Sanchez Valencia's reconstruction of syllogistic inference

Lexicon

- common nouns CN
- intransitive verbs IV
- copula IS
- operator THAT
- DET ={EVERY, SOME, A, NO, NOT EVERY}

Syntax rules

- If A is a CN, then DET A is an NP
- If B is a NP, then IS A B is a VP
- If A is a IV, then A is a VP
- If A is a VP, then THING THAT A is a CN
- If A is a NP and B is a VP, then A B is a Sentence

NP's as generalized quantifiers

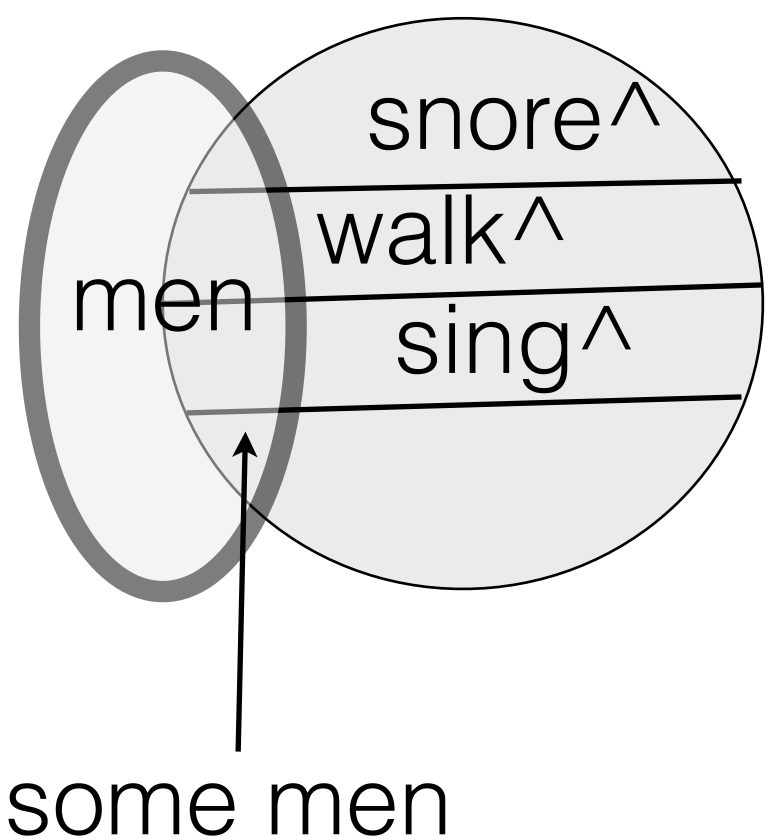
- Assume a set of individuals D_e and the classical set of truth values D_t
- CN will take their denotation in the set D_{et} of functions from D_e to D_t
- NP's take their denotations in the set $D_{(e,t)t}$ of functions from CN denotations to truth values.
- ⟦...⟧ are used for denotations.

