

# Advertising Prices in Equilibrium: Theory and Evidence

Matthew Gentzkow, *Stanford University and NBER*\*

Jesse M. Shapiro, *Brown University and NBER*

Frank Yang, *Stanford University*

Ali Yurukoglu, *Stanford University and NBER*

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## Abstract

Existing theories of media competition imply that advertisers will pay a lower price in equilibrium to reach consumers who multi-home. We generalize and extend this theoretical result and test it using data from television and social media advertising. We find that television outlets whose viewers watch more television charge a lower price per viewer to advertisers. This finding helps rationalize well-known stylized facts such as a premium for younger and more male audiences on television. Also consistent with the theory, we show that social media advertising markets feature a premium for older audiences. A quantitative version of our model whose only free parameter is a scale normalization can explain 44 percent of the variation in price per impression across owners of television networks.

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# 1 Introduction

Both digital and traditional media receive substantial revenues from selling consumer attention to advertisers (e.g., Statista 2021). Prices per unit of attention in these markets vary widely. On television, prices per impression can easily vary across programs or networks by a factor of three or more (e.g., Crupi 2009). Prices for online advertising exhibit similarly large variation (e.g., AdStage 2020). Which consumers' attention commands the highest prices is a key determinant of the incentive to produce content (Spence and Owen 1977, Wilbur 2008, Veiga and Weyl 2016). Pricing in advertising markets has become an important issue in antitrust policy (e.g., Competition and Markets Authority 2019).

Industry observers have long been puzzled by the large variation in the price of attention across different groups of consumers. Perhaps the most famous example is the premium paid to advertise on television programs with younger audiences. The premium attached to younger audiences—who are sometimes known as the “coveted” or “target” demographic (Dee 2002, Pomerantz 2006)—is widely regarded as a major influence on content and scheduling (Dee 2002, Surowiecki 2002, Einstein 2004, Pomerantz 2006, Goettler 2012, Gabler 2014), and persists despite the fact that older audiences tend to have greater purchasing power than younger audiences (Dee 2002, Surowiecki 2002, Pomerantz 2006, Gabler 2014).<sup>1</sup> Other documented price premia include a premium for advertising to men relative to women (Papazian 2009) and (on a per-impression basis) for advertising on programs with larger relative to smaller audiences (Chwe 1998, Phillips and Young 2012, Goettler 2012).

In this paper, we develop an equilibrium model of an advertising market with competing outlets. The model implies that the price that an outlet charges for its advertisements in equilibrium is decreasing in the activity level of the outlet's audience, i.e., in the extent to which members of its audience visit other outlets. We show that the model's predictions are borne out in data from the US television market, and can help explain well-known and potentially puzzling patterns such as a premium for younger, more male, and (on a per-impression basis) larger audiences. The predictions of the model are also in line with less-well-known facts that we document for social media advertising. A quantitative version of the model whose only free parameter is a scale normaliza-

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<sup>1</sup>Gabler (2014 pp. 3-4) writes that “We live in a culture of the young, for the young and by the young, and anyone over 49—the demographic breakpoint of old age for most television advertisers—is tossed onto the trash heap of history, all eighty million of them. In effect, these people, just under one-third of the American population, have been steadily disenfranchised by a ruthless, self-serving, myopic and ignorant dictator. That dictator is the eighteen to forty-nine demographic cohort, and it is the single most important factor in determining what we see, hear and read.”

tion can explain 44.4 percent of the variation in price per impression across owners of television networks.

Our model builds on a large theoretical literature on two-sided markets beginning with Rochet and Tirole (2003) and Anderson and Coate (2005), and extends Anderson et al.'s (2016) model of advertising pricing in markets with multi-homing. In the model, each of a set of owners may own multiple outlets, and each outlet may have multiple advertising slots. Owners simultaneously announce prices to advertise on the slots they own, after which a set of advertisers decide which slots to purchase. Advertisers have homogeneous value functions that are submodular in the set of outlets on which they advertise. The number of slots on each outlet exceeds the number of advertisers, so slots are not rationed in equilibrium, and because advertisers are homogeneous, equilibrium is efficient. In particular, equilibrium follows the incremental pricing principle of Anderson et al. (2016): the price an owner commands for its slots is determined by the difference in an advertiser's value from advertising on all outlets versus all outlets except those controlled by the given owner.

An important special case of a submodular value function arises when advertisers face diminishing returns from multiple impressions to a given viewer, and viewers multi-home in a pattern that is invariant to advertisers' choices. In this case, the incremental value of an outlet's advertising slots is determined by the overlap of its audience with those of other outlets. As a result, the price per viewer that an outlet can charge in equilibrium is decreasing in the overall activity level of its audience, and increasing in the overall size of its audience. In the extreme case where advertisers value only the first impression to a given viewer, each outlet's price per impression is determined solely by the fraction of its audience that is exclusive to that outlet.

We study the model's predictions empirically using data on television audience and advertising prices from Nielsen's Ad Intel database, and audience survey data from GfK MRI. Consistent with the predictions of the model, we show that outlets whose audiences watch more television charge a lower price per impression for their ads. Turning to the demographic patterns that have received significant attention in the industry, we find that the younger, more male audiences that command a price premium are also those that watch the least television. We also find, consistent with prior evidence and with the predictions of the model, that outlets with larger audiences command higher prices per impression, even after accounting for the viewing intensity of their audiences.

We also study social media advertising, using data on prices of Facebook advertisements collected as part of a series of experiments including our own. A key difference between social media

and television is that age patterns in activity levels are reversed: the young spend relatively little time watching television but they are the heaviest users of social media. Provided that television and social media advertisements are imperfect substitutes, this situation creates a sharp test of our model. If the forces we identify are a primary driver of advertising prices, older users should be the “coveted demographic” on social media. If the premium for young television viewers is instead driven by other forces—for example, a greater inherent value of young viewers due to their brand preferences being more malleable (Surowiecki 2002)—the pattern could be the opposite. We show that it is indeed more expensive to reach older than to reach younger users on Facebook.

Finally, we study the model’s predictions quantitatively. We consider a specification in which a given viewer’s probability of seeing an ad on a given outlet is proportional to the time that the viewer spends on the outlet. Based on this specification we use the audience survey data to calculate the incremental value of advertising on each outlet, which in turn yields a prediction for the equilibrium price charged by each owner for its advertising slots. We find that the model’s predictions are a good fit to observed prices. Across either outlets or owners, the slope of the line of best fit relating observed to predicted prices is statistically indistinguishable from one. Predicted prices explain 44.4 percent of the variation in price per impression across owners, and exhibit the same qualitative patterns as observed prices with respect to age, gender, and outlet size. This is true despite the fact that the only free parameter we fit to observed advertising prices is a scale normalization that does not affect our calculations of goodness-of-fit. We conclude the quantitative analysis by applying the model to predict the price effects of several recent content mergers. These predicted effects vary widely in ways that would be difficult to predict using standard concentration measures such as the Herfindahl-Hirschman Index (HHI).

The primary contribution of this paper is to show that the predictions of a model of a competitive advertising market with multi-homing are a good match, both qualitatively and quantitatively, to existing and novel facts about important real-world markets. In contrast to many prior studies of advertising markets (e.g., Kaiser and Wright 2006, Bel and Domènech 2009, Wilbur 2008, Fan 2013, Chandra and Kaiser 2014, Jeziorski 2014, Berry et al. 2016, Zubanov 2020), our quantitative model explicitly derives the price of advertising on a given outlet from a microfounded equilibrium model with a multi-homing audience.<sup>2</sup> Multi-homing is essential to the model’s implications.

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<sup>2</sup>Gentzkow et al. (2014) incorporate a microfounded model of advertising with multi-homing consumers into a structural model of newspapers’ choice of political affiliation, but allow for only a small number of outlets, and do not study the cross-sectional variation in advertising prices implied by their model. Prat and Valletti (forthcoming, Section 5) simulate effects of platform mergers under various assumptions about overlap in their audience, though using

In contrast to prior work that incorporates audience demographics into a model of advertiser demand (e.g., Wilbur 2008), our model can explain demographic premia in advertising prices without assuming that advertisers intrinsically value certain demographic characteristics.

Our analysis provides a unified explanation of several facts, some of which are new to the literature. There is a folk wisdom in television advertising that it is more expensive to advertise to groups that are harder to reach (Surowiecki 2002, Papazian 2009, Gabler 2014).<sup>3</sup> Some have questioned the logic of this proposition.<sup>4</sup> We provide what is to our knowledge the first systematic evidence on the relationship between an outlet’s advertising prices and the activity levels of its audience, and the first depiction of this relationship grounded in a quantitative economic model. We also systematically document advertising premia related to audience age, gender and size. In the case of social media, while some industry sources report a premium for older audiences on social media (e.g., Ampush 2014), we are not aware of prior evidence in the academic literature showing that transaction prices in the US are greater for Facebook ads targeted to older users.<sup>5</sup>

The paper also makes a contribution to the theoretical literature on advertising in two-sided markets with multi-homing. In particular, we generalize the incremental pricing result in Anderson et al. (2016) to allow for arbitrary submodular value and ownership structure. Unlike Ambrus et al. (2016) and Anderson and Peitz (2020), we do not model the determination of the number of advertising slots. Unlike Athey et al. (2018), we do not allow heterogeneity among advertisers in our baseline analysis, though in an extension we show that incremental pricing holds when the extent of heterogeneity is small or when owners can charge advertiser-specific prices. Unlike Prat and Valletti (forthcoming), we do not focus on the effects of the ad market on competition among advertisers, though we do allow for some interactions among advertisers in an extension. As in Anderson et al. (2016), our model allows for a very rich description of viewers’ choices of which outlets to watch, a feature we take advantage of when developing the model’s quantitative implications.

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a microfoundation different from ours.

<sup>3</sup>Papazian (2009) writes that, “As a rule, shows that pull higher proportions of easy-to-get heavy tube watchers come in at lower [cost per thousand impressions] than those that rely less on this preponderantly lowbrow segment and more on upscale audiences” (p. 134).

<sup>4</sup>Surowiecki (2002) writes that, “by this logic, advertisers ought to pay top dollar to reach shepherders in Uzbekistan.”

<sup>5</sup>Lambrecht and Tucker’s (2019, Table 7) analysis of average suggested bids for a STEM career information campaign on the Facebook platform across 191 countries indicates that average suggested bids are higher for ads targeted to females. The analysis does not show clear differences in average suggested bids by the age of the target users (columns 1 and 2) but does show evidence of interactions between age and gender (column 3). Our analysis differs in using transaction price data from campaigns in the US rather than suggested bid data from a campaign across 191 countries.

The remainder of the paper proceeds as follows. Section 2 presents our model and its implications. Section 3 describes our data and variable definitions. Section 4 presents our key findings about the determinants of advertising prices on television and social media. Section 5 presents our quantitative implementation of the model and discusses its implications. Section 6 concludes.

## 2 Model

There is a set of outlets  $\mathcal{J}$ . A given owner can own multiple outlets, and we define a partition  $\mathcal{Z}$  on the set of outlets that describes the ownership structure, using the notation  $Z \in \mathcal{Z}$  to refer both to a cell of the partition and to the owner of the outlets in that cell. Each outlet has available  $K$  advertising slots, each of which can be sold to one of the  $N$  advertisers in the set  $\mathcal{N}$ . We assume that  $N \leq K$ , i.e., that advertising slots are not scarce. Section 2.2 includes an extension with  $N > K$ . We let  $\mathcal{P}(\cdot)$  denote the power set operator.

The game proceeds as follows. Each owner  $Z$  simultaneously announces, for each bundle  $B \in \mathcal{P}(Z)$  of its outlets, a price  $p_B$  at which it will sell one slot on each outlet  $j \in B$  to any advertiser, with  $p_B = \infty$  denoting that a given bundle  $B$  is unavailable. Advertisers then move sequentially in random order and decide which, if any, bundles to buy. When all advertisers have moved, ads are shown and the game ends.

The payoff of an owner is given by the sum of the prices  $p_B$  of all bundles  $B$  that the owner sells. The payoff of an advertiser that buys slots in a set of bundles  $\mathcal{S} \subseteq \mathcal{P}(\mathcal{J})$  is given by

$$V(\{j : j \in B \in \mathcal{S}\}) - \sum_{B \in \mathcal{S}} p_B$$

where  $V(\cdot)$  is a non-negative value function that is monotonic in set-inclusion order.<sup>6</sup> We capture the idea that there are diminishing returns to advertising by assuming that  $V(\cdot)$  is submodular: an advertiser derives less incremental value from an outlet when adding it to a larger bundle.<sup>7</sup>

The following examples exhibit such a value function.

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<sup>6</sup>That is,

$$\mathcal{J}' \subseteq \mathcal{J}'' \subseteq \mathcal{J} \implies V(\mathcal{J}') \leq V(\mathcal{J}'').$$

<sup>7</sup>Formally,

$$\mathcal{J}' \subseteq \mathcal{J}'' \subseteq \mathcal{J}, j \in \mathcal{J} \setminus \mathcal{J}'' \implies V(\mathcal{J}' \cup \{j\}) - V(\mathcal{J}') \geq V(\mathcal{J}'' \cup \{j\}) - V(\mathcal{J}'').$$

**Example 1.** There is a set of viewers. Each viewer sees any ad slot on outlet  $j$  with some probability. For each viewer that views its ad  $M \in \mathbb{N}$  times, each advertiser gets value  $a \sum_{m=0}^M \beta_m$  where  $a > 0$ ,  $\beta_0 = 0$ ,  $\beta_1 > 0$ , and  $\beta_m \geq 0$  is non-increasing in  $m$  for  $m \geq 1$ . The following settings are nested in this one:

(a) (Anderson et al. 2016.) There is a set of viewers, each of which views all ad slots on a subset of the outlets.

(b) (Awareness advertising with forgetting.) For each viewer that remembers seeing its ad, each advertiser gets value  $a > 0$ . Each viewer remembers each ad they have seen with some probability, independently across ads.

**Example 2.** There is a set of viewers, each of which views all ads on a subset of the outlets. There is a partition  $\mathcal{C}$  that groups the outlets  $\mathcal{J}$  into categories. For each viewer that views its ad on an outlet in category  $C \in \mathcal{C}$ , each advertiser gets value  $a_C > 0$ .

**Example 3.** Each outlet consists of  $K$  programs. Each program has one ad slot. Each advertiser that purchases an ad slot on a given outlet is randomly assigned to the slot in one of the outlet's programs. There is a set of viewers. Each viewer views a subset of programs. Whether a given viewer views a given program depends on whether that program carries an ad, but not on whether other programs do. For each viewer that views its ad on  $M \in \mathbb{N}$  distinct outlets, each advertiser gets value  $u(M) \geq 0$  where  $u(\cdot)$  is nondecreasing and exhibits decreasing differences.

*Remark 1.* In Examples 1 and 2, viewers' choices of which outlets to view are not affected by the outcome of the game. We may interpret this either as a scenario in which viewers do not care about advertising or, following Anderson et al. (2016), as a scenario in which viewers make viewing decisions without knowing the outcome of the game. In Example 3, viewers' choices of which outlets to view are affected by the outcome of the game. We may interpret this as a scenario in which viewers care about advertising and make viewing decisions knowing the outcome of the advertising game.

*Remark 2.* Content owners such as television networks sometimes charge fees to viewers, either directly via "over the top" subscriptions or indirectly via bundlers like cable networks. Our model and analysis are compatible with the presence of such fees provided they are invariant to the outcome of the advertising game. This would be true if, for example, fees to viewers are set prior to the advertising game, or prior to viewers' knowledge of its outcome.

