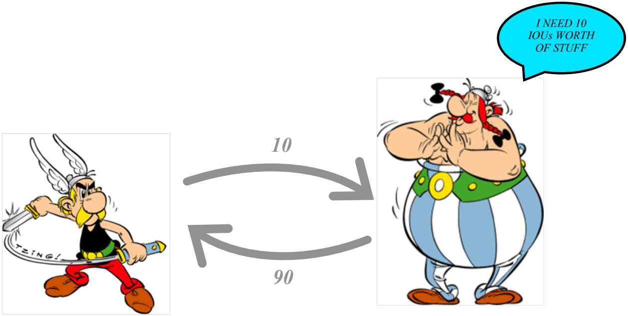

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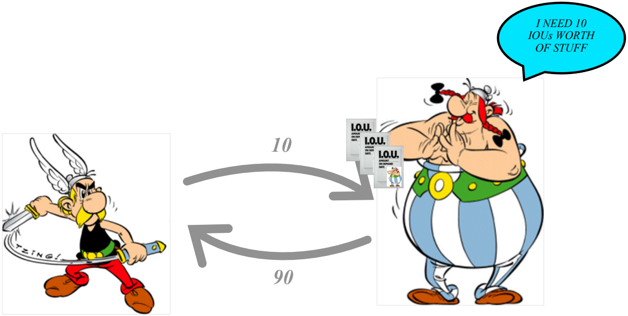

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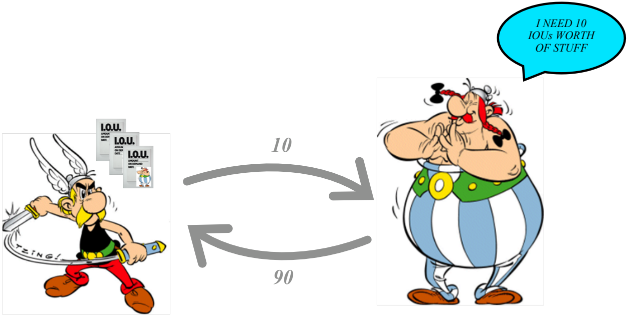
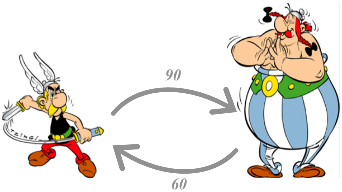
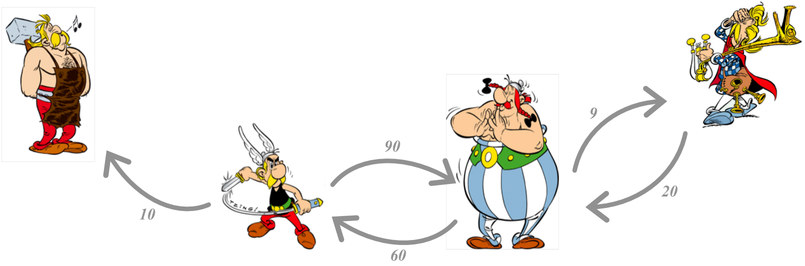


Illustration: Credit Networks

New Trust Values…



TZING!
90
60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10
9
20

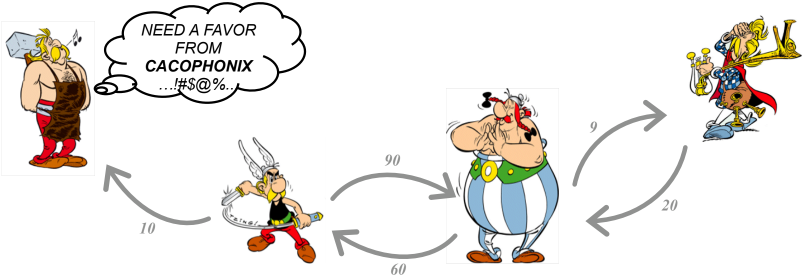
NEED A FAVOR FROM CACOPHONIX ...!#$@%..
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Illustration: Credit Networks