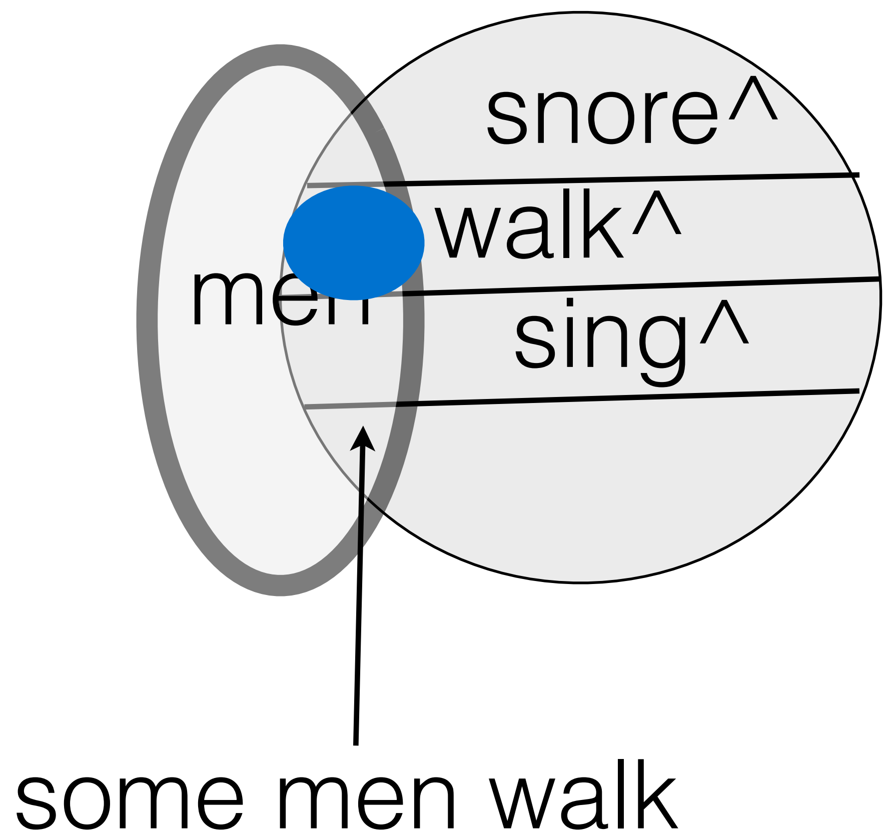
Sets and sets of sets

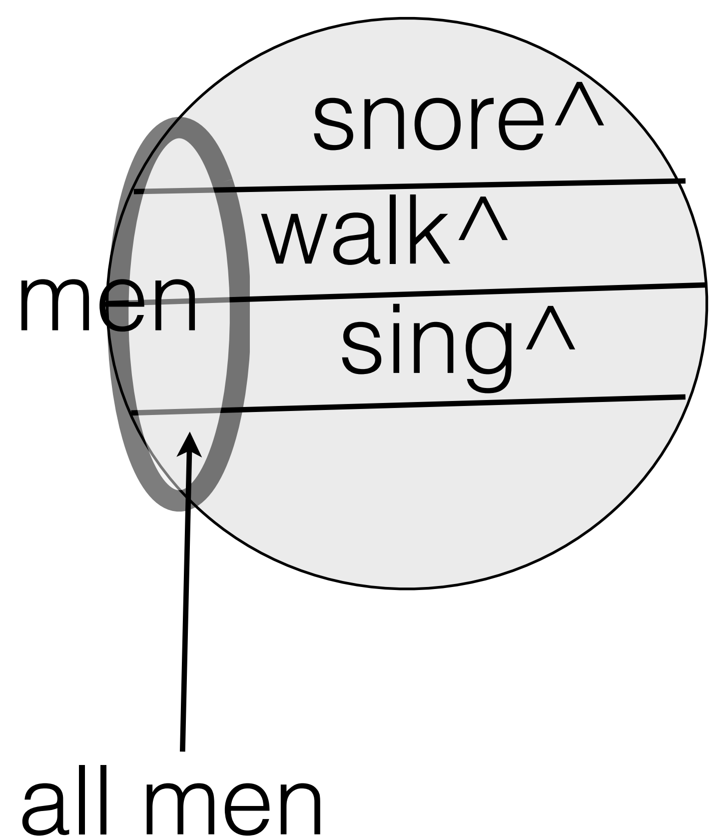
$D_t = \{0,1\}$
$D_e = \{j,m,b\}$
$D_e \times D_t$