Our main result is that each owner is able to extract the incremental value of the outlets it controls. To state this result, for a bundle  $B \subseteq \mathcal{J}$ , let the incremental value  $v_B$  be given by

$$v_B = V(\mathcal{J}) - V(\mathcal{J} \setminus B),$$

i.e., the value to an advertiser of advertising on all outlets rather than all outlets except those in  $B$ . We assume that every outlet in  $\mathcal{J}$  has positive incremental value,  $v_j > 0$  for all  $j \in \mathcal{J}$ . We take an equilibrium to be a subgame perfect equilibrium in pure strategies.

**Theorem 1.** (Incremental pricing) *There exists an equilibrium. In any equilibrium, for each owner  $Z \in \mathcal{Z}$ , the minimum price to buy slots on all of the outlets owned by  $Z$  is given by  $p_Z^* = v_Z$ , and all advertisers buy slots on all outlets.*

The following proposition shows that submodularity of  $V(\cdot)$  is in a sense necessary for the conclusion of Theorem 1.

**Proposition 1.** *If  $V(\cdot)$  is strictly monotonic but not submodular, then there exists a partition  $\mathcal{Z}'$  of some nontrivial  $\mathcal{J}' \subseteq \mathcal{J}$  such that, in the game with outlets  $\mathcal{J}'$  and ownership structure  $\mathcal{Z}'$ , there is no equilibrium in which  $p_{Z'}^* = v_{Z'}$  for all  $Z' \in \mathcal{Z}'$ .*

All proofs are given in Appendix A.

## 2.1 Comparative Statics

Consider a special case of Example 1 in which every owner owns a single outlet  $j \in \mathcal{J}$ , and diminishing returns are strict in the sense that  $\beta_2 < \beta_1$ . Suppose that there is a unit mass of viewers subdivided into a set  $G$  of mutually exclusive demographic groups, with group  $g$  having mass  $\mu_g$ , so that  $\sum_{g \in G} \mu_g = 1$ . Members of group  $g \in G$  see ads on outlet  $j$  with probability  $\eta_{gj} \in (0, 1)$ , independently across outlets. Let

$$\lambda_j = \sum_{g \in G} \mu_g \eta_{gj}, \quad \sigma_{gj} = \frac{\mu_g \eta_{gj}}{\lambda_j}$$

denote, respectively, the total mass of outlet  $j$ 's audience, and the share of this audience that comes from group  $g$ . Then  $p_j^*/\lambda_j$  is the price per viewer charged by the owner for an ad slot.

Applying Theorem 1 with the structure of the value function  $V(\cdot)$  in this case, we show two comparative statics results. The first result is that, all else equal, an outlet commands a larger price



premium for its viewers if its viewers come from less active groups.

**Proposition 2.** *Suppose that group  $g \in G$  is less active than group  $h \in G$  in the sense that  $\eta_{gj} \leq \eta_{hj}$  for all  $j \in \mathcal{J}$ . Suppose that outlet  $j \in \mathcal{J}$  draws a larger share of its audience from group  $g$  and a smaller share of its audience from group  $h$  than outlet  $k \in \mathcal{J}$ , in the sense that  $\sigma_{gj} \geq \sigma_{gk}$  and  $\sigma_{hj} \leq \sigma_{hk}$ , and that the two outlets have equal total audience sizes,  $\lambda_j = \lambda_k$ , and equal shares of audience from groups other than  $g$  and  $h$ ,  $\sigma_{g'j} = \sigma_{g'k}$  for all  $g' \neq g, h$ . Then outlet  $j$  has a higher equilibrium price per viewer than outlet  $k$ ,  $p_j^*/\lambda_j \geq p_k^*/\lambda_k$ .*

The inequality in the conclusion of Proposition 2 is strict if  $\eta_{gj'} < \eta_{hj'}$  for some  $j' \neq j, k$  and  $\sigma_{gj} > \sigma_{gk}$ .

Intuitively, Proposition 2 holds because, given diminishing returns, the incremental value of showing an ad to a viewer who watches more outlets is lower than the incremental value of showing an ad to a viewer who watches fewer outlets. Since more active viewers tend to watch more outlets, this force puts competitive pressure on the prices that outlets can charge to show ads to these viewers.

We next show that, all else equal, an outlet commands a larger price premium for its viewers if the outlet attracts a larger share of the total audience.

**Proposition 3.** *Suppose that outlet  $j$  has a larger audience than outlet  $k$  in the sense that for some  $\delta \geq 1$ ,  $\eta_{gj} = \delta \eta_{gk}$  for all  $g \in G$ . Then outlet  $j$  has a higher price per viewer than outlet  $k$ ,  $p_j^*/\lambda_j \geq p_k^*/\lambda_k$ .*

The inequality in the conclusion of Proposition 3 is strict if  $\delta > 1$ . Intuitively, Proposition 3 holds because viewers of the larger outlet tend to watch fewer other outlets, leading to less competitive pressure on the price that the larger outlet can charge to show ads to its viewers.

Appendix A.3 shows that statements analogous to Propositions 2 and 3 hold for multi-outlet owners when diminishing returns are perfect, i.e.  $\beta_m = 0$  for  $m \geq 2$ .

## 2.2 Extensions

*Unbundled pricing.* It is useful to be able to characterize the price of an owner's individual outlets. To do this we can imagine that some owners are not allowed to bundle slots on all of their outlets together. Formally, each owner  $Z$  is endowed with a partition  $\mathcal{F}_Z$  of  $Z$  such that they are only allowed to bundle outlets in the same cell of the partition. Denote by  $v_B^S = V(S) - V(S \setminus B)$  the

incremental value of bundle  $B$  in  $S \subseteq \mathcal{J}$ . We refine the notion of equilibrium by assuming that, when indifferent, owners break ties in favor of offering fewer bundles, and each advertiser breaks ties by favoring owners according to a prespecified ordering.

**Proposition 4.** *In any equilibrium satisfying the tie-breaking rule, each bundle sold has a price of  $p_B^* = v_B^S$ , where  $S \subseteq \mathcal{J}$  is the set of all outlets sold.*

In general, existence of an equilibrium is not guaranteed. Appendix A.3 establishes the existence of an equilibrium in a special case in which the comparative statics of Proposition 3 hold.

*Heterogeneous advertisers.* Suppose now that each advertiser  $n \in \mathcal{N}$  has a monotone and submodular value function  $V_n(\cdot)$ . If outlets can post advertiser-specific prices, then the result is parallel to that in Theorem 1, in the sense that the equilibrium price of owner  $Z$ 's bundle to advertiser  $n$  is given by  $v_{n,Z} = V_n(\mathcal{J}) - V_n(\mathcal{J} \setminus Z)$ . If outlets cannot post advertiser-specific prices, then incremental pricing holds if heterogeneity among the advertisers is sufficiently small compared to the incremental value of a single outlet. Let  $v_Z = \min_{n \in \mathcal{N}} v_{n,Z}$  and  $\bar{v}_Z = \max_{n \in \mathcal{N}} v_{n,Z}$  denote the minimum and maximum values of  $v_{n,Z}$ , respectively, with respect to  $n$ . Let  $\varphi(Z) = \min_{n \in \mathcal{N}, j \in Z} V_n((\mathcal{J} \setminus Z) \cup \{j\}) - V_n(\mathcal{J} \setminus Z)$  denote the minimal incremental value of any one of owner  $Z$ 's outlets. In the special case where  $Z$  is a single-outlet owner,  $\varphi(Z) = v_Z$ .

**Proposition 5.** *Suppose that heterogeneity in the value functions  $V_n(\cdot)$  is small in the sense that  $\bar{v}_Z - v_Z \leq \frac{1}{N} \varphi(Z)$  for all  $Z \in \mathcal{Z}$ . Then there exists an efficient equilibrium, and in any efficient equilibrium,  $p_Z^* = v_Z$  for all  $Z \in \mathcal{Z}$ , and all advertisers buy slots on all outlets.*

It is immediate that, in the setting of section 2.1, any efficient equilibrium obeys the comparative statics in Propositions 2 and 3.

The hypothesis of Proposition 5 restricts the incremental values rather than the level of  $V_n(\cdot)$ , in the sense that it allows for  $V_n(\cdot) = V(\cdot) + c_n$  for any  $c_n$  that preserves non-negativity.<sup>8</sup> The restriction on incremental values becomes more demanding as the number of advertisers,  $N$ , grows large.

*Rationing of ad slots.* Now suppose that we may have  $N > K$  and assume that bundle prices can only take on values in the set  $\{0, \Delta, 2\Delta, \dots\}$  where  $\Delta > 0$  is some fixed increment.

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<sup>8</sup>When there are two or more owners, it also allows for  $V_n(\emptyset) = V(\emptyset)$  and  $V_n(\mathcal{J}') = V(\mathcal{J}') + c_n$  for  $\emptyset \neq \mathcal{J}' \subseteq \mathcal{J}$ , where  $c_n \geq 0$ .

**Proposition 6.** *There exists a subgame perfect equilibrium, possibly in mixed strategies, and in any subgame perfect equilibrium each owner  $Z$  earns an expected revenue per slot between  $(v_Z - \Delta)/|Z|$  and  $\sum_{j \in Z} V(\{j\})/|Z|$ .*

Allowing for mixed strategies helps to guarantee existence of an equilibrium in this setting.

*Competition between advertisers.* Appendix A.3 provides additional conditions under which a modified incremental value pricing equilibrium exists when an advertiser’s value for advertising depends not only on the slots they purchase but also on those purchased by other advertisers.

### 3 Data

#### 3.1 Television Advertising Prices, Audience, and Ownership

We obtain our raw data on broadcast and cable television viewership and advertisement pricing in 2015 from Nielsen’s Ad Intel product (The Nielsen Company 2019). For each advertisement the data includes the telecast (e.g., NBC Nightly News, June 1), program (e.g., NBC nightly news), daypart (e.g., prime time), and network (e.g., NBC). It also includes the duration (e.g., 30 seconds) of the advertising spot, and estimates of its number of impressions (live viewers) and cost. We omit from all calculations any advertisements with zero cost or duration. We standardize cost to a 30-second-spot basis by dividing by the duration of the advertisement (in seconds) and multiplying by 30.

Advertising cost estimates in the AdIntel data are based on information obtained at the month-network-daypart level for cable television and at the month-program level for broadcast television (The Nielsen Company 2017). For consistency we therefore define our notion of an outlet  $j$  to be a network-daypart.<sup>9</sup> Appendix Figure 1 reports results when using network as our notion of an outlet, and also (for broadcast television) when using program.

For each outlet, we calculate total impressions across all advertisements and divide by the number of hours in the corresponding daypart in a 52-week year to get a measure of total impressions per hour, which we take as analogous to the concept  $\lambda_j$  defined in Section 2.1. For each outlet, we also calculate the total (standardized) cost of all advertisements and divide by the number of hours in the corresponding daypart in a 52-week year to get a measure of total cost per hour, which we take as analogous to the concept  $p_j^*$  defined in Section 2.1. Finally, for each outlet we divide

<sup>9</sup>In cases where a telecast spans multiple dayparts, we assign it to the daypart that contains the largest share of broadcast time.

total cost per hour by total impressions per hour to obtain the average price per impression of a 30-second spot on the outlet, which we take as analogous to the concept  $p_j^*/\lambda_j$  defined in section 2.1.

For each advertisement we also have information on the number of impressions by age (in bins) and gender.<sup>10</sup> From these we compute the share of impressions that are to adults (aged 18 and over) and the share among impressions to adults that are to females. We also compute the average age of adult impressions by imputing each bin to its midpoint value and imputing the oldest bin (65+) to age 75.

For a subset of advertisements representing 99.9 percent of all impressions, we also have information on the distribution of impressions across household income bins.<sup>11</sup> From these we compute the average household income of adult impressions (among those for which we measure income) by imputing each bin to its midpoint value, and imputing the highest-income bin (\$125,000+) to \$175,000.

We obtain from SNL Kagan, a product of S&P Global Market Intelligence, information on the ownership of cable networks in 2015 (S&P Global Market Intelligence 2019). We supplement this with other publicly available information including on the owners of broadcast networks. We form the ownership partition  $\mathcal{Z}$  by assigning each outlet to its majority owner, treating joint ventures as independent ownership groups. We perform analogous calculations to those at the outlet level to compute the price per impression and audience demographics of each owner  $Z \in \mathcal{Z}$ .

### 3.2 Social Media Advertising Prices

We obtain data on the cost of advertising to different audiences on Facebook via an original experiment conducted for this study and a separate advertising campaign conducted for a different study (Allcott et al. 2020a). In both cases advertisements were placed through Facebook’s Ad Manager. In the Facebook advertisement structure, an advertisement set is a group of one or more advertisements with a defined audience target, budget, schedule, bidding, and placement. An advertising

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<sup>10</sup>The age bins are 2-5, 6-8, 9-11, 12-14, 15-17, 18-20, 21-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-64, and 65+ years.

<sup>11</sup>The bins are 0-20, 20-30, 30-40, 40-50, 50-60, 60-75, 75-100, 100-125, and 125+, all in thousands of dollars. For programs representing 96.5 percent of all impressions, we have information on the distribution of impressions across household income bins for each month, from which we compute an annual average for the program. For programs representing 3.4 percent of all impressions, we have information on the distribution of impressions across household income bins for each of a subset of the program’s telecasts, from which we compute an average for the program. We associate each advertisement with the average distribution of impressions for its respective program.

campaign is a group of one or more advertisement sets corresponding to a single campaign objective. All advertisements targeted English speakers in the United States.

For our experiment, we administered an advertising campaign from July 15, 2017 through July 22, 2017 in partnership with GiveDirectly. The campaign consisted of 14 separate advertisement sets targeting each combination of gender and age group in  $\{\text{Men, Women}\} \times \{13-17, 18-24, 25-34, 35-44, 45-54, 55-64, 65+\}$ . Each advertisement set included just one advertisement, with fixed budgets of \$20 a day using automated cost-per-click bidding. For each advertisement set, we obtain the price per impression.

From Allcott et al. (2020b), we obtain data from 32 advertisement sets purchased on September 24, 2018: four each targeting each combination of gender and age group in  $\{\text{Men, Women}\} \times \{18-24, 25-44, 45-64, 65+\}$ . We compute the total cost and total number of impressions for each demographic group, and take the ratio of these to obtain the price per impression.

### 3.3 Audience Survey

From GfK MRI’s 2015 Survey of the American Consumer we obtain, for each of 23978 adult respondents, information on times of day spent watching television in the form of a week-long diary, as well as the implied total weekly television viewing time (GfK Mediamark Research and Intelligence 2017). We compute a measure of total viewing time in each daypart by allocating viewing time in each time slot to AdIntel dayparts in proportion to the share of the time slot that is contained within each daypart. We also obtain measures of viewership of each of 220 broadcast television programs,<sup>12</sup> and time spent watching each of 115 cable television networks in the preceding week. We successfully match 186 broadcast programs and 97 cable television networks to their counterparts in AdIntel.<sup>13</sup>

We use the data on viewership by daypart, broadcast program, and cable network to construct a measure of the time that each respondent viewed each outlet (network-daypart)  $j$ . To do this, we first allocate the viewing time of broadcast programs to their respective network-dayparts, assuming each program was watched for its full duration during the daypart in which its ads most

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<sup>12</sup>The data record the number of times a respondent watches a broadcast program in a typical week (for some broadcast programs) or month (for others). We convert the latter into weekly viewing by allocating monthly viewing time evenly across weeks.

