Credit Networks

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Credit Network

- Decentralized payment infrastructure introduced by [DeFigueiredo, Barr, 2005] and [Ghosh et. al., 2007]
- Do not need banks, common currency
- Models trust in networked interactions
- A robust “reputation system” for transaction oriented social networks
Barter and Currency

- Barter: If I need a goat from you, I had better have the blanket that you are looking for. Low liquidity.
- Centralized banks: Issue currencies, which are essentially IOUs from the bank. Very high liquidity; allows strangers to trade freely.
- Credit Networks: Bilateral exchange of IOUs among friends.
I NEED $10 WORTH OF STUFF
ILLUSTRATION: CENTRALIZED CURRENCY
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Diagram showing the flow of currency from a central bank to two figures, with amounts of 90 and 10.
ILLUSTRATION: CENTRALIZED CURRENCY
ILLUSTRATION: CENTRALIZED CURRENCY
OBELIX, I TRUST YOU FOR 10 IOUs
ILLUSTRATION: CREDIT NETWORKS

ASTERIX, I TRUST YOU FOR 90 IOUs
Illustration: Credit Networks

I NEED 10 IOUs WORTH OF STUFF
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New Trust Values...
ILLUSTRATION: CREDIT NETWORKS
INTERACTION AT A DISTANCE
ILLUSTRATION: CREDIT NETWORKS
INTERACTION AT A DISTANCE
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INTERACTION AT A DISTANCE

NEED A FAVOR
FROM
CACOPHONIX
...!#$@%...
ILLUSTRATION: CREDIT NETWORKS
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INTERACTION AT A DISTANCE
What is a Credit Network?

- Graph $G(V, E)$ represents a network (social network, p2p network, etc.)
- **Nodes:** (non-rational) agents/players; print their own currency
- **Edges:** credit limits $c_{uv} > 0$ extended by nodes to each other\(^1\)
- Payments made by passing IOUs along a chain of trust. Same as augmentation of *single-commodity* flow along a path from payee to payer
- Credit gets replenished when payments are made in the other direction

**Robustness:** Every node is vulnerable to default only from its own neighbors, and only for the amount it directly trusts them for.

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\(^1\) assume all currency exchange ratios to be unity
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Applications

- Insuring transactions in P2P commerce sites such as Craigs list
- Combating social spam (Facebook, LinkedIn)
- Email spam
- Question and Answer systems; Recommender systems
Research Questions

- Liquidity: Can credit networks sustain transactions for a long time, or does every node quickly get isolated?
- Network Formation: How do rational agents decide how much trust to assign to each other?
• Edges have integer capacity $c > 0$ (summing up both directions)

• Transaction rate matrix $\Lambda = \{\lambda_{uv} : u, v \in V, \lambda_{uu} = 0\}$

• Repeated transactions; at each time step choose $(s, t)$ with prob. $\lambda_{st}$

• Try to route a unit payment from $s$ to $t$ via the shortest feasible path; **update edge capacities** along the path

• Transaction fails if no path exists
**Definition**

Let $S$ and $S'$ be two states of the network. We say that $S'$ is **cycle-reachable** from $S$ if the network can be transformed from state $S$ to state $S'$ by routing a sequence of payments along feasible cycles (i.e. from a node to itself along a feasible path).
**Theorem**

Let \((s_1, t_1), (s_2, t_2), \ldots, (s_T, t_T)\) be the set of transactions of value \(v_1, v_2, \ldots, v_T\) respectively that succeed when the payment is routed along the shortest feasible path from \(s_i\) to \(t_i\). Then the same set of transactions succeed when the payment is routed along any feasible path from \(s_i\) to \(t_i\).

**Proof Sketch.**

Sending a unit of flow along two different paths from the same source to the same destination leads to two states that are cycle-reachable.
Cycle-reachability partitions all possible states of the credit network into equivalence classes.

**Theorem**

*If the transaction rates are symmetric, then the network has a uniform steady-state distribution over all reachable equivalence classes.*

Consequence: Yields a complete characterization of success probabilities in trees, cycles, or complete graphs; estimate for Erdös-Rényi graphs
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Assume capacity $c$. Then we have $c + 1$ states; each in a different equivalence class.

Success probability for a transaction is $c/(c + 1)$. 
No cycles. Hence, all states are equally likely.

Let $c_1, c_2, \ldots, c_L$ be the capacities along the path from $s$ to $t$ in the tree. Then, success probability is

$$\prod_{i=1}^{L} \frac{c_i}{(c_i + 1)}.$$
Assume capacity $c = 1$ on each edge, and the Markov chain is ergodic. Let $d_v$ denote the degree of node $v$. Then the stationary probability that $v$ is bankrupt is at most $1/(1 + d_v)$. 
NEED A FAVOR FROM CACOFONIX...!#@%...
Analysis
Centralized Payment Infrastructure
**Convert Credit Network → Centralized Model**

\[ \forall u, c_{ru} = \sum_v c_{vu} \]

⇒ Total credit in the system is conserved during conversion

Slight variant of the liquidity analysis gives steady state distribution and success probabilities.
Convert Credit Network → Centralized Model

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Convert Credit Network $\rightarrow$ Centralized Model

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Slight variant of the liquidity analysis gives steady state
distribution and success probabilities.
**Bankruptcy probability**

<table>
<thead>
<tr>
<th>Graph class</th>
<th>Credit Network</th>
<th>Centralized System</th>
</tr>
</thead>
<tbody>
<tr>
<td>General graphs</td>
<td>$\leq 1/(d_v + 1)$</td>
<td>$\approx 1/d_{AVG}$</td>
</tr>
</tbody>
</table>

**Transaction failure probability**

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</thead>
<tbody>
<tr>
<td>Star-network</td>
<td>$\Theta(1/c)$</td>
<td>$\Theta(1/c)$</td>
</tr>
<tr>
<td>Complete Graph</td>
<td>$\Theta(1/nc)$</td>
<td>$\Theta(1/nc)$</td>
</tr>
<tr>
<td>$G_c(n, p)$</td>
<td>$\Theta(1npc)$</td>
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</table>

**Table:** Steady-state Failure Probability in Credit Network v/s Centralized System

**Summary:** Credit networks have liquidity which is almost the same as that in centralized currency systems.
Dimitri B. DeFigueiredo and Earl T. Barr
Trustdavis: A non-exploitable online reputation system, CEC 2005

Arpita Ghosh, Mohammad Mahdian, Daniel M. Reeves, David M. Pennock, and Ryan Fugger
Mechanism design on trust networks, WINE 2007.