Interaction at a Distance

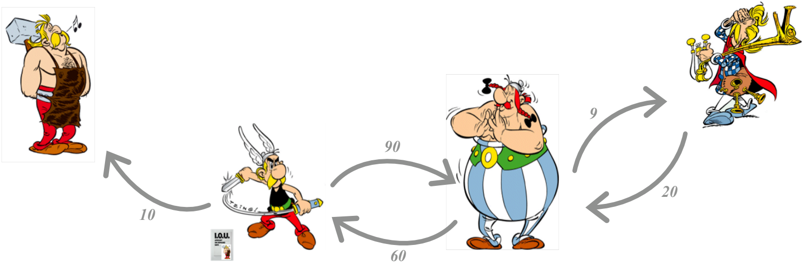

Illustration: Credit Networks

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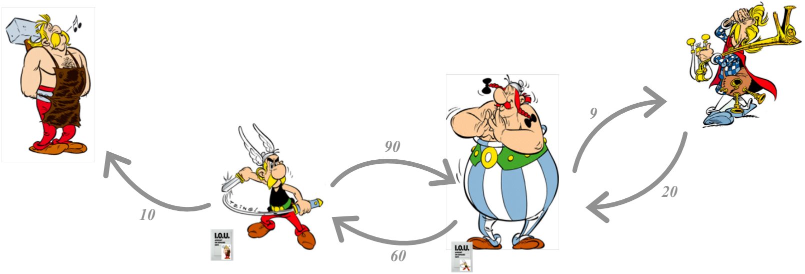

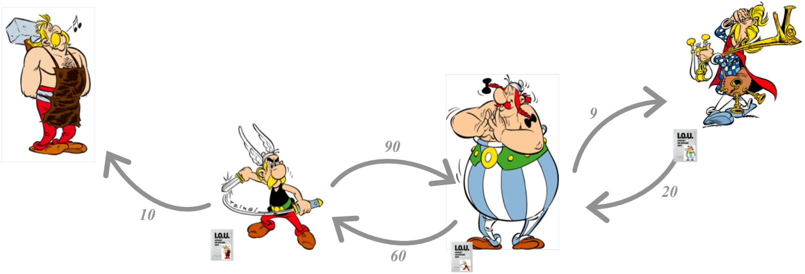

10
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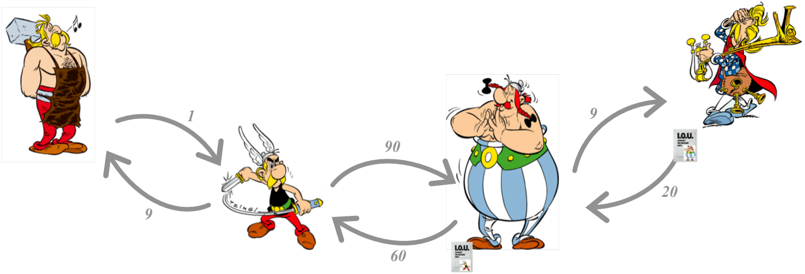
I.O.U.
I.O.U.
I.O.U.



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I.O.U.
I.O.U.

Illustration: Credit Networks

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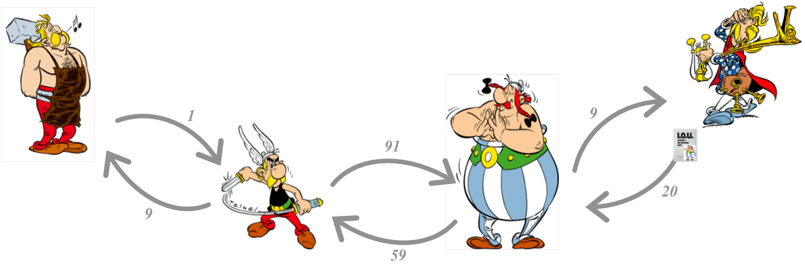

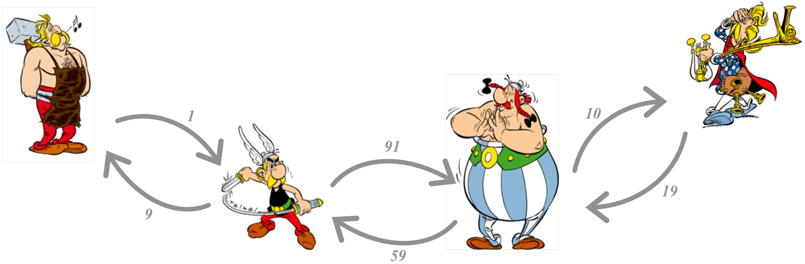

1
9
91
59
10
19

- Graph $G(V, E)$ represents a network (social network, p2p network, etc.)
- **Nodes:** (non-rational) agents/players; print their own currency
- **Edges:** credit limits $c_{uv} > 0$ extended by nodes to each other[1]
- Payments made by passing IOUs along a chain of trust. Same as augmentation of *single-commodity* flow along a path from payee to payer
- Credit gets replenished when payments are made in the other direction

Robustness: Every node is vulnerable to default only from its own neighbors, and only for the amount it directly trusts them for.