<sup>13</sup>The broadcast programs we match span 6 networks. Some programs (e.g. those on PBS) and some cable television networks (e.g., the Disney Channel, QVC) are excluded from AdIntel because they do not carry standard advertising spots.

commonly occur in AdIntel.<sup>14</sup> If in a given daypart there is viewing time that cannot be attributed to broadcast programs, we allocate that time to the cable networks in proportion to the respondent's reported viewing time of each network.<sup>15</sup>

We thus arrive at a measure of the time each respondent viewed each outlet  $j$ . We compute each respondent's total weekly viewing time by summing over outlets. For each outlet  $j$ , we compute the weighted average log of total weekly viewing time of its viewers, weighting each viewer by her viewing time on outlet  $j$ .<sup>16</sup> We treat average log total weekly viewing time as a measure of the overall activity level of outlet  $j$ 's audience.

We also obtain information for each respondent on the total time spent using the internet in an average week (calculated based on reported time spent on three recent days), and on the share of five social media sites (Facebook, Instagram, Reddit, Twitter, and Youtube) visited in the preceding 30 days. Lastly, we obtain information on each respondent's age (in bins) and gender.<sup>17</sup>

## 4 Evidence on the Determinants of Advertising Prices

Proposition 2 predicts that outlets with a more active audience will command a lower advertising price per impression. Figure 1 shows that this prediction is borne out in the data. Each panel shows a binned scatterplot of the outlet's log(price per impression) against the average log(weekly viewing time) of the outlet's audience. Panel A includes baseline controls including for daypart; Panel B additionally includes controls for log(impressions per hour).

Both panels of Figure 1 show a clear negative relationship between log(price per impression) and average log(weekly viewing time). The magnitude of the relationship is large: in Panel B, for example, moving from the bottom to the top decile of average log(weekly viewing time) corresponds to a decline in log(price per impression) of roughly 180 log points.

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<sup>14</sup>Specifically, we associate each program with the network-daypart that accounts for the largest share of ad duration among the network-dayparts in which the program appears in AdIntel. If the total duration of broadcast programs allocated to a given daypart exceeds the respondent's total viewing time of that daypart, we assume that all viewing during that daypart was to broadcast programs, and we allocate the respondent's viewing time during that daypart to broadcast networks in proportion to the respondent's viewing time of each broadcast network's programs. We assume that each broadcast program viewing has the same duration, and choose that duration so that the ratio of average total broadcast viewing hours and average total cable viewing hours is equal to the one in Nielsen Local TV View (The Nielsen Company, 2021).

<sup>15</sup>If the respondent reports zero viewing time for all cable networks, we instead allocate all viewing time during that daypart to broadcast networks in proportion to the respondent's viewing time of each network.

<sup>16</sup>We exclude from this calculation any respondent with zero total weekly viewing time.

<sup>17</sup>The age bins are 18, 19, 20, 21, 22-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74 and 75+ years.

Proposition 2 also makes predictions about which demographic groups should command a price premium in the advertising market. Appendix Figure 2 shows that older viewers watch more television than younger viewers and that female viewers watch more television than male viewers. The logic of Proposition 2 would lead us to expect that outlets with an older audience would command a lower advertising price than outlets with a younger audience, and likewise for outlets with a more female audience. Figure 2 shows that these predictions are borne out in the data: outlets with older, more female audiences tend to exhibit both lower  $\log(\text{price per impression})$  and higher average  $\log(\text{weekly viewing time})$ . The magnitude of the price differences between outlets with different audience demographics are large.

Proposition 3 predicts that outlets with a larger audience will command a higher advertising price per impression. Figure 3 shows that this prediction is borne out in the data. Panel A shows a binned scatterplot of  $\log(\text{price per impression})$  against  $\log(\text{impressions per hour})$  with baseline controls. Panel B additionally controls for average  $\log(\text{weekly viewing time})$ . Both plots show that a larger audience is associated with a higher price per impression, consistent with the logic of Proposition 3. The association is economically meaningful: in Panel B, for example, moving from the bottom to the top decile of  $\log(\text{impressions per hour})$  corresponds to an increase in  $\log(\text{price per impression})$  of roughly 44 log points.

Table 1 summarizes the patterns in Figures 1 through 3 in a regression, and exhibits sensitivity to controlling for the average income of an outlet's audience.

The logic of Proposition 2 suggests that the price premia for younger audiences in Figure 2 would reverse if older audiences were less active rather than more active media consumers. Comparing the television advertising market to the social media advertising market provides one way to test this hypothesis. Appendix Figure 3 shows that older people spend less time online (Panel A) and visit fewer social media sites (Panel B) than younger people. If television and social media ads are imperfect substitutes, then Proposition 2 would predict that older people command a premium in social media advertising, opposite to the pattern in television advertising. Figure 4 show that this prediction is borne out in our data from social media: ads shown to older audiences command a price premium on social media, both according to data we collected in our own experiment (Panel A) and according to data collected as part of Allcott et al.'s (2020a) study (Panel B). These differences are large, with the oldest group commanding a premium of 122 log points (Panel A) or 57 log points (Panel B) relative to the youngest group, on average across genders.

Appendix Figure 3 shows that measured gender differences in online activity depend on the measure of activity: on average, males report spending more time on the internet (Panel A), but visiting fewer social media sites (Panel B). Correspondingly, Figure 4 does not show a consistent price premium for advertising to male or female audiences on social media.

## 5 Quantification of the Forces in the Model

Consider a special case of Example 1 in which there is a finite collection of viewers, and each advertiser values each viewer reached (regardless of frequency) by some  $a > 0$ . Suppose further that, for a viewer  $i$  who spends time  $T_{ij}$  viewing outlet  $j$ , the probability of seeing an ad placed in one of that outlet's slots is given by  $\eta_{ij} = T_{ij}/T_j$  where  $T_j$  is outlet  $j$ 's total broadcast time. Given an ownership partition  $\mathcal{Z}$  and a value  $a$ , it is then possible to calculate the equilibrium price  $p_Z^* = V(\mathcal{J}) - V(\mathcal{J} \setminus Z)$  implied by Theorem 1, as well as the price per viewer  $p_Z^*/\lambda_Z$  defined analogously to section 2.1.<sup>18</sup>

We perform this calculation in the audience survey data, letting  $T_{ij}$  be the number of hours that respondent  $i$  spent watching outlet  $j$  in the last week. We calculate both predicted prices  $p_j^*/\lambda_j$  at the outlet level, treating outlets as independent owners, and predicted prices  $p_Z^*/\lambda_Z$  at the owner level. We calibrate  $a$  at each level so that the average value of the log(price per viewer) predicted by the model matches the average log(price per impression) observed in the data. The calibration of  $a$  is a scale normalization that does not affect any of the numerical results that we report in this section. Apart from the calibration of  $a$ , we do not use any data on advertising prices in calculating the log price per viewer predicted by the model.

Figure 5 shows that the predicted prices are a good fit to the observed prices. Panel A shows a scatterplot of the observed log(price per impression) against the predicted log(price per viewer) at the outlet level. Panel B shows the analogous scatterplot at the owner level. At the outlet level, predicted prices explain 21.0 percent of the variation in actual prices. At the owner level, predicted prices explain 44.4 percent of the variation. At both levels, the slope of the line of best fit is not statistically distinguishable from 1. At both levels, the model is able to rationalize very large

<sup>18</sup>In particular, results in Appendix A.3 imply that in this case

$$p_Z^* = a \frac{1}{I} \sum_{i=1}^I \eta_{iZ} \prod_{Z' \neq Z} (1 - \eta_{iZ'}), \quad \lambda_Z = \frac{1}{I} \sum_{i=1}^I \eta_{iZ}, \quad \eta_{iZ} = 1 - \prod_{j \in Z} (1 - \eta_{ij}),$$

where  $I$  is the total number of viewers.



differences in advertising prices between outlets or owners.

Table 2 shows the model’s implications for the patterns summarized in Table 1. In particular, Table 2 re-estimates the same regression models as in Table 1, replacing the observed  $\log(\text{price per impression})$  with the predicted  $\log(\text{price per viewer}) \ln(p_j^*/\lambda_j)$ . In general, the model matches the qualitative patterns in the data well, but predicts weaker relationships than those observed in the data. A possible interpretation is that the independent variables in the regression influence advertising prices through channels other than those captured in the model.

We can also consider the model’s implications for the effects of competition. According to the model, changing the ownership partition  $\mathcal{Z}$  from the factual partition to one in which each network has a different owner would reduce total advertising revenues by 5.6 log points. In this sense, in terms of owners’ ability to extract surplus in the advertising market, the factual ownership partition is meaningfully, but not dramatically, different from one in which each network has a different owner.

To further explore the role of competition, we next consider the effects of specific mergers between network owners. In particular, Figure 6 visualizes the implications of each possible pairwise merger among the top 10 owners by audience. For each merger, we calculate the log of the predicted change in revenue, the log of the predicted change in the Hirschman-Herfindahl index (HHI) of audience shares, and the log of the size of the overlapping audience between the two merging owners. To remove the effects of scale, we residualize each of these with respect to the log of the combined pre-merger audience of the merging owners. Panel A plots the log of the predicted change in revenue against the log of the predicted change in HHI. Panel B plots the log of the predicted change in revenue against the log of the overlapping audience. In both panels, we highlight three pairs that merged since 2015: Discovery and Scripps (2018), CBS and Viacom (2019), Disney and Fox (2019).

Panel A of Figure 6 shows that, relative to scale, the Disney-Fox merger is predicted to have had an especially large effect on advertising revenues, and the CBS-Viacom merger to have had an especially small effect. Panel A shows that the difference between these two mergers is not accounted for by differences in their effects on concentration, as measured by the HHI. Rather, as Panel B shows, the difference between these two mergers is accounted for by the fact that Disney and Fox had much greater overlap in their audience, relative to their scale, than did CBS and Viacom.

## 6 Conclusions

We extend existing theoretical results on competitive advertising markets with a multi-homing audience. Our model predicts that the equilibrium price per viewer that an outlet charges for its ads is lower the more active is the outlet's audience. We show that this prediction is borne out in data on television advertising. The prediction can help us understand why there is a premium for younger viewers on television and a premium for older viewers on social media. A disciplined, quantitative implementation of the model rationalizes a meaningful portion of the variation in advertising prices across television outlets and owners, and of the premia associated with specific demographic groups. The model implies that the revenue effect of a merger of network owners is not well-predicted by the effect of the merger on the HHI. Rather, overlap in the owners' audiences plays a central role.

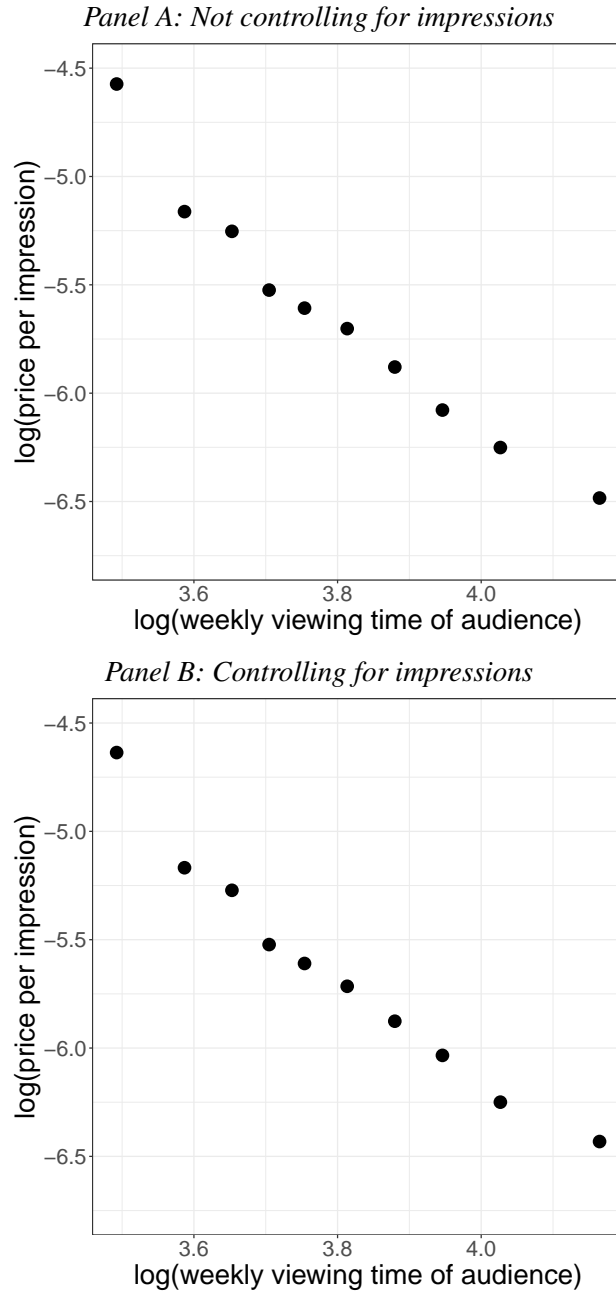
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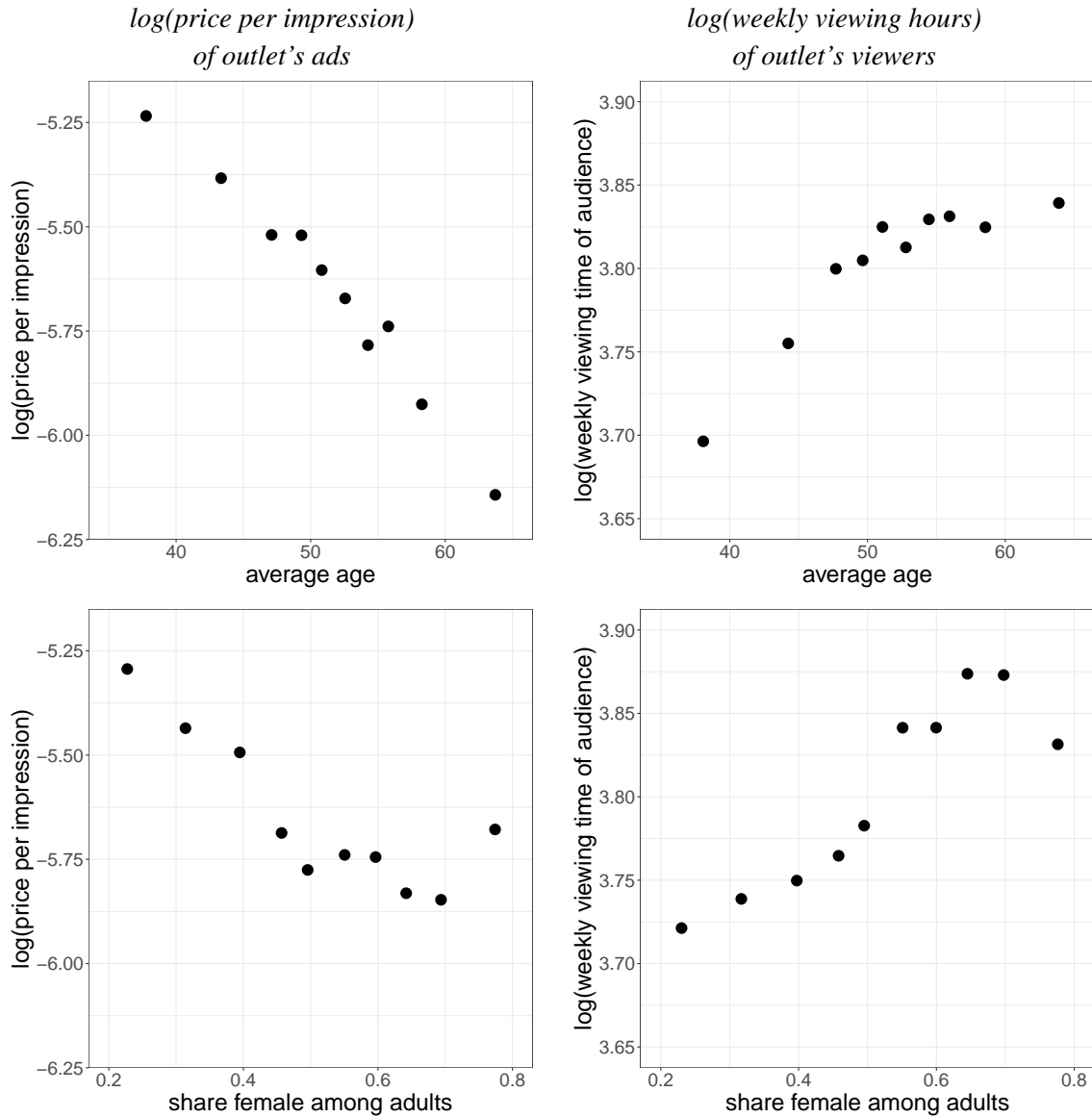
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Figure 1: Advertising prices and audience activity levels of television outlets



Notes: Each plot is a binned scatterplot of a dependent variable against an independent variable of interest. To construct each plot, we regress the dependent variable on indicators for deciles of the independent variable of interest and a set of controls. The unit of analysis in the regression is an outlet. The y-axis values in the plot are the coefficients on the decile indicators, recentered by adding a scalar so that their mean value is equal to the sample mean value of the dependent variable. The x-axis values in the plot are the mean values of the independent variable of interest within the corresponding decile. In both plots, the dependent variable is the log price per impression of a 30-second spot on the outlet, the independent variable of interest is the weighted average log weekly viewing time of the outlet's viewers, and the controls include the share of the outlet's impressions that are to adults, and indicators for the outlet's daypart. In Panel B, the controls additionally include deciles for the log of the outlet's impressions per hour.