All women:
{1,1,1}
{0,1,1}
{0,1,0}
{1,1,0}

{j,m,b}
{0,0,0}
{0,0,1}
{0,1,1}
{1,1,1}
{1,1,0} John and Mary sing
{1,0.0}
{0,1,0} Mary walks
{1,0,1}


snore^
walk^
men
sing^
some men


snore^
walk^
men
sing^
some men walk

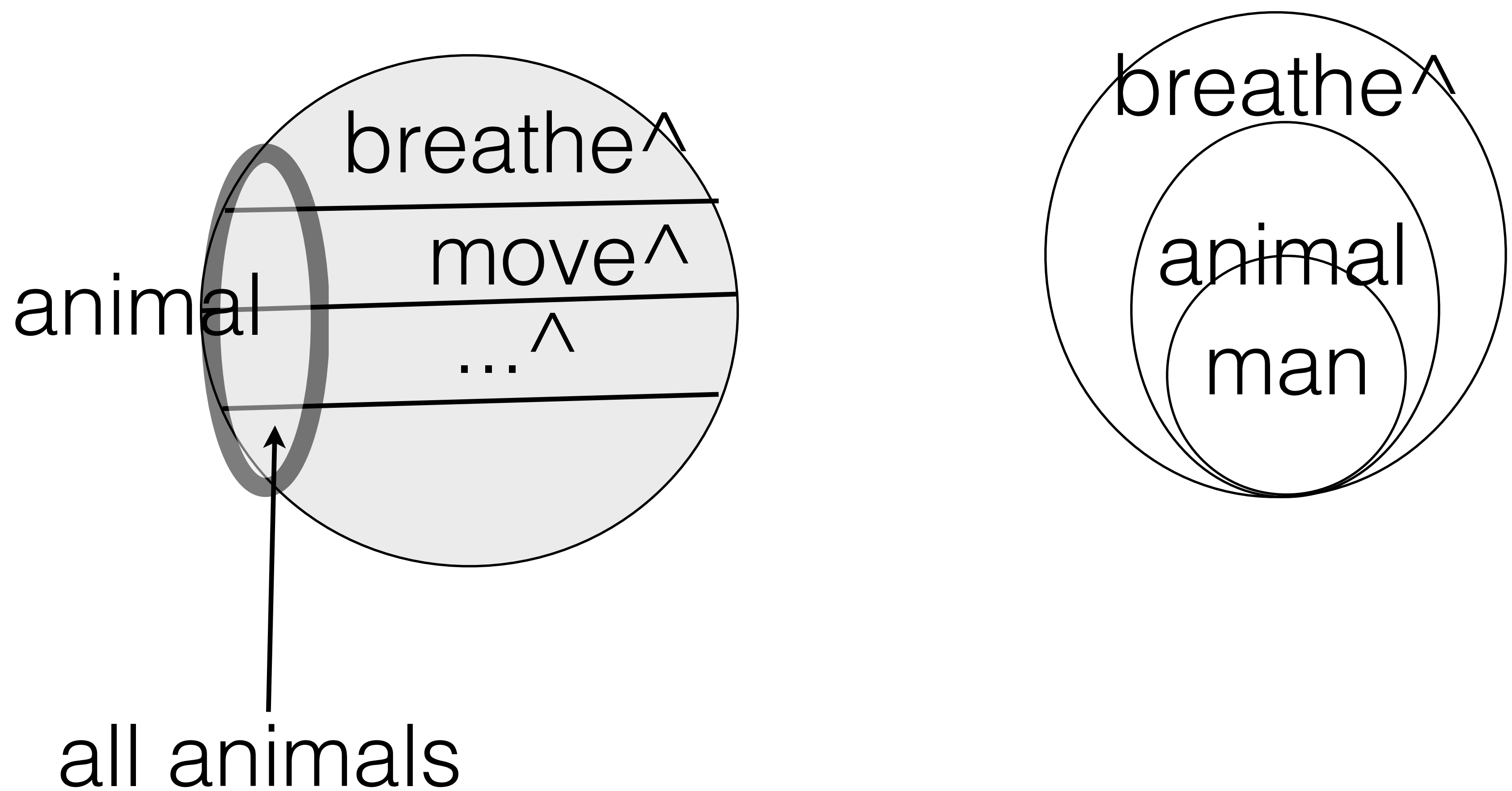

snore^
walk^
men
sing^
all men

Denotations

- if $B \in CN$ or $B \in IV$ then $[\![B]\!] \subseteq De,t$
- if $B \in CN$, then $[\![\text{IS A B}]\!] = [\![B]\!]$
- if $B \in CN$, then
  - $[\![\text{EVERY B}]\!] = \{X_{e,t} \mid [\![B]\!] \subseteq X\}$
  - $[\![\text{NO B}]\!] = \{X_{e,t} \mid [\![B]\!] \cap X = 0\}$
  - $[\![\text{NOT EVERY B}]\!] = \{X_{e,t} \mid [\![B]\!] \not\subseteq X\}$
  - $[\![\text{SOME B}]\!] = \{X_{e,t} \mid [\![B]\!] \cap X \neq 0\}$

Denotations

- if B ∈ VP, then ⟦THING THAT B⟧ = ⟦B⟧
- ⟦NP VP⟧ = 1 ⇔ ⟦VP⟧ ∈ ⟦NP⟧ example: every woman is a logician is true if ⟦logician⟧ ∈ ⟦every woman⟧
- ⟦NOT EVERY B⟧ = $\{X_{e,t} \mid X \notin$ ⟦EVERY B⟧$\}$ = D(e,t)t - ⟦EVERY B⟧
- ⟦NO B⟧ = $\{X_{e,t} \mid X \notin$ ⟦SOME B⟧$\}$ = D(e,t)t - ⟦SOME B⟧


breathe^
move^
...^
animal
all animals
breathe^
animal
man

Syllogisms and monotonicity

- Monotonicity: $x \in y$ is upward monotone in y, i.e. for all z with $y \subseteq z$, $x \in y$ entails $x \in z$.

