From Syntax to Natural Logic

Lauri Karttunen, Annie Zaenen
Stanford

- Syllogisms: Aristotle and Medieval
- What is a logical system?
- Extensions of syllogistic reasoning: De Morgan and Leibniz
- Extensions of syllogistic reasoning: Middle Ages

Aristotelian and Medieval Thought

Syllogisms

All poodles are *dogs*
All *dogs* are animals
All poodles are animals

Major premise, minor premise, conclusion
Terms:
minor term: subject of the conclusion,
major term: predicate of the conclusion,
middle term: somewhere in each of the premises

Aristotelian and medieval arrangements

A is predicated of all B	All B are A
B is predicated of all C	All C are B
A is predicated of all C	All C are A

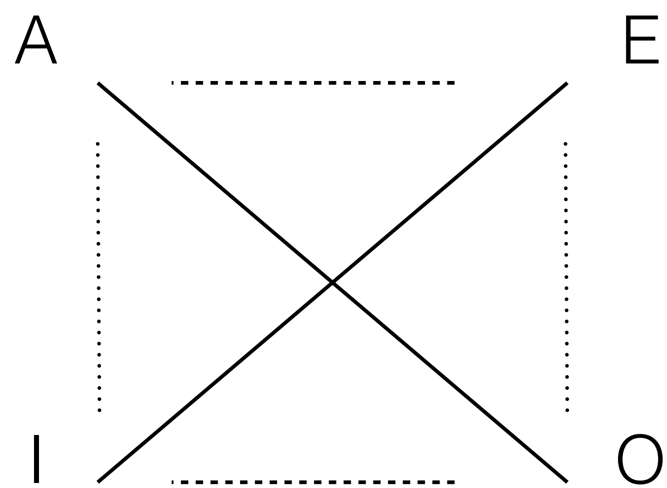
nicely captures transitivity in the basic ‘perfect’ figures, the Latin and later translations loose that.

- Four figures in Aristotelian arrangement:
  1. the middle term is the predicate of the major and the subject of the minor (AB, BC)
  2. the middle term is the subject in both premises (BA, BC)
  3. the middle term is the predicate in both premises (AB, CB)
  4. (the middle term is the subject in the major and the predicate in the minor) (BA, CB)
- Four figures in medieval arrangement:
  1. the middle term is the subject of the major and the predicate of the minor (AB, CA)
  2. the middle term is the predicate in both premises (AB,CB)
  3. the middle term is the subject in both premises (AB,AC)
  4. (the middle term is the predicate in the major and the subject in the minor) (AB,BC)

Mood of Syllogism

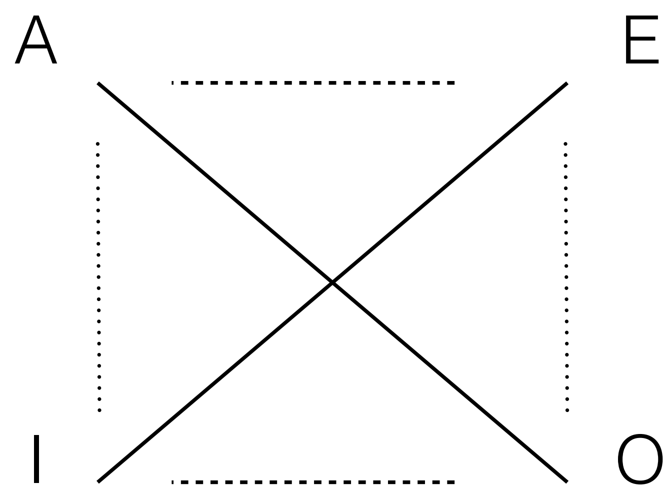
Mood	Mood
A = A is predicated of all B (AaB)	A = all A are B (AaB)
I = A is predicated of some B (AiB)	I = some A is B (AiB)
E = A is not predicated of any B (AeB)	E = No A is B (AeB)
O = A is not predicated of some B (AoB)	O = some A are not B (AoB)

Square of Opposition



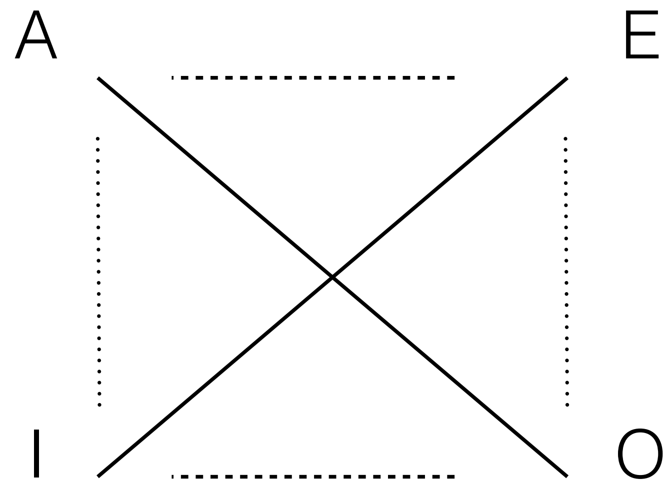
Diagonal : contradictories: p,q not both true and either p or q true
A,E : contraries: p,q not both true; p,q can both be false
I,O : subcontraries: p and q cannot be both false and p and q can be true
A,I and E,O : subalternations (implications if existential import assumed)