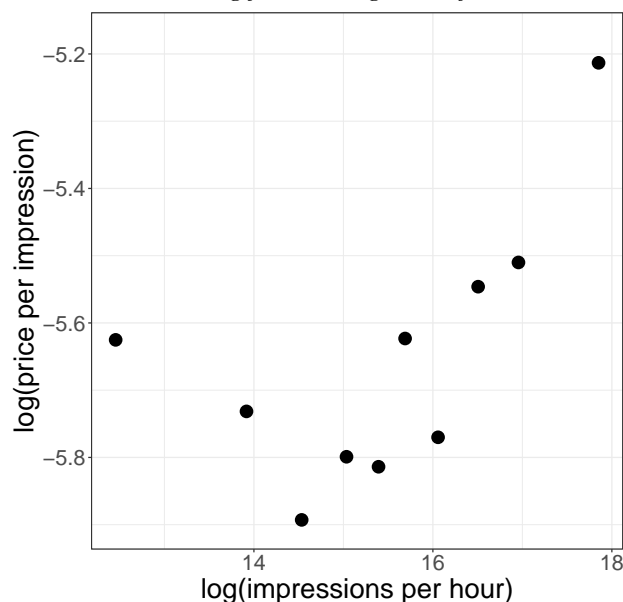
Figure 2: Advertising prices and activity levels by audience demographics of television outlets



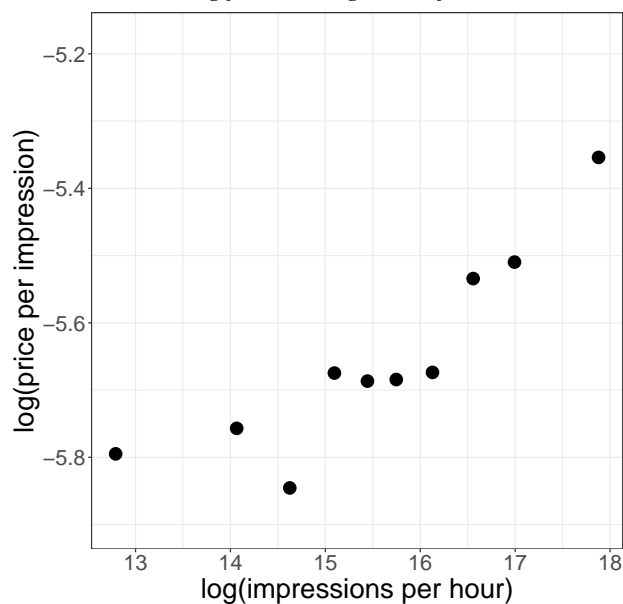
Notes: Each plot is a binned scatterplot of a dependent variable against an independent variable of interest. To construct each plot, we regress the dependent variable on indicators for deciles of the independent variable of interest and a set of controls. The unit of analysis in the regression is an outlet. The y-axis values in the plot are the coefficients on the decile indicators, recentered by adding a scalar so that their mean value is equal to the sample mean value of the dependent variable. The x-axis values in the plot are the mean values of the independent variable of interest within the corresponding decile. In all plots, controls include the share of the outlet’s impressions that are to adults, and indicators for the outlet’s daypart. In the upper row of plots, the independent variable of interest is the average age of the outlet’s adult impressions, and the controls additionally include indicators for deciles of the share of the outlet’s adult impressions that are to females. In the lower row of plots, the independent variable of interest is the share of the outlet’s adult impressions that are to females, and the controls additionally include indicators for deciles of the average age of the outlet’s adult impressions. In the left column of plots, the dependent variable is the log price per impression of a 30-second spot on the outlet. In the right column of plots, the dependent variable is the weighted average log weekly viewing time of the outlet’s viewers.

Figure 3: Advertising prices and audience size of television outlets

Panel A: Not controlling for viewing time of outlet's audience



Panel B: Controlling for viewing time of outlet's audience

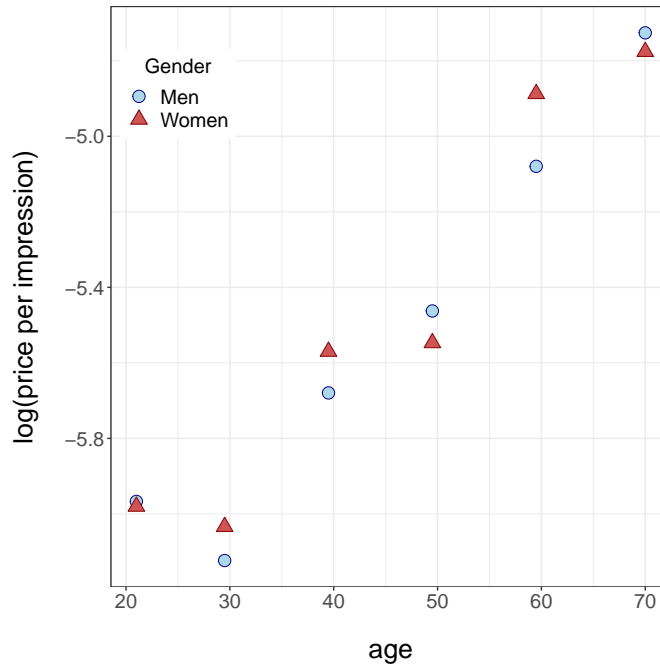


Notes: Each plot is a binned scatterplot of a dependent variable against an independent variable of interest. To construct each plot, we regress the dependent variable on indicators for deciles of the independent variable of interest and a set of controls. The unit of analysis in the regression is an outlet. The y-axis values in the plot are the coefficients on the decile indicators, recentered by adding a scalar so that their mean value is equal to the sample mean value of the dependent variable. The x-axis values in the plot are the mean values of the independent variable of interest within the corresponding decile. In both plots, the dependent variable is the log price per impression of a 30-second spot on the outlet, the independent variable of interest is the log of the impressions per hour of the outlet, and the controls include the share of the outlet's impressions that are to adults, and indicators for the outlet's daypart. In Panel B, the controls additionally include deciles for the weighted average log weekly viewing time of the outlet's viewers.

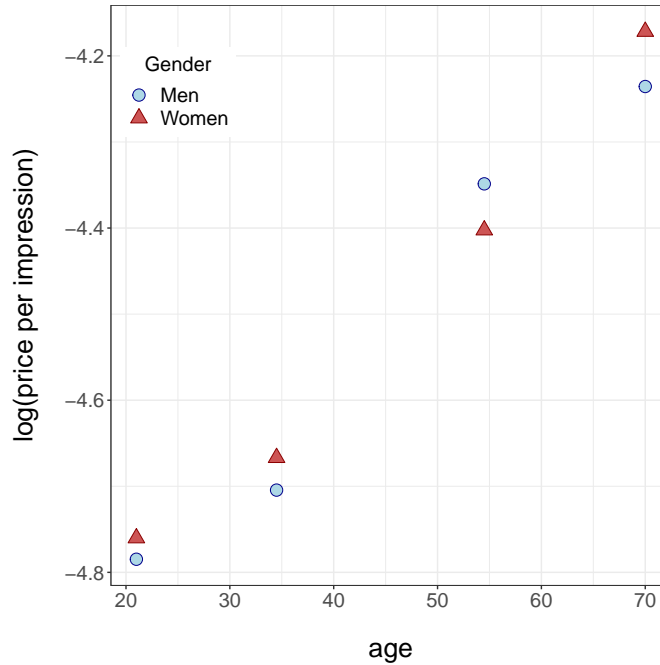


Figure 4: Demographic premia and viewing time on Facebook

Panel A: Data from our experiment

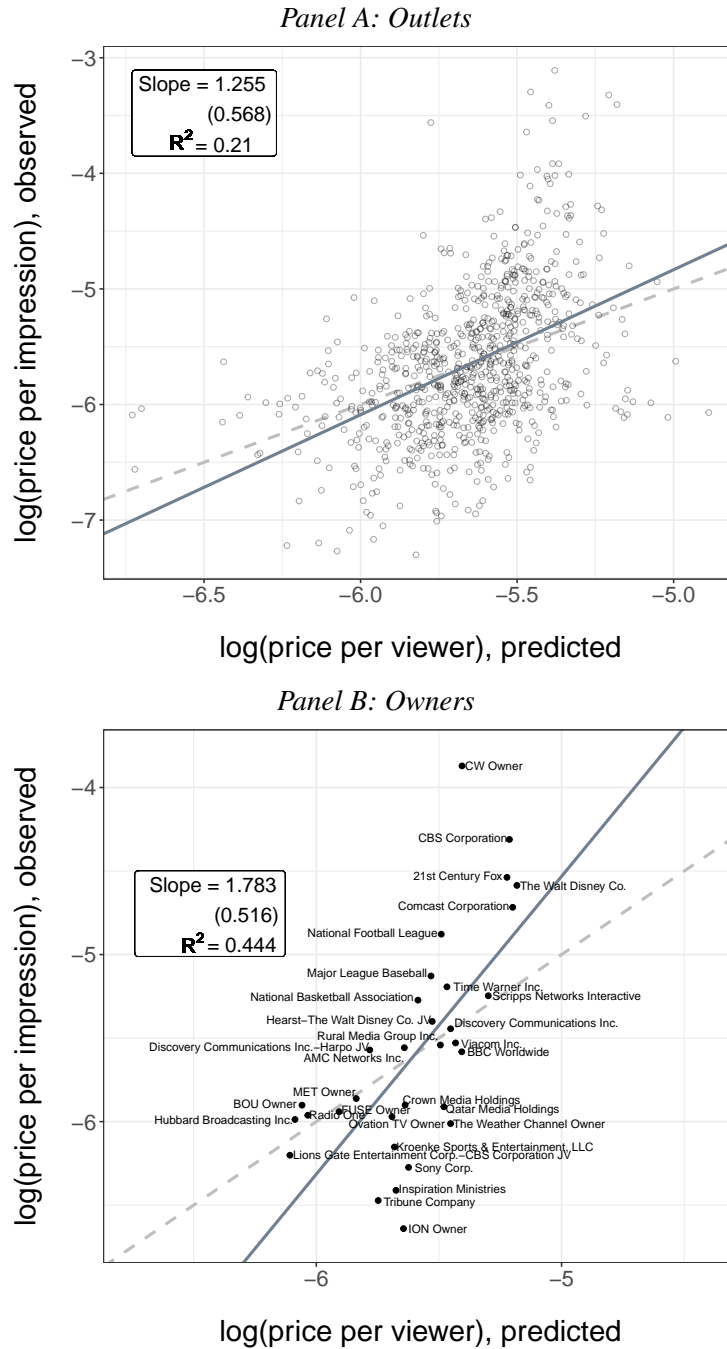


Panel B: Data from Allcott et al. (2020b)



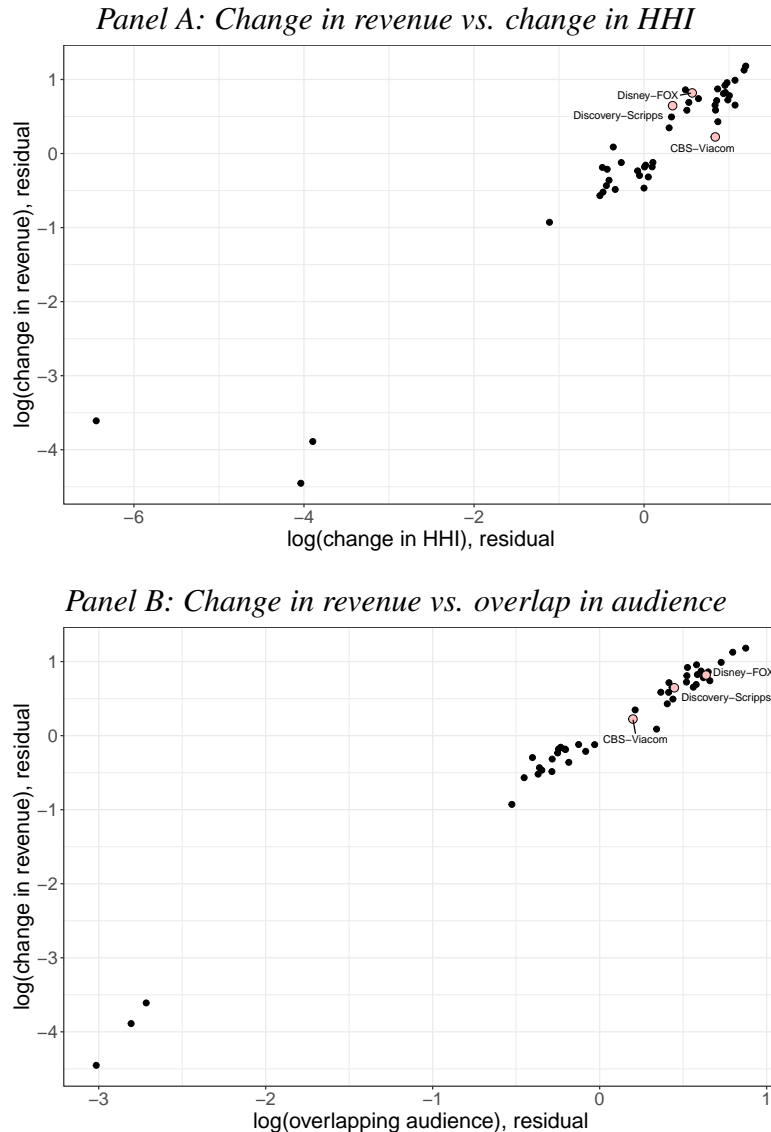
Notes: The plot shows the log(price per impression) for advertisement sets targeted to a given gender and age group. In Panel A, the data are taken from our own experiment, and the groups are  $\{\text{Men, Women}\} \times \{18-24, 25-34, 35-44, 45-54, 55-64, 65+\}$ . In Panel B, the data are taken from Allcott et al. (2020b), and the groups are  $\{\text{Men, Women}\} \times \{18-24, 25-44, 45-64, 65\text{-plus}\}$ . In both panels, the y-axis value is the log(price per impression) for advertisement sets targeting the given group, and the x-axis value is the midpoint of the age range for the given group, treating 70 as the midpoint for ages 65+.