- Structural: NP VP is monotone in NP: if ⟦NP VP⟧ denotes the truth, then ⟦NP VP⟧ corresponds to an expression of the form $x \in y$

- Lexical:

  - ⟦every B⟧ and ⟦some B⟧ are closed under supersets: if $x \in$ ⟦every B⟧ and $x \subseteq y$ implies $y \in$ ⟦every B⟧ **if every woman is a logician, every woman is a philosopher**

  - ⟦not every B⟧ and ⟦no B⟧ are closed under subsets: if $x \in$ ⟦not every B⟧ and $y \subseteq x$ implies $y \in$ ⟦not every B⟧ **if no woman is a philosopher, no woman is a logician**

Every A VP1 $[\![VP1]\!] \subseteq [\![VP2]\!]$
-- M1
Every A VP2

Some A VP1 $[\![VP1]\!] \subseteq [\![VP2]\!]$
-- M2
Some A VP2

No A VP1 $[\![VP2]\!] \subseteq [\![VP1]\!]$
-- M3
No A VP2

Not every A is a C $[\![B]\!] \subseteq [\![C]\!]$
-- M4
Not every A VP2

Every A VP1 ⟦VP1⟧⊆⟦VP2⟧
-------------------------------------- M1
Every A VP2
every is ↑ in its second argument

Some A VP1 ⟦VP1⟧⊆⟦VP2⟧
-------------------------------------- M2
Some A VP2
some is ↑ in its second argument

No A VP1 ⟦VP2⟧⊆⟦VP1⟧
-------------------------------------- M3
No A VP2
no is ↓ in its second argument

Not every A is a C ⟦B⟧⊆⟦C⟧
-------------------------------------- M4
Not every A VP2
not every is ↓ in its second argument