Square of Opposition



Diagonal : contradictories: p,q not both true and either p or q true
A: All A are B
E: No A is B
I: Some A is B
O: Some A are not B

Square of Opposition



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http://en.wikipedia.org/wiki/File:Square_of_opposition,_set_diagrams.svg

Combination of syllogisms in moods

Four moods and four figures get us 256 combinations
How many are valid deductions? 24

1	2	3	4
Barbara	Cesare	Datisi	Calemes
Celerent	Camestres	Disamis	Dimatis
Darii	Festino	Ferison	Fresison
Ferio	Baroco	Bocardo	Calemos
Barbari	Cesaro	Felapton	Fesapo
Celaront	Camestrop	Darapti	Barnalip

First figure

Barbara (AAA-1)

- All men are mortal. (MaP)
- All Greeks are men. (SaM)
- ∴ All Greeks are mortal. (SaP)

Celarent (EAE-1)

- No reptiles have fur. (MeP)
- All snakes are reptiles. (SaM)
- ∴ No snakes have fur. (SeP)

Darii (AII-1)

- All rabbits have fur. (MaP)
- Some pets are rabbits. (SiM)
- ∴ Some pets have fur. (SiP)

Ferio (EIO-1)

- No homework is fun. (MeP)
- Some reading is homework. (SiM)
- ∴ Some reading is not fun. (SoP)

First figure

Barbari (AAI-1)

- All men are mortal. (MaP)
- All Greeks are men. (SaM)
- ∴ Some Greeks are mortal. (SiP)

Celaront (EAO-1)

- No reptiles have fur. (MeP)
- All snakes are reptiles. (SaM)
- ∴ Some snakes have no fur. (SoP)

Bamalip is like Barbari with S and P exchanged

All Greeks are men. (PaM)
All men are mortal. (MaS)
∴ Some mortals are Greek. (SiP)

Dimatis is like Darii with S and P exchanged.

- Some pets are rabbits. (PiM)
- All rabbits have fur. (MaS)
- ∴ Some fur bearing animals are pets. (SiP)

Darii (AII-1)

- All rabbits have fur. (MaP)
- Some pets are rabbits. (SiM)
- ∴ Some pets have fur. (SiP)

Calemes is like Celarent with S and P exchanged.

- All snakes are reptiles. (PaM)
- No reptiles have fur. (MeS)
- ∴ No fur bearing animal is a snake. (SeP)

Celarent (EAE-1)

- No reptiles have fur. (MeP)
- All snakes are reptiles. (SaM)
- ∴ No snakes have fur. (SeP)

A = all A are B (AaB)
I = some A is B (AiB)
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Premise-introduction: first figure
- Barbara: From AaB and BaC, infer AaC
- Celerent: From AeB and BaC, infer AeC
- Darii: From AaB and BiC, infer AiC
- Ferio: From AeB and BiC, infer AoC

Conversion
- From AeB infer BeA
- From AiB infer BiA
- From AaB infer BiA

Reductio ad impossibile: (RAP)
- r may be inferred from {p,q} if either the contradictory or the contrary of q has been inferred from {p, contradictory r}

Ekthesis
- AiB, therefore AaC, BaC (C not occurring previously)
- AoB, therefore, AeC, BaC (C not occurring previously)
- AaC, BaC, therefore, AiB
- AeC, BaC, therefore, AoB

Baroco : proof by reductio ad impossibile

- Every L is M
- Some S is not M,
- Some S is not L.

First, we assume the contradictory of the conclusion. The conclusion is "Some S is not L," and so the contradictory of the conclusion is:

- Every S is L.

Next, we add this to Premise 1:

- 1. Every L is M
- 2. Every S is L

Lines 1 and 2 match the premises Barbara (Every L is M, Every S is L, Every S is M)

We deduce:

- Every S is M

But this is the contradictory of the other premise of Baroco. That is, this is the contradictory of Some S is not M. This means that if one premise is true and the contradictory of the conclusion is true, then the other premise must be false, which means that if one were to assert both premises while also asserting the denial or contradictory of the conclusion, one would contradict oneself. This shows that it would be impossible for the premises to be true while the conclusion is false. Which shows that if the premises are true, then the conclusion must be true. The argument is valid.