Figure 5: Observed and predicted advertising prices



Notes: Each plot is a scatterplot of the log(price per impression) of a 30-second spot observed in the data (y-axis), as described in Section 3.1, against the log(price per viewer) predicted by the model (x-axis), as described in Section 5. In Panel A, the unit of analysis is an outlet  $j$ , and variables are residualized with respect to the share of the outlet's impressions that are to adults, and indicators for the outlet's daypart. In Panel B, the unit of analysis is an owner  $Z$ , and variables are residualized with respect to the share of the owner's impressions that are to adults. In both panels, residualized variables are recentered by adding a scalar so that the mean value of each recentered variable is equal to the sample mean of the log(price per impression) observed in the data. The dashed line depicts a 45-degree line. The solid line depicts the line of best fit. The box reports the slope of the line of best fit and its standard error, along with the  $R^2$  of the associated linear model. In Panel A, the standard error is clustered by network.

Figure 6: **Simulated effects of mergers on advertising revenue**



Notes: We construct the plots as follows. For each owner we compute the audience size as the probability of an average viewer seeing an ad on at least one of the owner’s outlets, as described in Section 5. We select the top 10 owners by this metric, excluding joint ventures, and form all possible pairs of these 10. For each pair, we compute the overlapping audience, defined as the share of the audience seeing an ad on both owners’ outlets. For each pair we also simulate the effect of a pairwise merger on the pair’s total advertising revenue, using our model of advertising-market equilibrium as described in Section 5. We also simulate the effect of the merger on the Herfindahl-Hirschman Index (HHI), where the HHI is computed with respect to the probability of an average viewer seeing an ad on at least one of the owner’s outlets. For simulated mergers between two owners each of which owns one of {ABC, CBS, FOX, NBC}, we assume that the broadcast networks of the owner with a smaller total audience size become an independent entity post-merger. We include the revenues of this independent entity when calculating the effect of the merger on the pair’s total advertising revenue, and exclude the audience of this independent entity when computing the overlapping audience pre-merger. Panel A plots the log of the simulated change in revenue (y-axis) against the log of the simulated change in HHI (x-axis). Panel B plots the log of the simulated change in revenue (y-axis) against the log of the overlapping audience (x-axis). All plotted variables are residualized with respect to the log of the combined pre-merger audience of the two owners. The highlighted points are those pairs that have merged since 2015.

Table 1: Advertising prices, audience demographics, and audience activity levels of television outlets

	<i>Dependent variable: log(price per impression)</i>				
	(1)	(2)	(3)	(4)	(5)
Average log(weekly viewing hours) of audience	-2.6067 (0.3811)	-2.5275 (0.3831)	-1.9391 (0.3433)		
Average age of impressions				-0.0373 (0.0073)	-0.0309 (0.0077)
Share female among adult impressions				-0.8658 (0.2941)	-0.4692 (0.2622)
log(impressions per hour)		0.0671 (0.0350)	0.0924 (0.0288)		0.0834 (0.0300)
Average household income of impressions (\$1000)			0.0107 (0.0031)		0.0152 (0.0033)
Number of networks	103	103	103	108	108
Number of network-dayparts	810	810	809	850	849

Notes: Each column reports estimates of a linear regression. The unit of analysis is an outlet (network-daypart). The dependent variable is the log price per impression of a 30-second spot on the outlet. Standard errors are clustered by network. All models include controls for the share of the outlet's impressions that are to adults, and indicators for the outlet's daypart. Number of network-dayparts changes across columns because average income is unavailable for one outlet and because some outlets cannot be matched to the audience survey for the purpose of obtaining the average log(weekly viewing hours) of the outlet's audience.

Table 2: **Predicted advertising prices, audience demographics, and audience activity levels of television outlets**

*Dependent variable: log(price per viewer) predicted by model*

	(1)	(2)	(3)	(4)	(5)
Average log(weekly viewing hours) of audience	-1.6823 (0.0518)	-1.6698 (0.0492)	-1.6651 (0.0576)		
Average age of impressions				-0.0071 (0.0028)	-0.0046 (0.0025)
Share female among adult impressions				-0.4425 (0.0837)	-0.2918 (0.0985)
log(impressions per hour)		0.0106 (0.0040)	0.0110 (0.0042)		0.0383 (0.0100)
Average household income of impressions (\$1000)			0.0001 (0.0004)		0.0055 (0.0016)
Number of networks	103	103	103	103	103
Number of network-dayparts	810	810	809	810	809

Notes: Each column reports estimates of a linear regression. The unit of analysis is an outlet (network-daypart). The dependent variable is the log(price per viewer) of the outlet predicted by the model, as described in Section 5. Standard errors are clustered by network. All models include controls for the share of the outlet’s impressions that are to adults, and indicators for the outlet’s daypart. Number of network-dayparts changes across columns because average income is unavailable for one outlet. Number of networks and network-dayparts is smaller than in Table 1 because log(price per viewer) predicted by model is available only for outlets that can be matched to the audience survey.

## A Proofs and Additional Theoretical Results

### A.1 Preliminaries

**Lemma 1.** *Write SPEPS to abbreviate subgame perfect equilibrium in pure strategies.*

- (i) *For any  $V(\cdot)$ , not necessarily monotone or submodular, in any SPEPS all advertisers make the same total payment to any given owner. This holds even if each owner  $Z$  is endowed with an arbitrary partition  $\mathcal{F}_Z$  of  $Z$  such that they are only allowed to bundle outlets in the same cell of the partition.*
- (ii) *If  $V(\cdot)$  is either monotone and submodular or strictly monotone and not necessarily submodular, then in any SPEPS each advertiser buys slots on all outlets.*

*Proof.* Fix any SPEPS. First, observe that because the advertisers are homogeneous, they must have the same equilibrium payoff (say  $W$ ).

For part (i), suppose for contradiction that there exists some owner  $Z \in \mathcal{Z}$  such that not all advertisers make the same total payment to  $Z$ . Let  $n$  be an advertiser who pays the most to owner  $Z$ . Let  $S_c$  be the set of outlets that advertiser  $n$  buys slots on in the cell  $c \in \mathcal{F}_Z$ . Let owner  $Z$  deviate by offering the bundles  $\mathcal{B} := \{S_c : S_c \neq \emptyset, c \in \mathcal{F}_Z\}$  with prices  $\{p_{S_c}^* - \varepsilon : S_c \neq \emptyset, c \in \mathcal{F}_Z\}$  for any  $\varepsilon > 0$ , where  $p_S^*$  denotes the minimum price to buy slots on the outlets in a given set  $S$  in the SPEPS. Note that  $|\mathcal{B}| \geq 1$  since advertiser  $n$  pays positive amount to owner  $Z$ . By buying all the bundles offered in this deviation of  $Z$  (and imitating advertiser  $n$ 's choices in the original SPEPS), any advertiser can obtain a payoff of  $W + \varepsilon|\mathcal{B}|$ . Note that any set of outlets an advertiser wants to buy slots on after this deviation is also a valid choice in the original equilibrium. Therefore, if an advertiser does not buy all the bundles in  $\mathcal{B}$ , then the advertiser gets at most  $W + \varepsilon(|\mathcal{B}| - 1)$ . Hence after this deviation, all advertisers buy the bundles in  $\mathcal{B}$  offered by  $Z$ . But then this is a profitable deviation for owner  $Z$  when  $\varepsilon$  is small enough. Contradiction.

For part (ii), suppose for contradiction that there exist some owner  $Z \in \mathcal{Z}$ , some outlet  $j \in Z$ , and some advertiser  $n$  who does not buy a slot on outlet  $j$ . By part (i), all advertisers pay the same total amount to owner  $Z$  (say  $t$ ). Let  $T \subset Z$  be the set of outlets that advertiser  $n$  buys slots on from owner  $Z$ . Let  $R$  be the set of outlets in  $\mathcal{J} \setminus Z$  that advertiser  $n$  buys slots on. The equilibrium payoff for each advertiser is thus given by

$$W = V(T \cup R) - p_R^* - t.$$

Let owner  $Z$  offer a single bundle  $Z$  with a price  $\tilde{p}_Z = t + \varepsilon$ , for some  $\varepsilon > 0$  (recall that here we assume the owner can bundle anything). If  $V(\cdot)$  is strictly monotone, then we clearly have

$V(Z \cup R) - V(T \cup R) > 0$ . If  $V(\cdot)$  is submodular and monotone, then we also have

$$V(Z \cup R) - V(T \cup R) \geq V(\mathcal{J}) - V(\mathcal{J} \setminus (Z \setminus T)) \geq V(\mathcal{J}) - V(\mathcal{J} \setminus \{j\}) = v_j > 0$$

where the strict inequality is due to our assumption that every outlet has positive incremental value. Therefore, for  $\varepsilon$  small enough, we have

$$V(Z \cup R) - p_R^* - (t + \varepsilon) > V(T \cup R) - p_R^* - t = W.$$

Pick any such  $\varepsilon$ . Every advertiser would buy the bundle  $Z$  at the price  $t + \varepsilon$ , because any strategy not doing so is a feasible strategy in the equilibrium and generates a payoff less than or equal to  $W$ . But this is then a profitable deviation for owner  $Z$ . Contradiction.  $\square$

## A.2 Proofs Omitted From the Main Text

**Example 1.** Let  $i$  denote a viewer uniformly drawn from the set of viewers. Let  $X_S$  denote the random number of outlets in  $S \subseteq \mathcal{J}$  watched by  $i$ . We can write

$$V(S) = a \mathbb{E} \left[ \sum_{m=0}^{X_S} \beta_m \right].$$

To show  $V$  is monotone and submodular, it suffices to fix a realization of viewer  $i$ 's decision, and show the realized value function

$$\tilde{V}(S) := a \sum_{m=0}^{X_S} \beta_m$$

is monotone and submodular, since averaging preserves monotonicity and submodularity. For any  $S \subseteq \mathcal{J}$  and  $j \in \mathcal{J} \setminus S$ , we have

$$\tilde{V}(S \cup \{j\}) - \tilde{V}(S) = a \mathbf{1}_{i \rightarrow j} \beta_{X_S+1}$$

where  $i \rightarrow j$  denotes the event that viewer  $i$  views outlet  $j$ . This shows monotonicity as  $a > 0$  and  $\beta_m \geq 0$  for all  $m$ . For submodularity, fix any  $S \subseteq T \subseteq \mathcal{J}$ , and  $j \in \mathcal{J} \setminus T$ . Since  $S \subseteq T$ , we have  $1 \leq X_S + 1 \leq X_T + 1$ . Since  $\beta_m$  is non-increasing in  $m$  for  $m \geq 1$ , it follows immediately that

$$\tilde{V}(S \cup \{j\}) - \tilde{V}(S) = a \mathbf{1}_{i \rightarrow j} \beta_{X_S+1} \geq a \mathbf{1}_{i \rightarrow j} \beta_{X_T+1} = \tilde{V}(T \cup \{j\}) - \tilde{V}(T),$$

which shows submodularity.

**Example 2.** We can write

$$V(S) = \sum_{C \in \mathcal{C}} V(S \cap C; a_C)$$

where  $V(\cdot; a)$  is the value function given in the proof for Example 1 with  $\beta_m = 0$  for  $m \geq 2$ . Since  $V(\cdot; a)$  is monotone and submodular for any  $a > 0$ , and both monotonicity and submodularity are preserved under restriction and addition, we have that  $V(\cdot)$  is monotone and submodular.

**Example 3.** Let  $\mathcal{K}$  be the set of programs with a generic element denoted by  $k$ , and let  $\mathcal{K}_j \subseteq \mathcal{K}$  be the programs associated with outlet  $j$ . Let  $i$  denote a viewer uniformly drawn from the set of viewers. Let  $i \rightarrow k; A$  denote the event that viewer  $i$  watches program  $k$  and program  $k$  carries an ad. For a set of programs  $\mathcal{K}' \subseteq \mathcal{K}$ , let

$$X_{\mathcal{K}'} = \sum_{k \in \mathcal{K}'} \mathbf{1}_{i \rightarrow k; A}$$

be the number of programs in  $\mathcal{K}'$  watched by  $i$  when each program carries an ad. Let

$$\mathcal{R} = \{ \mathcal{K}' \subseteq \mathcal{K} : |\mathcal{K}' \cap \mathcal{K}_j| = 1 \text{ for all } j \in \mathcal{J} \}$$

consist of sets of representative programs (i.e., one program for each outlet). Let  $\mathcal{K}_S = \cup_{j \in S} \mathcal{K}_j$ . Note that for each advertiser, the value of a set of outlets  $S \subseteq \mathcal{J}$  can be written as

$$V(S) = \mathbb{E} \left[ \frac{1}{|\mathcal{R}|} \sum_{\mathcal{K}' \in \mathcal{R}} u(X_{\mathcal{K}' \cap \mathcal{K}_S}) \right]$$

where the expectation is taken over a fixed probability distribution (regardless of the choice of  $S$ ) which specifies that every program carries an ad. As in Example 1, it suffices to show that for any realization and any  $\mathcal{K}' \in \mathcal{R}$  fixed, we have

$$\tilde{V}(S) := u(X_{\mathcal{K}' \cap \mathcal{K}_S})$$

is monotone and submodular. It is clear that  $\tilde{V}(\cdot)$  is monotone since  $u(\cdot)$  is nondecreasing. For submodularity, note that for any  $S \subseteq T \subseteq \mathcal{J}$  and any  $j \in \mathcal{J} \setminus T$ ,

$$\begin{aligned} \tilde{V}(S \cup \{j\}) - \tilde{V}(S) &= u(X_{\mathcal{K}' \cap \mathcal{K}_{S \cup \{j\}}}) - u(X_{\mathcal{K}' \cap \mathcal{K}_S}) \\ &\geq u(X_{\mathcal{K}' \cap \mathcal{K}_{T \cup \{j\}}}) - u(X_{\mathcal{K}' \cap \mathcal{K}_S} + (X_{\mathcal{K}' \cap \mathcal{K}_{T \cup \{j\}}} - X_{\mathcal{K}' \cap \mathcal{K}_{S \cup \{j\}}})) \\ &= u(X_{\mathcal{K}' \cap \mathcal{K}_{T \cup \{j\}}}) - u(X_{\mathcal{K}' \cap \mathcal{K}_T}) \\ &= \tilde{V}(T \cup \{j\}) - \tilde{V}(T) \end{aligned}$$



where the second line follows from the assumption that  $u(\cdot)$  has decreasing differences.

**Proof of Theorem 1.** We first construct a SPEPS. Let each owner  $Z$  offer a single bundle consisting of all outlets in  $Z$  with a price  $p_Z = v_Z$ . For any profile of posted prices (including off-the-equilibrium-path histories), let every advertiser solve in the second stage the problem,

$$\max_{S \subseteq \mathcal{J}} V(S) - p_S^*,$$

where  $p_S^*$  denotes the minimum price to buy slots on all of the outlets in  $S$  for a given profile of prices  $p$ . The problem may have multiple solutions. Pick a solution  $S^*$  such that  $|S^*|$  is the largest. Let advertisers buy slots on all outlets in  $S^*$ .