- Any positive sentence implies an inclusion relation
- Inference rules

$$\frac{\text{Every A is a B}}{[\![\text{EveryB}]\!] \subseteq [\![\text{EveryA}]\!]}\text{P1} \qquad \frac{\text{Every A is a B}}{[\![\text{Some A}]\!] \subseteq [\![\text{Some B}]\!]}\text{P2} \qquad \frac{\text{Some A is a B}}{[\![\text{Every A}]\!] \subseteq [\![\text{Some B}]\!]}\text{P3}$$

$$\frac{\text{Some A is a B}}{[\![\text{Every B}]\!] \subseteq [\![\text{Some A}]\!]}\text{P4} \qquad \frac{\text{Every A is a B}}{[\![\text{Not every A}]\!] \subseteq [\![\text{Not every B}]\!]}\text{P5} \qquad \frac{\text{Every A is a B}}{[\![\text{Not B}]\!] \subseteq [\![\text{No A}]\!]}\text{P6}$$

$$\frac{\text{Some A is a B}}{[\![\text{No B}]\!] \subseteq [\![\text{Not every A}]\!]}\text{P7} \qquad \frac{\text{Some A is a B}}{[\![\text{No A}]\!] \subseteq [\![\text{Not every B}]\!]}\text{P8} \qquad \frac{\text{Every A VP}}{[\![\text{is a A}]\!] \subseteq [\![\text{VP}]\!]}\text{P9}$$

- Any positive sentence implies an inclusion relation

- Inference rules

  Every A is a B

  -------------------------P1

  ⟦EveryB⟧⊆⟦EveryA⟧

  The denotation of every B is the set of properties that every B has.

  Every man is an animal

  the set of properties that every animal has are included in the set of properties that every man has.

  Some A is a B

  -------------------------P4

  ⟦Every B⟧⊆⟦Some A⟧

  Some animals are wild beasts

  the set of properties of all wild beasts are included in the set of properties that some animals have.

First figure syllogisms and monotonicity

Barbara:

Every M VP,
Every S is an M,
Every S VP

```
                         Every S is an M
                         -------------------- P1
Every M VP  ⟦Every M⟧⊆⟦ Every S⟧
--------------------------------------------- M
Every S VP
```

Darii:

Every M VP,
Some S is an M,
Some S VP

```
                         Some S is an M
                         -------------------- P4
Every M VP  ⟦Every M⟧⊆⟦ Every S⟧
--------------------------------------------- M
Every S VP
```

Second figure syllogisms and monotonicity

Camestres

Every P VP,
No S VP
No S is a P

```
                              Every P VP
                              -------------------P9
No M VP              ⟦is a P⟧⊆⟦ VP⟧
---------------------------------------------------------- M
No S VP
```

Baroco

Every P VP,
Not every S VP,
Not every S is a P

```
                                  EveryP VP
                                -------------------P9
Not every S VP         ⟦is a P⟧⊆⟦ VP⟧
---------------------------------------------------------- M
Not every S VP
```

Second figure syllogisms and monotonicity with conversion

Cesare
No P VP,
Every S VP
No S is a P

```
                        Every P VP
                        ------------------P9
No P VP      [[is a P]]⊆[[ VP]]
---------------------------------------------- M
No P is an S
--------------C2
No S is a P
```

Conversion rules

```
Some A VP                           No A VP
--------------------------------C1  --------------------------------C2
Some thing that VP is an A          No thing that VP is an A
```

Monotonic extensions of syllogistic logic

- Relative terms
  - Every horse is an animal →Every tail of a horse is a tail of an animal
  - Every man is an animal →He who kills a man kill an animal

De Morgan

- De Morgan's upward monotonicity

  Every X is Y   F(X)

  F(Y) provided some of the denotation of X is spoken of in F(X)

- De Morgan's downward monotonicity

  Every X is Y   F(Y)

  F(X) provided all of the denotation of Y is spoken of in F(Y)

De Morgan

- De Morgan's upward monotonicity

Every X is Y   F(X)

F(Y) provided some of the denotation of X is spoken of in F(X)

Every man is an animal. He who kills a man kills a man.