[1] assume all currency exchange ratios to be unity

What is a Credit Network?

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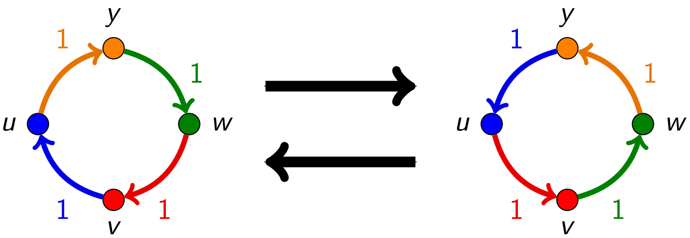
- Barter/Exchange economies like P2P networks.
- Insuring transactions in P2P commerce sites such as Craigs list
- Combating social spam (Facebook, LinkedIn)
- Email spam
- Question and Answer systems; Recommender systems

- Liquidity: Can credit networks sustain transactions for a long time, or does every node quickly get isolated?
- Network Formation: How do rational agents decide how much trust to assign to each other?

- Edges have integer capacity $c > 0$ (summing up both directions)
- Transaction rate matrix $\Lambda = \{\lambda_{uv} : u, v \in V, \lambda_{uu} = 0\}$
- Repeated transactions; at each time step choose (s, t) with prob. λ_{st}
- Try to route a unit payment from s to t via the shortest feasible path; **update edge capacities** along the path
- Transaction fails if no path exists



Definition

Let $\mathcal{S}$ and $\mathcal{S}'$ be two states of the network. We say that $\mathcal{S}'$ is **cycle-reachable** *from $\mathcal{S}$ if the network can be transformed from state $\mathcal{S}$ to state $\mathcal{S}'$ by routing a sequence of payments along feasible cycles (i.e. from a node to itself along a feasible path).*

Theorem

Let $(s_1, t_1), (s_2, t_2), \ldots, (s_T, t_T)$ be the set of transactions of value $v_1, v_2, \ldots, v_T$ respectively that succeed when the payment is routed along the shortest feasible path from s_i to t_i. Then the same set of transactions succeed when the payment is routed along any feasible path from s_i to t_i.

Proof Sketch.

Sending a unit of flow along two different paths from the same source to the same destination leads to two states that are cycle-reachable.

Cycle-reachability partitions all possible states of the credit network into equivalence classes.

Theorem

If the transaction rates are symmetric, then the network has a uniform steady-state distribution over all reachable equivalence classes.

Consequence: Yields a complete characterization of success probabilities in trees, cycles, or complete graphs; estimate for Erdös-Rényi graphs

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Assume capacity c. Then we have $c+1$ states; each in a different equivalence class.

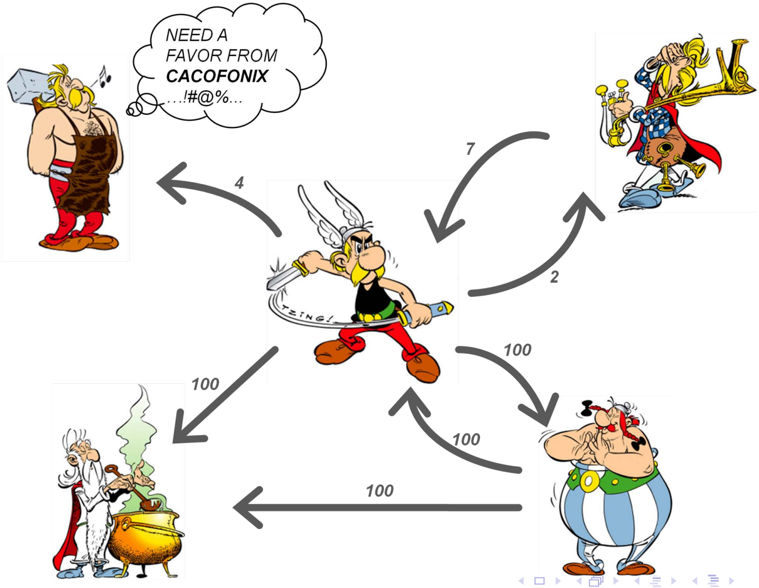
Success probability for a transaction is $c/(c+1)$.