It remains to verify that no owner has a profitable deviation. Observe that if  $p_Z = v_Z$  is offered by some owner  $Z$  and there is no proper subset  $W \subset Z$  being offered, then any advertiser will buy the bundle  $Z$  regardless of the prices  $p_{-Z}$  of other owners' bundles. This is because for any  $S \subseteq \mathcal{J} \setminus Z$ , submodularity of  $V(\cdot)$  implies

$$V(S \cup Z) - V(S) \geq V(\mathcal{J}) - V(\mathcal{J} \setminus Z) = v_Z.$$

Fix any owner  $Z$ . Suppose all other players follow the proposed strategy. Fix any advertiser. By the above observation we know that the advertiser would always buy slots on all outlets not in  $Z$ . Then the maximal amount that owner  $Z$  can extract from this advertiser is  $v_Z$  because for any  $S \supseteq \mathcal{J} \setminus Z$ , monotonicity of  $V(\cdot)$  implies

$$V(\mathcal{J}) - V(\mathcal{J} \setminus Z) \geq V(S) - V(\mathcal{J} \setminus Z)$$

where the right hand side is the maximal price that the advertiser is willing to pay for the slots on outlets in  $S \setminus (\mathcal{J} \setminus Z)$ . Therefore, following the proposed strategy is optimal for owner  $Z$ . Since  $Z$  is an arbitrary owner, our construction is a SPEPS.

To prove the second part of the statement, fix any SPEPS of the game. By Lemma 1(ii), all advertisers buy slots on all outlets in  $\mathcal{J}$ . Therefore, each advertiser pays  $p_Z^*$  to each owner  $Z$ . If  $p_Z^* > v_Z$  for any owner  $Z$ , then any advertiser can profitably deviate by only buying slots on all outlets in  $\mathcal{J} \setminus Z$ . If  $p_Z^* < v_Z$  for any owner  $Z$ , then, by the earlier observation, owner  $Z$  can profitably deviate by offering a single bundle  $Z$  with a price  $v_Z - \varepsilon$  for  $\varepsilon > 0$  sufficiently small to extract  $v_Z - \varepsilon > p_Z^*$  from each advertiser. Thus  $p_Z^* = v_Z$  for all  $Z \in \mathcal{Z}$ .

**Proof of Proposition 1.** Since  $V(\cdot)$  is not submodular, there exist  $S \subset T \subset \mathcal{J}$  and  $j \in \mathcal{J} \setminus T$  such that

$$V(T \cup \{j\}) - V(T) > V(S \cup \{j\}) - V(S).$$

Let  $\mathcal{J}' = T \cup \{j\}$ , and  $R = T \setminus S$ . Consider the partition  $\mathcal{Z}' = \{S, R, \{j\}\}$ . Suppose for contradiction that there is a SPEPS in which  $p_{Z'}^* = v_{Z'}$  for all  $Z' \in \mathcal{Z}'$ . Since  $V(\cdot)$  is strictly monotone, by Lemma 1(ii), all advertisers buy slots on all outlets in  $\mathcal{J}'$ . Fix any advertiser. Consider the deviation of not buying any slot from  $R$  and  $\{j\}$ . This deviation decreases the payment by

$$\begin{aligned} v_R + v_j &= [V(\mathcal{J}') - V(\mathcal{J}' \setminus R)] + [V(\mathcal{J}') - V(\mathcal{J}' \setminus \{j\})] \\ &= [V(T \cup \{j\}) - V(S \cup \{j\})] + [V(T \cup \{j\}) - V(T)] \\ &> V(T) - V(S) + [V(T \cup \{j\}) - V(T)] \\ &= V(T \cup \{j\}) - V(S), \end{aligned}$$

where the strict inequality is due to  $V(T \cup \{j\}) - V(T) > V(S \cup \{j\}) - V(S)$ . Since  $V(T \cup \{j\}) - V(S)$  is the payoff that the advertiser gives up for not buying any slot on the outlets in  $R$  and  $\{j\}$ , this deviation is profitable. Contradiction.

**Proof of Proposition 2.** With a slight abuse of notation, for any group  $g$ , let  $g$  denote both the group and a randomly sampled viewer from the group. By Theorem 1, we can write

$$p_j^* = a \sum_g \mu_g \mathbb{E}[\mathbf{1}_{g \rightarrow j} \beta_{X_j^g + 1}]$$

where  $g \rightarrow j$  denotes the event that a randomly sampled viewer  $g$  views outlet  $j$  and  $X_j^g$  counts the random number of outlets viewed by  $g$  that are not  $j$ . For any  $g' \neq g, h$ , we have

$$\eta_{g'j} = \frac{\lambda_j \sigma_{g'j}}{\mu_{g'}} = \frac{\lambda_k \sigma_{g'k}}{\mu_{g'}} = \eta_{g'k}.$$

Therefore, for any  $g' \neq g, h$ , by independence and symmetry,

$$\mathbb{E}[\mathbf{1}_{g' \rightarrow j} \beta_{X_j^{g'} + 1}] = \mathbb{E}[\mathbf{1}_{g' \rightarrow k} \beta_{X_k^{g'} + 1}].$$

To prove  $p_j^* / \lambda_j \geq p_k^* / \lambda_k$ , it then suffices to show

$$\mu_g \mathbb{E}[\mathbf{1}_{g \rightarrow j} \beta_{X_j^g + 1} - \mathbf{1}_{g \rightarrow k} \beta_{X_k^g + 1}] \geq \mu_h \mathbb{E}[\mathbf{1}_{h \rightarrow k} \beta_{X_k^h + 1} - \mathbf{1}_{h \rightarrow j} \beta_{X_j^h + 1}].$$

Using independence, we can write the above as

$$\mu_g [\eta_{gj}(1 - \eta_{gk}) - \eta_{gk}(1 - \eta_{gj})] \mathbb{E}[\beta_{X^g+1}] \geq \mu_h [\eta_{hk}(1 - \eta_{hj}) - \eta_{hj}(1 - \eta_{hk})] \mathbb{E}[\beta_{X^h+1}]$$

where  $X^g$  counts the random number of outlets viewed by viewer  $g$  that are not in  $\{j, k\}$ . Since  $\lambda_j = \lambda_k$ , this reduces to

$$(\sigma_{gj} - \sigma_{gk}) \mathbb{E}[\beta_{X^g+1}] \geq (\sigma_{hk} - \sigma_{hj}) \mathbb{E}[\beta_{X^h+1}].$$

It follows easily from our assumptions that  $\sigma_{gj} - \sigma_{gk} = \sigma_{hk} - \sigma_{hj} \geq 0$ . So it suffices to show  $\mathbb{E}[\beta_{X^g+1}] \geq \mathbb{E}[\beta_{X^h+1}]$ . Since  $\eta_{gj} \leq \eta_{hj}$  for all  $j \in \mathcal{J}$  and viewing decisions are independent across outlets for both  $g$  and  $h$ , there exists a monotone coupling of the viewing decisions by  $g$  and  $h$  in the sense that for all  $j \in \mathcal{J}$ ,

$$\mathbf{1}_{g \rightarrow j} \leq \mathbf{1}_{h \rightarrow j}.$$

Under this coupling, we have  $X^g \leq X^h$  pointwise. The claim then follows directly by noting that  $\beta_m$  is non-increasing in  $m$  for  $m \geq 1$ .

Now suppose  $\sigma_{gj} > \sigma_{gk}$  and  $\eta_{gj'} < \eta_{hj'}$  for some  $j' \neq j, k$ . Using integration by parts, we have

$$\begin{aligned} \mathbb{E}[\beta_{X^g+1}] - \mathbb{E}[\beta_{X^h+1}] &= \int_0^\infty \mathbb{P}(\beta_{X^g+1} > s) ds - \int_0^\infty \mathbb{P}(\beta_{X^h+1} > s) ds \\ &= \sum_{m=1}^\infty (\beta_m - \beta_{m+1}) \left( \mathbb{P}(X^g + 1 \leq m) - \mathbb{P}(X^h + 1 \leq m) \right) > 0 \end{aligned}$$

where the strict inequality follows from the fact that each term in the summation is nonnegative,  $\beta_1 > \beta_2$ , and  $\mathbb{P}(X^g = 0) = \prod_{l \neq j, k} (1 - \eta_{gl}) > \prod_{l \neq j, k} (1 - \eta_{hl}) = \mathbb{P}(X^h = 0)$ . Since  $\sigma_{gj} > \sigma_{gk}$ , we then have  $(\sigma_{gj} - \sigma_{gk}) \mathbb{E}[\beta_{X^g+1}] > (\sigma_{hk} - \sigma_{hj}) \mathbb{E}[\beta_{X^h+1}]$  and hence  $p_j^*/\lambda_j > p_k^*/\lambda_k$ .

**Proof of Proposition 3.** We follow the same notation as in the proof of Proposition 2. By Theorem 1, we can write

$$\begin{aligned} p_j^* &= a \sum_g \mu_g \mathbb{E}[\mathbf{1}_{g \rightarrow j} \beta_{X_j^g+1}] \\ &= a \sum_g \mu_g (\eta_{gj} \eta_{gk} \mathbb{E}[\beta_{X^g+2}] + \eta_{gj}(1 - \eta_{gk}) \mathbb{E}[\beta_{X^g+1}]) \\ &= a \sum_g \mu_g (\eta_{gk} \eta_{gj} \mathbb{E}[\beta_{X^g+2}] + \delta \eta_{gk} (1 - \frac{1}{\delta} \eta_{gj}) \mathbb{E}[\beta_{X^g+1}]) \\ &= a \sum_g \mu_g (\eta_{gk} \eta_{gj} \mathbb{E}[\beta_{X^g+2}] + \eta_{gk} (1 - \eta_{gj}) \mathbb{E}[\beta_{X^g+1}] + \eta_{gk} (\delta - 1) \mathbb{E}[\beta_{X^g+1}]) \end{aligned}$$

$$\begin{aligned}
&= a \sum_g \mu_g \eta_{gk} (\mathbb{E}[\beta_{X_k^g+1}] + (\delta - 1)\mathbb{E}[\beta_{X^g+1}]) \\
&\geq a \sum_g \mu_g \eta_{gk} (\mathbb{E}[\beta_{X_k^g+1}] + (\delta - 1)\mathbb{E}[\beta_{X_k^g+1}]) = \delta p_k^* = \frac{\lambda_j}{\lambda_k} p_k^*
\end{aligned}$$

where we have used independence,  $\delta \geq 1$ ,  $X_k^g \geq X^g$ , and  $\beta_m$  is non-increasing for  $m \geq 1$ . Now suppose  $\delta > 1$ . For any group  $g$ , using integration by parts, we have

$$\begin{aligned}
\mathbb{E}[\beta_{X^g+1}] - \mathbb{E}[\beta_{X_k^g+1}] &= \int_0^\infty \mathbb{P}(\beta_{X^g+1} > s) ds - \int_0^\infty \mathbb{P}(\beta_{X_k^g+1} > s) ds \\
&= \sum_{m=1}^\infty (\beta_m - \beta_{m+1}) (\mathbb{P}(X^g + 1 \leq m) - \mathbb{P}(X_k^g + 1 \leq m)) > 0
\end{aligned}$$

where the strict inequality follows from that each term in the summation is nonnegative,  $\beta_1 > \beta_2$ , and  $\mathbb{P}(X^g = 0) - \mathbb{P}(X_k^g = 0) = \eta_{gj} \prod_{l \neq j, k} (1 - \eta_{gl}) > 0$ . Since  $\delta > 1$ , we then have  $(\delta - 1)\mathbb{E}[\beta_{X^g+1}] > (\delta - 1)\mathbb{E}[\beta_{X_k^g+1}]$  and hence  $p_j^*/\lambda_j > p_k^*/\lambda_k$ .

**Proof of Proposition 4.** Let  $\mathcal{O}_n$  denote advertiser  $n$ 's tie-breaking ordering over owners; that is, if indifferent among one or more sets of bundles, advertiser  $n$  chooses in a manner that maximizes the payoffs of the owners according to a lexicographic preference over owners defined by  $\mathcal{O}_n$ .

Fix any SPEPS. Fix any owner  $Z$  and any advertiser  $n$ . Let  $S_c$  be the set of outlets that advertiser  $n$  buys slots on in the cell  $c \in \mathcal{F}_Z$ . Consider owner  $Z$  offering the bundles  $\mathcal{B} := \{S_c : S_c \neq \emptyset, c \in \mathcal{F}_Z\}$  with prices  $\{p_{S_c}^* : S_c \neq \emptyset, c \in \mathcal{F}_Z\}$ , where  $p_S^*$  denotes the minimum price to buy slots on the outlets for any set  $S \subseteq \mathcal{J}$  in the SPEPS. We claim that owner  $Z$  weakly increases the payoff with this strategy. Note that for each advertiser, this change restricts the set of possible choices while keeping at least one choice that maintains the equilibrium payoff (imitating the choice of advertiser  $n$  in the original SPEPS). Because in the original SPEPS all advertisers pay the same total amount to  $Z$  by Lemma 1(i), this change can only decrease owner  $Z$ 's payoff if there is some advertiser  $n'$  (not necessarily different from  $n$ ) who now breaks ties in favor of some owner that ranks higher than  $Z$  in  $\mathcal{O}_{n'}$ . However, that choice must also be made in the original SPEPS by advertiser  $n'$  due to the tie-breaking rule. But then advertiser  $n'$  pays strictly less than advertiser  $n$  to owner  $Z$  in the original SPEPS, contradicting Lemma 1(i).

Because an owner chooses to offer fewer number of bundles when indifferent, the above observation implies that every advertiser must buy the same set of bundles from any given owner  $Z$  and that owner  $Z$  offers at most one bundle from each cell in  $\mathcal{F}_Z$ . (Otherwise, owner  $Z$  may simply pick an advertiser  $n$  who buys the smallest number of bundles from  $Z$  and offer the set of bundles  $\mathcal{B}$  as defined above to strictly decrease the total number of bundles offered without decreasing

payoff.) Then all advertisers buy slots on the same set of outlets (say  $S$ ) and any owner  $Z$  offers  $\mathcal{B}_Z := \{S \cap B : S \cap B \neq \emptyset, B \in \mathcal{F}_Z\}$  as the available bundles.

Therefore, in the second stage, the set of feasible bundles that advertisers can choose is a partition of  $S$ . In particular, bundles not contained in  $S$  are not offered by the owners. For any bundle  $B$  offered by any owner  $Z$ , by rationality of the advertisers,

$$p_B \leq V(S) - V(S \setminus B) = v_B^S.$$

Now, for contradiction, suppose there exist some owner  $Z$  and some bundle  $B' \in \mathcal{F}_Z$ ,  $B' \subseteq S$  such that  $p_{B'} < v_{B'}^S$ . Consider the following deviation. Let owner  $Z$  offer all bundles in  $\mathcal{B}_Z$  as in the equilibrium but change the price for each bundle  $B$  to  $\tilde{p}_B = v_B^S - \varepsilon$  for some  $\varepsilon > 0$ . We claim that after this deviation, all advertisers continue buying slots on the same outlets from owner  $Z$  as in the equilibrium. Indeed, if an advertiser stops buying some bundle  $B \in \mathcal{B}_Z$ , then the advertiser can only choose  $S' \subseteq S \setminus B$  since the set of available bundles is a partition of  $S$ . But submodularity of  $V(\cdot)$  implies

$$V(S' \cup B) - V(S') \geq V(S) - V(S \setminus B) = v_B^S > \tilde{p}_B.$$

Therefore owner  $Z$  can extract  $v_B^S - \varepsilon$  for each bundle  $B \in \mathcal{B}_Z$  from each advertiser. For  $\varepsilon$  sufficiently small, this is then a profitable deviation for owner  $Z$  since in the equilibrium we have  $p_B \leq v_B^S$  for all  $B \in \mathcal{B}_Z$  and  $p_{B'} < v_{B'}^S$  for some bundle  $B' \in \mathcal{B}_Z$ . Contradiction.