He who kills a man kills an animal

- De Morgan’s upward monotonicity
- Every X is Y   F(X)
- → F(Y) provided some of the denotation of X is spoken of in F(X)
- Every man is an animal. He who kills a man kills a man → He who kills a man kills an animal
- Every man is an animal. He who kills a man kills a man → He who kills an animal kills a man.
- No way to handle the difference between a relative clause and a main clause occurrence.

Leibniz

- Jungius (1587-1657) enumerated a number of non syllogistic valid inference patterns. E.g.
    - Salomon is the son of David --> David is the father of Salomon.
    - Bill is taller than John --> John is less tall than Bill
- Leibniz tried to tackle some of these, unfortunately not very successfully in spite of the fact he was one of the first philosophers to stress the importance of precise formulations.

Leibniz's view on the relative term problem

- L↑: given the sentence Every S is P, P is substitutable for S in any affirmative sentence in which S occurs as a predicate.

  Example: Every horse is an animal,
  the thing that is a horse,
  the thing that is an animal

- PR: if A is an expression with B as one of its non-subject terms, then B is equivalent to the complex expression 'thing that is A'. So if we find horse or man or animal in a non subject position we replace it by "thing that is a horse, a man, an animal'

Every tail of a horse is a tail of a horse

---PR

Every tail of a horse is the tail of a thing that is a horse. Every horse is an animal.

---L↑

Every tail of a horse is a tail of a thing that is an animal

--PR

Every tail of a horse is a tail of an animal

Every tail of a horse is a tail of a horse

--PR

Every tail of a thing that is a horse is the tail of a horse. Every horse is an animal.

---L↑

Every tail of a thing that is an animal is a tail of a horse

---PR

Every tail of an animal is a tail of horse

Back to the Middle Ages

An aside

- De Morgan’s laws in a medieval guise
- From the negation of a conjunctive proposition to the disjunction of the negation of its parts, and conversely.
- NOT (P AND Q) = (NOT P) OR (NOT Q)
- From the negation of a disjunctive proposition to the conjunction of the negation of its parts and conversely
- NOT (P OR Q) = (NOT P) AND (NOT Q)
- Kneale and Kneale p.294

- The main thing to take away from the slides that follow is that the medieval logicians had the notion of quantifier scope. specifically that existentials with wide and narrow scope were interpreted differently

- Also: it is assumed that the Latin word order indicates the scope.

- If you are interested in the suppositio theory, an approach that looks at it with a modern logico/linguistic perspective can be found at: http://www.humnet.ucla.edu/humnet/phil/faculty/tparsons/Medieval%20Book/contents.htm

Supposition

- A word supposits for = stands for
  - material supposition (word itself) **Man** is a noun
  - formal supposition
    - simple supposition (concept/form) **Man**(kind) is a species
    - personal supposition (indirect signification)
      - determinata: A **man** is running
      - confusa
        - confusa tantum: every man is an **animal**
        - confusa et distributiva: every **man** is an animal

- suppositio confusa: depends on number of individuals that a term could stand for: either universally quantified or existential in the scope of a universal

- suppositio determinata: stands for a single individual: existential with wide scope

- Every distributive sign (‘all’, ‘no’) gives *suppositio confusa et distributiva* to the term to which it is adjoined; a negative sign does the same for the remote term but an affirmative sign gives *suppositio confusa tantum* to the remote term,

- Inference rules: There is no valid inference from *supposition confusa tantum* to *suppositio confusa et distributiva.* But there is a valid inference from *suppositio confusa et distributiva* to *suppositio determinata*

  - No man is an ass --> no man is this ass.

  - *Every man is an animal --> every man is this animal.