No cycles. Hence, all states are equally likely.

Let $c_1, c_2, \ldots, c_L$ be the capacities along the path from s to t in the tree. Then, success probability is

$$\prod_{i=1}^{L} c_i/(c_i+1).$$

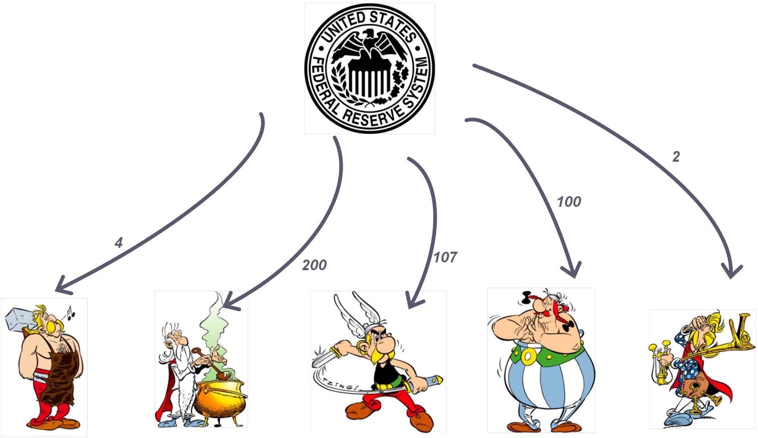
Assume capacity $c = 1$ on each edge, and the Markov chain is ergodic. Let d_v denote the degree of node v. Then the stationary probability that v is bankrupt is at most $1/(1 + d_v)$.

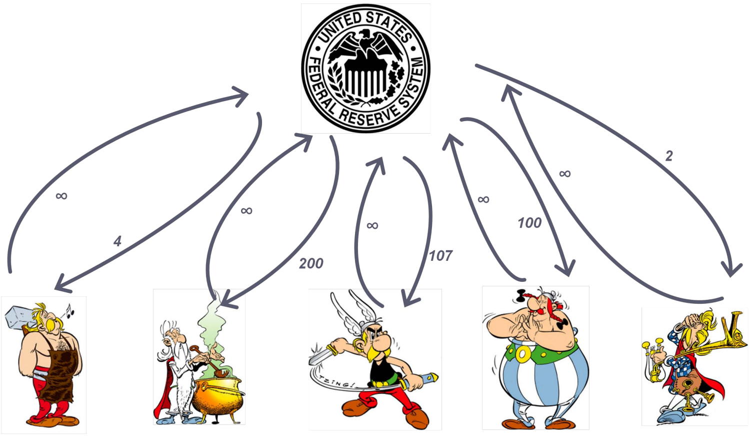


NEED A
FAVOR FROM
CACOFONIX
...!#@%...
7
4
2
TZING!
100
100
100
100

Analysis

Centralized Payment Infrastructure





UNITED STATES
FEDERAL RESERVE SYSTEM
∞
4
∞
200
∞
107
∞
100
∞
2

Convert Credit Network → Centralized Model

$$\forall u, c_{ru} = \sum_v c_{vu}$$

$\implies$ Total credit in the system is conserved during conversion

Slight variant of the liquidity analysis gives steady state distribution and success probabilities.

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Bankruptcy probability

Graph class	Credit Network	Centralized System
General graphs	$\leq 1/(d_v + 1)$	$\approx 1/d_{AVG}$

Transaction failure probability

Graph class	Credit Network	Centralized System
Star-network	$\Theta(1/c)$	$\Theta(1/c)$
Complete Graph	$\Theta(1/nc)$	$\Theta(1/nc)$
$G_c(n, p)$	$\Theta(1/npc)$	$\Theta(1/npc)$

Table: Steady-state Failure Probability in Credit Network v/s Centralized System

Summary: Credit networks have liquidity which is almost the same as that in centralized currency systems.

Dimitri B. DeFigueiredo and Earl T. Barr
Trustdavis: A non-exploitable online reputation system, CEC 2005

Arpita Ghosh, Mohammad Mahdian, Daniel M. Reeves, David M. Pennock, and Ryan Fugger
Mechanism design on trust networks, WINE 2007.