**Proof of Proposition 5.** As in the proof of Theorem 1, we first construct a SPEPS. Let each owner  $Z$  offer a single bundle consisting of all outlets in  $Z$  with a price  $p_Z = \underline{v}_Z$ . For any profile of posted prices (including off-the-equilibrium-path histories), let each advertiser  $n$  solve the following problem in the second stage

$$\max_{S \subseteq \mathcal{J}} V_n(S) - p_S^*$$

where  $p_S^*$  denotes the minimum price to buy slots on all of the outlets in  $S$  for a given profile of prices  $p$ . Pick a solution  $S_n^*$  such that  $|S_n^*|$  is the largest. Let advertiser  $n$  buy slots on outlets in  $S_n^*$ .

We only need to check each owner has no profitable deviation. Observe that if  $p_Z = \underline{v}_Z$  is offered by some owner  $Z$  and there is no proper subset  $W \subset Z$  being offered, then any advertiser will buy the bundle  $Z$  regardless of  $p_{-Z}$ . This is because for any  $S \subseteq \mathcal{J} \setminus Z$ , submodularity of  $V_n(\cdot)$  implies

$$V_n(S \cup Z) - V_n(S) \geq V_n(\mathcal{J}) - V_n(\mathcal{J} \setminus Z) \geq \min_{n' \in \mathcal{N}} V_{n'}(\mathcal{J}) - V_{n'}(\mathcal{J} \setminus Z) = \underline{v}_Z.$$

Fix an owner  $Z$ . Suppose all other players follow the proposed strategy. We claim that offering a single bundle  $Z$  with a price  $\underline{v}_Z$  is an optimal strategy for owner  $Z$ . To see this, consider two cases.

Case 1: Suppose  $Z$  offers some set of bundles  $\mathcal{B}_Z$  such that every advertiser buys a slot on every outlet in  $Z$ . Then the minimal price to buy all outlets in  $Z$  must be no more than  $\underline{v}_Z$  because otherwise there is one advertiser who can profitably deviate by simply not buying anything in  $\mathcal{B}_Z$ . Hence the owner cannot do better than simply offering the bundle  $Z$  with a price  $\underline{v}_Z$ .

Case 2: Suppose  $Z$  offers some set of bundles  $\mathcal{B}_Z$  such that there exist at least one outlet  $j \in Z$  and one advertiser  $n \in \mathcal{N}$  who does not buy a slot on outlet  $j$ . We claim that the total revenue that owner  $Z$  extracts is no more than

$$\max_{n,j} \left\{ \sum_{n' \neq n} v_{n',Z} + v_{n,Z \setminus \{j\}} \right\}.$$

Indeed, this is the maximal revenue that owner  $Z$  can possibly get, even if the owner price discriminates using the identities of the advertisers but is subject to the constraint that at least one advertiser does not buy on some outlet  $j \in Z$ . Now note that

$$\begin{aligned} \max_{n,j} \left\{ \sum_{n' \neq n} v_{n',Z} + v_{n,Z \setminus \{j\}} \right\} &\leq \max_{n,j} \{ N\bar{v}_Z - (v_{n,Z} - v_{n,Z \setminus \{j\}}) \} \\ &= N\bar{v}_Z - \min_{n,j} \{ V_n((\mathcal{J} \setminus Z) \cup \{j\}) - V_n(\mathcal{J} \setminus Z) \} \\ &= N\bar{v}_Z - \varphi(Z) \\ &\leq N\bar{v}_Z - N(\bar{v}_Z - \underline{v}_Z) = N\underline{v}_Z \end{aligned}$$

where we have used the assumption that  $\bar{v}_Z - \underline{v}_Z \leq \frac{1}{N}\varphi(Z)$ . Hence the owner also cannot do better than simply offering the bundle  $Z$  with a price  $\underline{v}_Z$ .

Thus the construction is a SPEPS. The outcome is efficient because all advertisers buy slots on all outlets.

To prove the second part of the statement, fix any efficient SPEPS. Note that all outlets must sell  $N$  slots, because the preferences for each player are quasilinear in money and thus the total surplus is maximized only if all potential trades are realized (recall  $K \geq N$ ). Then by the argument in Case 1, we know that  $p_Z^* \leq \underline{v}_Z$  for all  $Z \in \mathcal{Z}$ . Moreover,  $p_Z^*$  cannot be strictly lower than  $\underline{v}_Z$  for any owner  $Z$ , because if so it is a profitable deviation for owner  $Z$  to offer a single bundle  $Z$  with a price  $\underline{v}_Z - \varepsilon$  for  $\varepsilon > 0$  small enough. Hence  $p_Z^* = \underline{v}_Z$  for all  $Z \in \mathcal{Z}$ .

**Proof of Proposition 6.** We prove the second part of the statement first. Fix any subgame perfect equilibrium allowing for mixed strategies (SPEMS) and any owner  $Z$ . Suppose, for contradiction, the expected revenue per slot  $r_Z > \sum_{j \in Z} V(\{j\})/|Z|$ . Then the expected total revenue is strictly higher than  $K \sum_{j \in Z} V(\{j\})$ . Thus with positive probability, the owner earns a realized revenue

strictly higher than  $K \sum_{j \in Z} V(\{j\})$ . In these events, there is at least one advertiser who buys slots on a set of outlets  $B \subseteq Z$  and pays strictly more than  $\sum_{j \in B} V(\{j\})$  to the owner. Let  $S \subseteq \mathcal{J}$  be the set of outlets that the advertiser buys slots on. Since any non-negative submodular function is also sub-additive, we have

$$V(S) - V(S \setminus B) \leq V(B) - V(\emptyset) \leq \sum_{j \in B} V(\{j\}).$$

So simply not buying anything from  $Z$  is a profitable deviation for the advertiser. Contradiction.

Now, for contradiction, suppose  $r_Z < (v_Z - \Delta)/|Z|$ . Then the expected total revenue is strictly lower than  $K(v_Z - \Delta)$ . Let the owner deviate by offering a single bundle  $Z$  with a price  $\tilde{p}_Z = \lceil v_Z - \Delta \rceil$ , where  $\lceil x \rceil$  denotes the operator that rounds  $x$  up to the closest value in  $\{0, \Delta, 2\Delta, \dots\}$ . Note that

$$v_Z - \Delta \leq \lceil v_Z - \Delta \rceil < v_Z.$$

Since  $\tilde{p}_Z < v_Z$ , by the argument in the proof of Theorem 1, submodularity of  $V(\cdot)$  implies that, in any realization, the owner would be able to sell all the slots and secure revenue  $K\tilde{p}_Z$ . But this is then a profitable deviation. Contradiction.

To show the existence of a SPEMS, we construct an auxiliary finite game in normal form, apply the standard existence result, and then recover a SPEMS in the original game. Consider a simultaneous-move game between all the owners. Let

$$\mathcal{A}(Z) = \{0, \Delta, 2\Delta, \dots, \lceil V(\mathcal{J}) \rceil, \infty\}^{|\mathcal{P}(Z)|}$$

be the set of pure strategies that an owner can choose from. Clearly,  $\mathcal{A}(Z)$  is finite for any  $Z$ . For each pure strategy profile  $p$ , draw a random order for the advertisers and then let the advertisers, in that order, choose which slots to buy given the posted prices specified in  $p$ . Then assign the resulting expected revenue (averaged over different orders) for owner  $Z$  as the payoff to owner  $Z$  in the auxiliary game given the pure strategy profile  $p$ . This constructs a finite normal-form game among the owners, and thus a Nash equilibrium (possibly in mixed strategies) exists (say  $\mathcal{E}$ ). Now let each owner play the strategy prescribed by  $\mathcal{E}$  in the original game, followed by advertisers choosing which slots to buy in the same way as before. Evidently, this constructs a SPEMS for the original game.

### A.3 Additional Results

**Comparative statics for multi-outlet owners.** Consider a setting identical to that of Section 2.1 except that each owner  $Z$  may own multiple outlets. Suppose diminishing returns are perfect in the

sense that  $\beta_m = 0$  for  $m \geq 2$ . Members of group  $g \in G$  see ads on outlet  $j$  with probability  $\eta_{gj}$ , independently across outlets. For a given owner  $Z$ , let

$$\eta_{gZ} = 1 - \prod_{j \in Z} (1 - \eta_{gj}), \quad \lambda_Z = \sum_{g \in G} \mu_g \eta_{gZ}, \quad \sigma_{gZ} = \frac{\mu_g \eta_{gZ}}{\lambda_Z}$$

denote the share of group  $g$  that is in the owner's audience, the total mass of the owner's audience, and the share of this audience that comes from group  $g$ , respectively. Then  $p_Z^*/\lambda_Z$  is the price per viewer collected by owner  $Z$  for an ad slot on each of its outlets. By Theorem 1, we know that

$$p_Z^* = a\beta_1 \sum_g \mu_g \eta_{gZ} \prod_{j \notin Z} (1 - \eta_{gj}) = a\beta_1 \sum_g \mu_g \eta_{gZ} \prod_{Z' \neq Z} (1 - \eta_{gZ'}).$$

Note that the equilibrium prices above are identical to those in the setting of section 2.1, replacing outlets with owners, if we specify perfect diminishing returns. Therefore our results on comparative statics apply immediately.

**Proposition 7.** *Suppose that group  $g \in G$  is less active than group  $h \in G$  in the sense that  $\eta_{gZ} \leq \eta_{hZ}$  for all  $Z \in \mathcal{Z}$ . Suppose that owner  $Y \in \mathcal{Z}$  draws a larger share of its audience from group  $g$  and a smaller share of its audience from group  $h$  than owner  $Z \in \mathcal{Z}$ , in the sense that  $\sigma_{gY} \geq \sigma_{gZ}$  and  $\sigma_{hY} \leq \sigma_{hZ}$ , and that the two owners have equal total audience sizes,  $\lambda_Y = \lambda_Z$ , and equal shares of audience from groups other than  $g$  and  $h$ ,  $\sigma_{g'Y} = \sigma_{g'Z}$  for all  $g' \neq g, h$ . Then owner  $Y$  has a higher equilibrium price per viewer than owner  $Z$ ,  $p_Y^*/\lambda_Y \geq p_Z^*/\lambda_Z$ .*

**Proposition 8.** *Suppose that owner  $Y$  has a larger audience than owner  $Z$  in the sense that for some  $\delta \geq 1$ ,  $\eta_{gY} = \delta \eta_{gZ}$  for all  $g \in G$ . Then owner  $Y$  has a higher price per viewer than owner  $Z$ ,  $p_Y^*/\lambda_Y \geq p_Z^*/\lambda_Z$ .*

**Existence of unbundled pricing equilibrium.** Consider the setting of Section 2.1 with  $G = 1$ ,  $F_Z$  the finest possible partition (i.e., the owners are not allowed to bundle), and  $\beta_m = \beta^{m-1}$  for some constant  $\beta \geq 0$ . We construct the set of outlets  $S$  that all advertisers buy slots on. Let  $i$  denote a viewer uniformly drawn from the population. Let  $X_T$  denote the random number of outlets in  $T$  viewed by viewer  $i$ . Since  $\beta_m = \beta^{m-1}$ , for any  $T \subseteq \mathcal{J}$  we can write

$$V(T) = a\mathbb{E} \left[ \sum_{m=1}^{X_T} \beta^{m-1} \right]$$

where the sum is interpreted as zero when  $X_T = 0$ . Fix an owner  $Z$ . Let  $F \subseteq Z$  denote the menu  $Z$  offers. Since we assume no owner can bundle any outlets, every outlet  $j$  in  $F$  is sold individually



at some price. Let  $Z$  solve

$$\max_{F \subseteq Z} \sum_{j \in F} V(F) - V(F \setminus \{j\}).$$

Let  $F_Z^*$  be a maximizer of the above problem. For any  $T \subseteq \mathcal{J} \setminus Z$ , we claim that  $F_Z^*$  also solves

$$\max_{F \subseteq Z} \sum_{j \in F} V(T \cup F) - V((T \cup F) \setminus \{j\}).$$

Indeed, for any  $F \subseteq Z$  and any  $j \in F$ ,

$$\begin{aligned} V(T \cup F) - V((T \cup F) \setminus \{j\}) &= a \mathbb{E} \left[ \sum_{m=1}^{X_{T \cup F}} \beta^{m-1} - \sum_{m=1}^{X_{(T \cup F) \setminus \{j\}}} \beta^{m-1} \right] \\ &= a \mathbb{E} [\mathbf{1}_{i \rightarrow j} \beta^{X_{T \cup F} - 1}] \\ &= a \mathbb{E} [\mathbf{1}_{i \rightarrow j} \beta^{X_T + X_F - 1}] \\ &= a \mathbb{E} [\beta^{X_T}] \mathbb{E} [\mathbf{1}_{i \rightarrow j} \beta^{X_F - 1}] \\ &= \mathbb{E} [\beta^{X_T}] (V(F) - V(F \setminus \{j\})) \end{aligned}$$

where we have used the fact that viewing probabilities are independent across outlets. Therefore,

$$\max_{F \subseteq Z} \sum_{j \in F} V(T \cup F) - V((T \cup F) \setminus \{j\}) = \mathbb{E} [\beta^{X_T}] \max_{F \subseteq Z} \sum_{j \in F} V(F) - V(F \setminus \{j\})$$

and thus is also solved by  $F_Z^*$ . Now let  $S = \bigcup_{Z \in \mathcal{Z}} F_Z^*$ . We can then construct a SPEPS using the set  $S$  and equipping outlets in  $S$  with the prices identified in Proposition 4:  $p_j = v_j^S$  for all  $j \in S$  and  $p_j = \infty$  for all  $j \notin S$ . By the proof of Proposition 4, we know that with the proposed bundles and prices, it is optimal for any advertiser to buy slots on all outlets in  $S$ . We also know that given these prices, for any owner  $Z$ , regardless what owner  $Z$  does, the advertisers buy slots on outlets in  $S \setminus Z$  (in this construction, for any advertiser  $n$ , we may let the tie-breaking ordering  $\mathcal{O}_n$  be any complete ordering over the owners). Therefore, owner  $Z$  simply solves the problem

$$\max_{F \subseteq Z} \sum_{j \in F} V((S \setminus Z) \cup F) - V((S \setminus Z) \cup F \setminus \{j\})$$

which has a maximizer  $F_Z^*$  as shown earlier. So it is optimal for owner  $Z$  to offer menu  $F_Z^*$  in which each outlet  $j \in F_Z^*$  has a price  $v_j^S$ . Since this holds for any owner, the construction is a SPEPS.

To see how Proposition 2 and 3 extend, note that

$$p_j^* = V(S) - V(S \setminus \{j\}) = a \sum_g \mu_g \mathbb{E} [\mathbf{1}_{g \rightarrow j} \beta_{X_j^g + 1}]$$

where we use the notations in the proof of Proposition 2, except that  $X_j^g$  now counts the number of outlets in  $S \setminus \{j\}$  that are viewed by  $g$ . The rest follows verbatim.