  - Socrates does not see a man --> A man is not seen by Socrates

  - *Every man sees a man --> a man is seen by every man

- There is no valid inference from suppositio determinata to suppositio confusa et distributiva but only to suppositio confusa tantum
  - A man is seen by every man --> every man sees a man
  - *A man is not seen by Socrates --> Socrates does not see a man

Summarizing suppositio for the varous moods

- Mood
  - A = all S are P (S distributed, P not)
  - I = some S are P (S and P undistributed)
  - E = no S are P (S and P distributed)
  - O = some S are not P (P distributed S not distributed)
  - distributed: refers to everything it signifies

Scope of existentials

- Asinum omnis homo videt (An ass, every man sees): suppositio determinata
- Omnis homo videt asinum (Every man sees an ass): suppositio confusa tantum

  the first sentence implies the second but not vice versa

De Morgan

- De Morgan's upward monotonicity

  Every X is Y   F(X)

  F(Y) provided some of the denotation of X is spoken of in F(X)

Revising De Morgan in the light of medieval suppositio theory

- De Morgan's upward monotonicity

  Every X is Y   F(X)

  F(Y) provided some of the denotation of X is spoken of in F(X)

  Every X is Y   F(X)

  F(Y) provided X occurs in F(X) with suppositio non-distributiva

- De Morgan’s downward monotonicity

  Every X is Y   F(Y)

  F(X) provided all of the denotation of Y is spoken of in F(Y)

  Every X is Y   F(Y)

  F(X) provided Y occurs in F(Y) with supposition confusa et distributiva

Every horse is a animal. Every animal sees a man

--↓

Every horse sees a man

Every horse is a animal. A horse sees a man

--↑

An animal sees a man

And the tail of the horse?

- Suppositio was restricted to subjects and objects as a whole: in ‘tail of a horse’, ‘tail’ and ‘horse’ do not have any suppositio.
- No good derivations but also no false conclusions.
- But there is the insight that terms have to be marked depending on where they occur in the sentence.

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- suppositio determinata: stands for a single individual: existential with wide scope

- A man is seeing every horse
- Brunellus is a horse
- →A man is seeing Brunellus
- Every man is an animal
- Some one is a killer of a man
- →Some one is a killer of an animal

Consequentiae

- formal
  - antecedent: single categorical proposition
  - antecedent: complex categorical proposition
- material
  - good simply; in view of the special meanings of the terms (extra premises)
  - good as things are now; contingently valid

From any proposition implying formal contraction follows every other proposition as a formal consequence
From an impossible proposition there follows every other proposition as a material consequence good simple
From any proposition a necessary proposition follows good simple
From any false proposition follows every other proposition as material consequence good as things are now
Every true proposition follows from any other proposition as a material consequence good as things are now

Abelard: dici de omni et nullo

- If something A is predicated of something else B universally and a third thing C places the subject B under it universally, then the same thing C also places the predicate A under it with the same mode, namely universally. (If all B A and all C B ,then all C A)

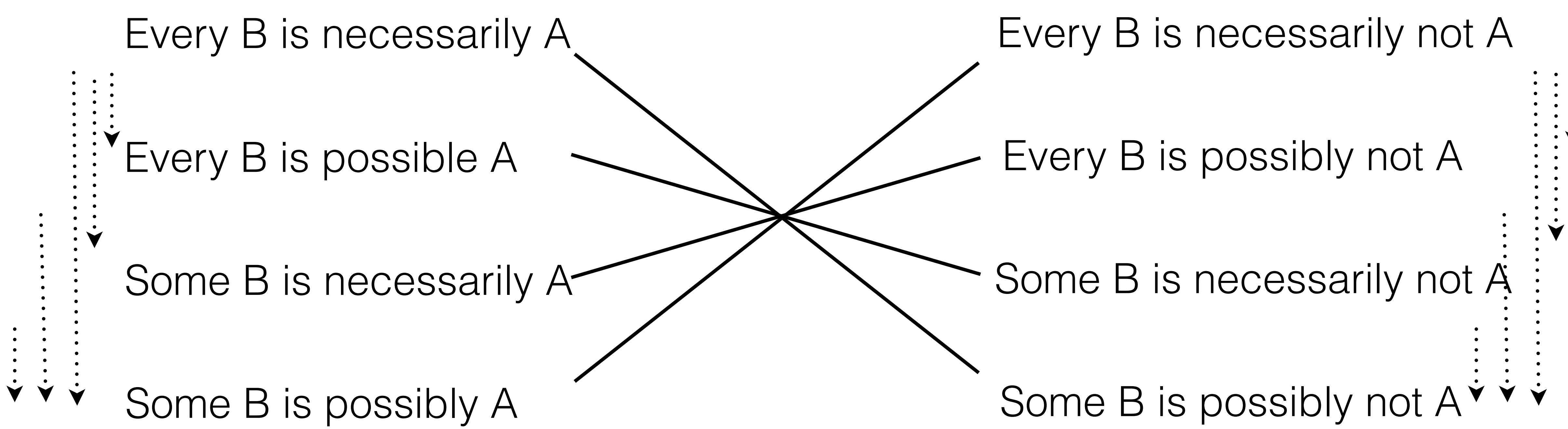
- If something A is removed from something else B universally and a third thing C places the subject B under it universally, then the first predicate A is removed from the second subject C universally. (If no B A and all C B, then no C A)