A = all A are B (AaB)
I = some A is B (AiB)
E = No A is B (AeB)
O = some A are not B (AoB)

Bocardo

Step	Justification	Aristotle's Text
1. MaN		*Next, if M belongs to every N,*
2. MoX		*but to no X,*
To prove: NoX		*then it is necessary for N not to belong to some X*
3. NaX	Contradictory of the desired conclusion	*For if it belongs to all,*
4. MaN	Repetition of premise 1	*and M is predicated of every N,*
5. MaX	(4, 5, Barbara)	*then it is necessary that M belongs to every X.*
6. MoX	(5 is the contradictory of 2)	*But it was assumed not to belong to some.*

A modern partial interpretation

Syllogistic Logic of *All* and *Some*

Syntax: Start with a collection of unary atoms (for nouns).
Sentences: *All p are q*, *Some p are q*

Semantics: A model $\mathcal{M}$ is a set M,
and for each noun p we have an interpretation $[\![p]\!] \subseteq M$.

$$\begin{array}{lll} \mathcal{M} \models \textit{All p are q} & \text{iff} & [\![p]\!] \subseteq [\![q]\!] \\ \mathcal{M} \models \textit{Some p are q} & \text{iff} & [\![p]\!] \cap [\![q]\!] \neq \emptyset \end{array}$$

Proof system is based on the following rules:

$$\frac{}{\textit{All p are p}} \qquad \frac{\textit{All p are n} \quad \textit{All n are q}}{\textit{All p are q}}$$

$$\frac{\textit{Some p are q}}{\textit{Some q are p}} \qquad \frac{\textit{Some p are q}}{\textit{Some p are p}} \qquad \frac{\textit{All q are n} \quad \textit{Some p are q}}{\textit{Some p are n}}$$

Semantic and proof-theoretic notions

If Γ is a set of sentences, we write $\mathcal{M} \models \Gamma$ if for all $\varphi \in \Gamma$, $\mathcal{M} \models \varphi$.

$\Gamma \models \varphi$ means that every $\mathcal{M} \models \Gamma$ also has $\mathcal{M} \models \varphi$.

A proof tree over Γ is a finite tree $\mathcal{T}$
whose nodes are labeled with sentences,
and each node is either an element of Γ,
or comes from its parent(s) by an application of one of the rules.

$\Gamma \vdash \varphi$ means that there is a proof tree $\mathcal{T}$ for over Γ
whose root is labeled φ.

Example of a derivation

If there is an n, and if all n are p and also q, then some p are q.

Some n are n, All n are p, All n are q $\vdash$ *Some p are q*.

The proof tree is

$$\dfrac{\mathit{All\ n\ are\ q} \qquad \dfrac{\dfrac{\mathit{All\ n\ are\ p} \quad \mathit{Some\ n\ are\ n}}{\mathit{Some\ n\ are\ p}}}{\mathit{Some\ p\ are\ n}}}{\mathit{Some\ p\ are\ q}}$$

The languages $\mathcal{S}$ and $\mathcal{S}^\dagger$ add noun-level negation

Let us add complemented atoms $\overline{p}$ on top of
the language of All and Some,
with interpretation via set complement: $[\![\overline{p}]\!] = M \setminus [\![p]\!]$.

So we have

$$
\left.
\begin{array}{ll}
\mathcal{S} & \left\{
\begin{array}{l}
\textit{All } p \textit{ are } q \\
\textit{Some } p \textit{ are } q \\
\textit{All } p \textit{ are } \overline{q} \equiv \textit{No } p \textit{ are } q \\
\textit{Some } p \textit{ are } \overline{q} \equiv \textit{Some } p \textit{ aren't } q
\end{array}
\right. \\
& \\
& \quad \textit{Some non-}p \textit{ are non-}q
\end{array}
\right\} \mathcal{S}^\dagger
$$

A SYLLOGISTIC SYSTEM FOR $\mathcal{S}^{\dagger}$

$$\frac{}{\textsf{All } p \textsf{ are } p} \qquad \frac{\textsf{Some } p \textsf{ are } q}{\textsf{Some } p \textsf{ are } p} \qquad \frac{\textsf{Some } p \textsf{ are } q}{\textsf{Some } q \textsf{ are } p}$$

$$\frac{\textsf{All } p \textsf{ are } n \quad \textsf{All } n \textsf{ are } q}{\textsf{All } p \textsf{ are } q} \qquad \frac{\textsf{All } n \textsf{ are } p \quad \textsf{Some } n \textsf{ are } q}{\textsf{Some } p \textsf{ are } q}$$

$$\frac{\textsf{All } q \textsf{ are } \overline{q}}{\textsf{All } q \textsf{ are } p}\ \textsf{Zero} \qquad \frac{\textsf{All } \overline{q} \textsf{ are } q}{\textsf{All } p \textsf{ are } q}\ \textsf{One}$$

$$\frac{\textsf{All } p \textsf{ are } \overline{q}}{\textsf{All } q \textsf{ are } \overline{p}}\ \textsf{Antitone} \qquad \frac{\textsf{Some } p \textsf{ are } \overline{p}}{\varphi}\ \textsf{Ex falso quodlibet}$$

Logic: correct reasoning

- Logic: language + deductive system and/or model theoretic semantics
- language: formulas with syntax
- argument: premises + conclusion
- argument is *derivable* when there is a deduction from the premises to the conclusion
- argument is *valid* if there is no interpretation where all the premises are true and the conclusion is false
- an argument is valid only when it is derivable: **completeness**
- an argument is derivable only when it is valid: **soundness**

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Deduction

- Many systems possible, example: natural deduction: inference rules to introduce and eliminate the connectives and the quantifiers.