**Competitors' ad effect.** We consider a setting in which each owner owns a single outlet, and modify the value function  $V(\cdot)$  as follows. Let advertiser  $n$ 's value for buying ad on the set of outlets  $S_n$  be  $V(S_n, \vec{S}_{-n})$ , where  $\vec{S}_{-n}$  is the vector of outlets bought by other advertisers. We say  $\vec{S}_{-n} \leq \vec{S}'_{-n}$  if each entry of the vector is smaller in the set-inclusion order. Since all owners are single-outlet owners, we use  $j$  to denote both an outlet and the owner associated with the outlet. Let  $\vec{\mathcal{J}}$  be the vector of length  $N - 1$  with  $\mathcal{J}$  in each entry, and  $\overrightarrow{\mathcal{J} \setminus \{j\}}$  be the vector of length  $N - 1$  with  $\mathcal{J} \setminus \{j\}$  in each entry. We impose two assumptions:

$$V(\mathcal{J}, \vec{S}_{-n}) - V(\mathcal{J} \setminus \{j\}, \vec{S}_{-n}) \geq V(\mathcal{J}, \vec{S}'_{-n}) - V(\mathcal{J} \setminus \{j\}, \vec{S}'_{-n}) \text{ for any } \vec{S}_{-n} \leq \vec{S}'_{-n} \text{ and } j; \quad (\text{A1})$$

$$V(\mathcal{J}, \overrightarrow{\mathcal{J} \setminus \{j\}}) - V(\mathcal{J} \setminus \{j\}, \overrightarrow{\mathcal{J} \setminus \{j\}}) \leq (1 + \frac{1}{N}) \left( V(\mathcal{J}, \vec{\mathcal{J}}) - V(\mathcal{J} \setminus \{j\}, \vec{\mathcal{J}}) \right) \text{ for any } j. \quad (\text{A2})$$

Let  $\tilde{v}_j = V(\mathcal{J}, \vec{\mathcal{J}}) - V(\mathcal{J} \setminus \{j\}, \vec{\mathcal{J}})$  denote the modified incremental value of outlet  $j$  in this setting.

**Proposition 9.** *Suppose  $V(\cdot, \vec{S})$  is monotone and submodular for any  $\vec{S}$ , and  $V(\cdot, \cdot)$  satisfies (A1) and (A2). Then there exists a SPEPS in which the price for outlet  $j$  is  $p_j^* = \tilde{v}_j$ , and all advertisers buy slots on all outlets.*

*Proof.* We construct a SPEPS as follows. Let each owner  $j$  announce price  $\tilde{v}_j$ . For each profile of prices  $p$  announced (including off-the-equilibrium-path histories), the subgame in the second stage is a finite extensive-form game and hence admits a SPEPS by backward induction. When doing the backward induction, if an advertiser is indifferent between different sets of outlets to buy slots on, we pick one with the largest cardinality. Now we verify no owner has a profitable deviation.

Observe that if  $p_j = \tilde{v}_j$  is offered by an owner, then any advertiser will buy a slot on outlet  $j$  regardless of  $p_{-j}$  and what other advertisers do. This is because for any  $S \subseteq \mathcal{J} \setminus \{j\}$  and any  $\vec{S}_{-n} \leq \vec{\mathcal{J}}$ ,

$$V(S \cup \{j\}, \vec{S}_{-n}) - V(S, \vec{S}_{-n}) \geq V(\mathcal{J}, \vec{S}_{-n}) - V(\mathcal{J} \setminus \{j\}, \vec{S}_{-n}) \geq V(\mathcal{J}, \vec{\mathcal{J}}) - V(\mathcal{J} \setminus \{j\}, \vec{\mathcal{J}})$$

where we have used submodularity of  $V(\cdot, \vec{S}_{-n})$  and assumption (A1). Further, when all other advertisers buy slots on all outlets, the incremental value for an advertiser to buy a slot on some outlet  $j$  is exactly  $V(\mathcal{J}, \vec{\mathcal{J}}) - V(\mathcal{J} \setminus \{j\}, \vec{\mathcal{J}})$ . Therefore, at the proposed price profile, for any outlet  $j$ , each advertiser is indifferent between buying and not buying a slot on outlet  $j$  holding everything else fixed (including other advertisers' decisions).

Now fix any owner  $j$ . Suppose all other players follow the proposed strategy. Note that owner  $j$  is selling  $N$  slots by announcing price  $\tilde{v}_j$  and clearly has no incentive to decrease the

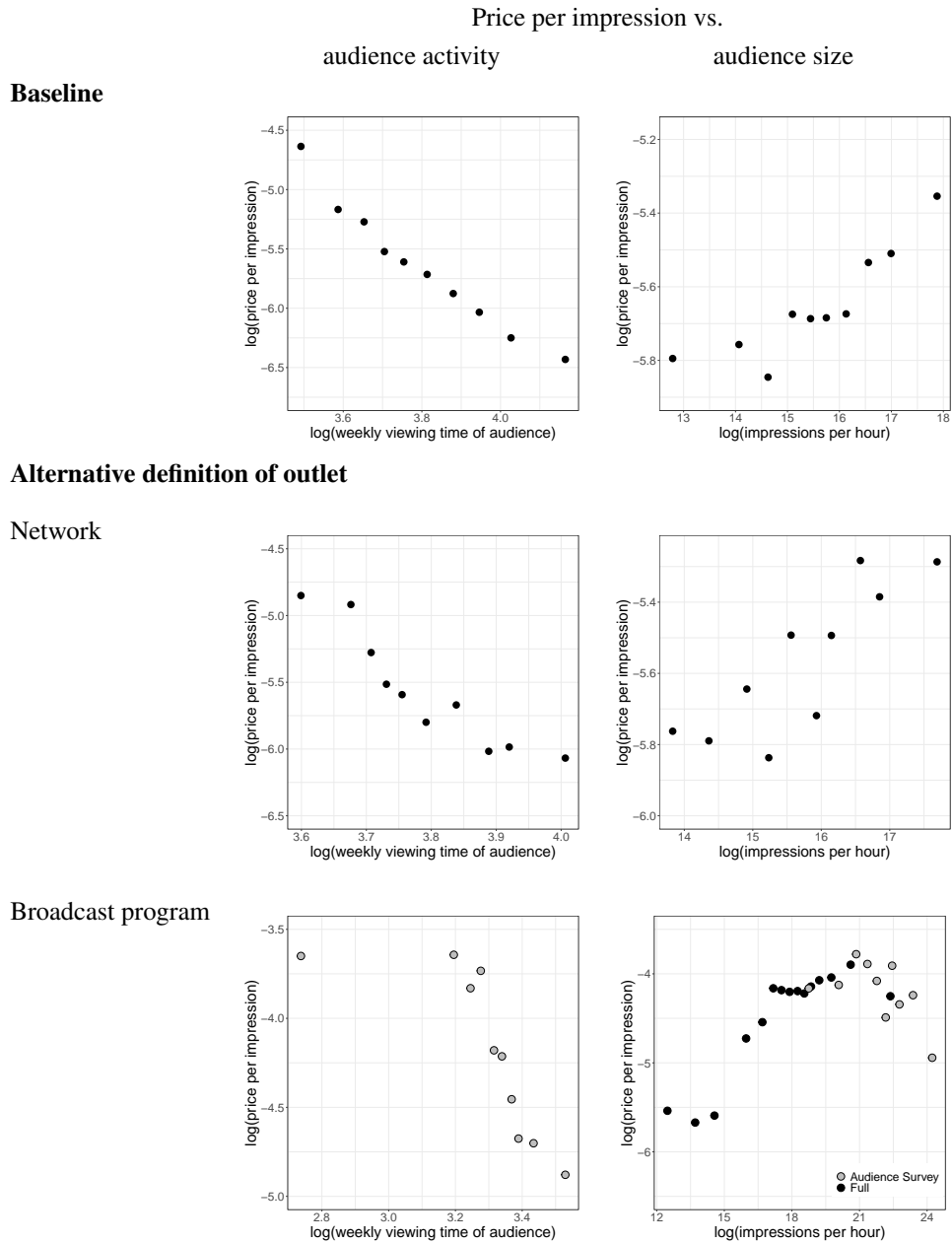
price. Consider the deviation of raising the price. By the earlier observation, all advertisers would continue buying slots on outlets in  $\mathcal{J} \setminus \{j\}$ . Therefore, by (A1), the maximal amount owner  $j$  can extract from an advertiser is at most  $V(\mathcal{J}, \overrightarrow{\mathcal{J} \setminus \{j\}}) - V(\mathcal{J} \setminus \{j\}, \overrightarrow{\mathcal{J} \setminus \{j\}})$ . Further, we claim that at least one advertiser would stop buying the slot on outlet  $j$  after raising the price. Suppose not. Then all advertisers buy slots on all outlets. But the last advertiser moving in sequence has a profitable deviation of buying only the slots on outlets in  $\mathcal{J} \setminus \{j\}$ . Thus there are at most  $N - 1$  advertisers buying a slot on outlet  $j$ . Hence owner  $j$ 's revenue is at most

$$(N - 1) \left( V(\mathcal{J}, \overrightarrow{\mathcal{J} \setminus \{j\}}) - V(\mathcal{J} \setminus \{j\}, \overrightarrow{\mathcal{J} \setminus \{j\}}) \right) \leq (N - 1) \left( 1 + \frac{1}{N} \right) \left( V(\mathcal{J}, \overrightarrow{\mathcal{J}}) - V(\mathcal{J} \setminus \{j\}, \overrightarrow{\mathcal{J}}) \right) \leq N \tilde{v}_j$$

where the first inequality is due to (A2). So there is no profitable deviation for owner  $j$ . Since this holds for any owner, the construction is a SPEPS.  $\square$

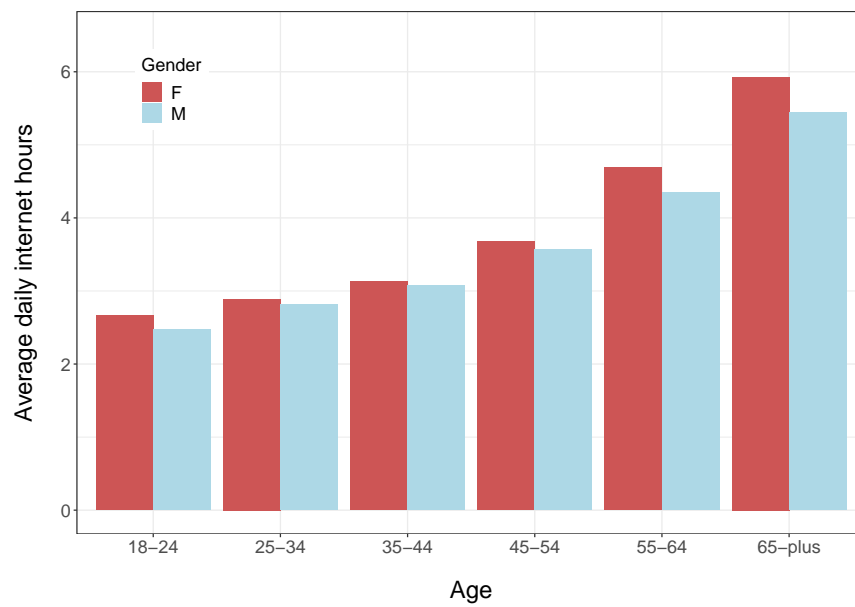
## **B Additional Empirical Results**

Appendix Figure 1: **Sensitivity to alternative definitions and assumptions**



Notes: Each row corresponds to a different specification. Each column corresponds to a different plot. The row labeled “Baseline” corresponds to the main specification in the paper, with the plot “Price per impression vs. audience activity” corresponding to Panel B of Figure 1 and the plot “Price per impression vs. audience size” corresponding to Panel C of Figure 3. The rows under the header “Alternative definition of outlet” consider different outlet definitions. In the row labeled “Network” an outlet  $j$  is a network. In the “Network” row, the “Price per impression vs. audience activity” specification includes controls for the share of total impressions that are to adults and for indicators of deciles of audience size, and the “Price per impression vs. audience size” specification includes controls for the share of total impressions that are to adults and for indicators of deciles of audience activity. In the row labeled “Broadcast program” an outlet  $j$  is a broadcast program, with bins corresponding to 15 quantiles of the full sample of broadcast programs (3058 programs) colored black and bins corresponding to deciles of the subsample of broadcast programs included in the audience survey (164 programs) colored gray. In the “Broadcast program” row, the “Price per impression vs. audience activity” specification includes controls for the share of total impressions that are to adults and for indicators of deciles of audience size, and the “Price per impression vs. audience size” specification includes a control for the share of total impressions that are to adults.

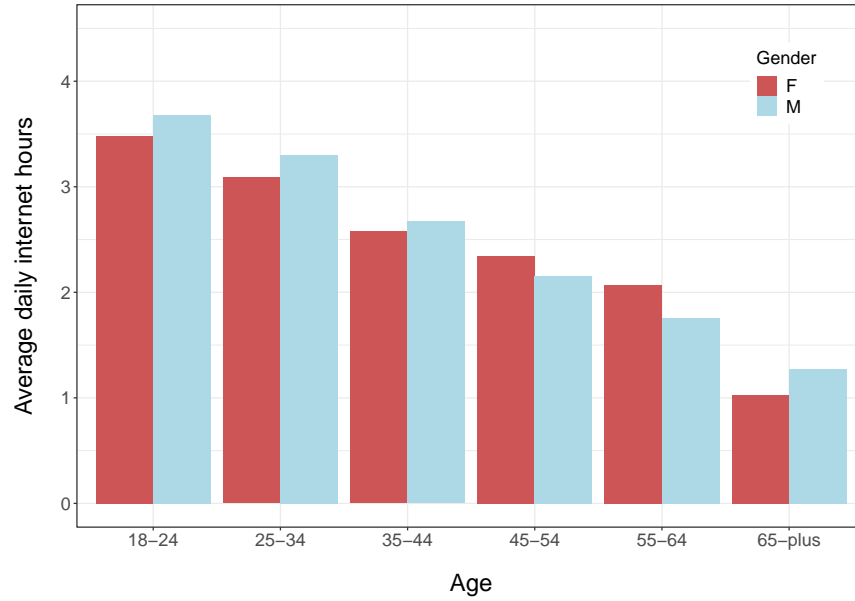
Appendix Figure 2: Average television viewing hours per day by age and gender



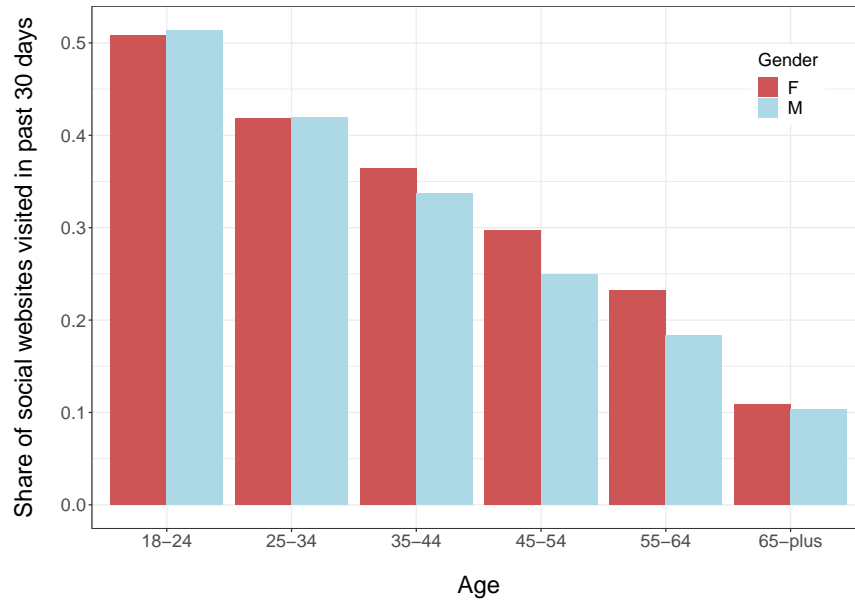
Notes : The figure shows the average daily viewing hours spent on television across age groups by gender.

Appendix Figure 3: Measures of online activity by age and gender

Panel A: Average internet hours per day



Panel B: Share of social media sites visited in the past 30 days



Notes: Panel A shows average daily hours spent on the internet across age groups by gender. Panel B shows the average share of five social media sites (Facebook, Instagram, Reddit, Twitter and Youtube) visited in the past 30 days across age groups by gender.