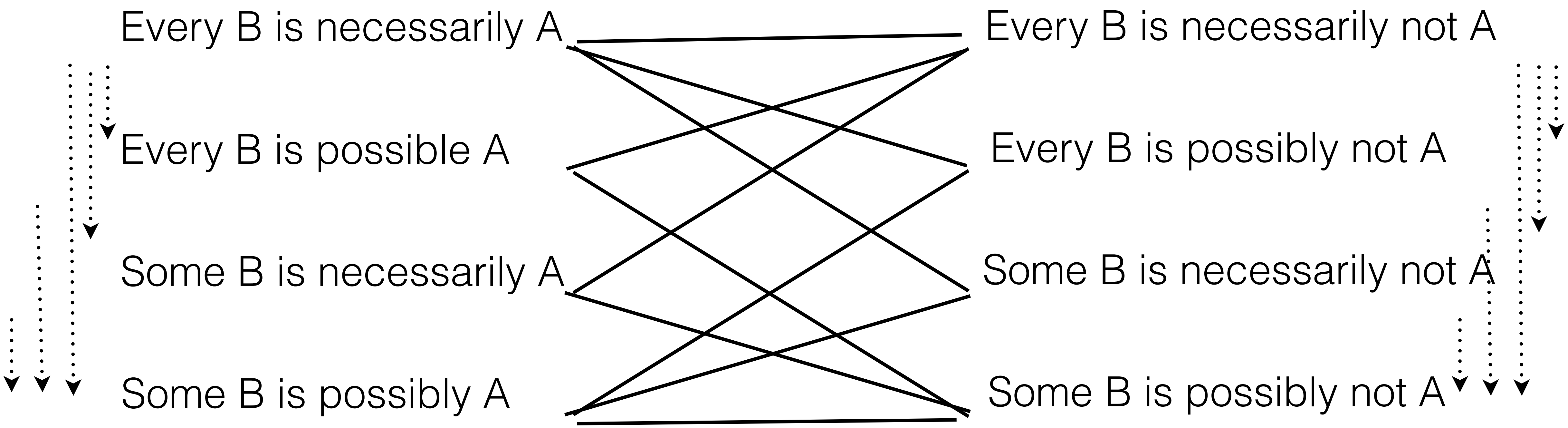
- (more rules for the other figures)

Medieval principles

- Dictum De Omni: whatever is true of every X is true of what is X
- A term is distributed in a sentence if the sentence is true about all of the predicate

- de re de dicto
- temporal interval logic
- modal logic





Every B is possibly A Some A is possibly B
Some B is possibly A = Some A is possibly B
Evert B is necessarily not A = Every A is necessarily not B
B is contingently A = B is contingently not A

Signification