Sanchez Valencia’s reconstruction of syllogistic inference
Lexicon

• common nouns CN

• intransitive verbs IV

• copula IS

• operator THAT

• DET = {EVERY, SOME, A, NO, NOT EVERY}
Syntax rules

• If A is a CN, then DET A is an NP
• If B is a NP, then IS A B is a VP
• If A is a IV, then A is a VP
• If A is a VP, then THING THAT A is a CN
• If A is a NP and B is a VP, then A B is a Sentence
NP’s as generalized quantifiers

- Assume a set of individuals $D_e$ and the classical set of truth values $D_t$
- CN will take their denotation in the set $D_{et}$ of functions from $D_e$ to $D_t$
- NP's take their denotations in the set $D_{(e,t)t}$ of functions from CN denotations to truth values.
- $\lbrack \ldots \rbrack$ are used for denotations.
Sets and sets of sets

$D_t = \{0, 1\}$

$D_e = \{j, m, b\}$

$D_e \times D_t$

All women:

- $\{1,1,1\}$
- $\{0,1,1\}$
- $\{0,1,0\}$
- $\{1,1,0\}$

John and Mary sing

- $\{1,1,0\}$

Mary walks

- $\{0,1,0\}$
- $\{1,0,1\}$
some men

men

sing
walk
snore
some men walk
snore
walk
sing

men

all men

all men
Denotations

• if \(B \in \text{CN}\) or \(B \in \text{IV}\) then \([B] \subseteq \text{De,t}\)

• if \(B \in \text{CN}\), then \([\text{IS A B}] = [B]\)

• if \(B \in \text{CN}\), then
  
  • \([\text{EVERY B}] = \{X_{e,t} | [B] \subseteq X\}\)
  
  • \([\text{NO B}] = \{X_{e,t} | [B] \cap X = 0\}\)
  
  • \([\text{NOT EVERY B}] = \{X_{e,t} | [B] \not\subseteq X\}\)
  
  • \([\text{SOME B}] = \{X_{e,t} | [B] \cap X \neq 0\}\)
Denotations

- if $B \in VP$, then $\llbracket \text{THING THAT } B \rrbracket = \llbracket B \rrbracket$

- $\llbracket NP \ VP \rrbracket = 1 \iff \llbracket VP \rrbracket \in \llbracket NP \rrbracket$ example: every woman is a logician is true if $\llbracket \text{logician} \rrbracket \in \llbracket \text{every woman} \rrbracket$

- $\llbracket \text{NOT EVERY } B \rrbracket = \{ X_{e,t} \mid X \notin \llbracket \text{EVERY } B \rrbracket \} = D(e,t)t - \llbracket \text{EVERY } B \rrbracket$

- $\llbracket \text{NO } B \rrbracket = \{ X_{e,t} \mid X \notin \llbracket \text{SOME } B \rrbracket \} = D(e,t)t - \llbracket \text{SOME } B \rrbracket$
breathe
move
...
all animals

animal

man

breathe
animal

all animals
Syllogisms and monotonicity

• Monotonicity: $x \in y$ is upward monotone in $y$, i.e. for all $z$ with $y \subseteq z$, $x \in y$ entails $x \in z$.

• Structural: NP VP is monotone in NP: if $[\text{NP VP}]$ denotes the truth, then $[\text{NP VP}]$ corresponds to an expression of the form $x \in y$

• Lexical:
  • $[\text{every B}]$ and $[\text{some B}]$ are closed under superset: if $x \in [\text{every B}]$ and $x \subseteq y$ implies $y \in [\text{every B}]$ if every woman is a logician, every woman is a philosopher
  • $[\text{not every B}]$ and $[\text{no B}]$ are closed under subset: if $x \in [\text{not every B}]$ and $y \subseteq x$ implies $y \in [\text{not every B}]$ if no woman is a philosopher, no woman is a logician
Every A VP1 [VP1]⊂[VP2]  
--------------------------------------- M1
  Every A VP2

Some A VP1 [VP1]⊂[VP2]  
--------------------------------------- M2
  Some A VP2

No A VP1 [VP2]⊂[VP1]  
--------------------------------------- M3
  No A VP2

Not every A is a C [B]⊂[C]  
--------------------------------------- M4
  Not every A VP2
Every A VP1 \([\text{VP1}] \subseteq [\text{VP2}]\)

--------------------------------------- M1

Every A VP2
every is ↑ in its second argument

Some A VP1 \([\text{VP1}] \subseteq [\text{VP2}]\)

--------------------------------------- M2

Some A VP2
some is ↑ in its second argument

No A VP1 \([\text{VP2}] \subseteq [\text{VP1}]\)

--------------------------------------- M3

No A VP2
no is ↓ in its second argument

Not every A is a C \([B] \subseteq [C]\)

----------------------------------------- M4

Not every A VP2
not every is ↓ in its second argument
• Any positive sentence implies an inclusion relation

• Inference rules

Every A is a B
----------------------------------P1
⟦EveryB⟧⊆⟦EveryA⟧

Every A is a B
----------------------------------P2
⟦Some A⟧⊆⟦Some B⟧

Some A is a B
----------------------------------P3
⟦Every A⟧⊆⟦Some B⟧

Some A is a B
----------------------------------P4
⟦Every B⟧⊆⟦Some A⟧

Every A is a B
----------------------------------P5
⟦Not every A⟧⊆⟦Not every B⟧

Every A is a B
----------------------------------P6
⟦Not B⟧⊆⟦No A⟧

Some A is a B
----------------------------------P7
⟦No B⟧⊆⟦Not every A⟧

Some A is a B
----------------------------------P8
⟦No A⟧⊆⟦Not every B⟧

Every A VP
----------------------------------P9
⟦is a A⟧⊆⟦VP⟧
• Any positive sentence implies an inclusion relation

• Inference rules

Every A is a B
-------------------------P1
⟦EveryB⟧ ⊆ ⟦EveryA⟧
The denotation of every B is the set of properties that every B has.
Every man is an animal
the set of properties that every animal has are included in the set of properties that every man has.

Some A is a B
-------------------------P4
⟦Every B⟧ ⊆ ⟦Some A⟧
Some animals are wild beasts
the set of properties of all wild beasts are included in the set of properties that some animals have.
First figure syllogisms and monotonicity

Barbara:
Every M VP,
Every S is an M,
Every S VP

Every S is an M

P1

Even M VP \[\text{[Every M]} \subseteq [\text{Every S}]\]
Every S VP

M

Darii:
Every M VP,
Some S is an M,
Some S VP

Some S is an M

P4

Every M VP \[\text{[Every M]} \subseteq [\text{Every S}]\]
Every S VP

M
Second figure syllogisms and monotonicity

Camестres
Every P VP,
No S VP
No S is a P

\[ \text{Every P VP} \quad \frac{\text{No M VP}}{\text{Not } \text{is } \text{a P}} \quad \frac{\text{No S VP}}{\text{ Barsco }} \]

\[ \text{Not every S VP,} \quad \text{Not every S is a P} \]

\[ \text{Baroco} \quad \text{Every P VP} \quad \frac{\text{Not every S VP}}{\text{Not every S is a P}} \]

\[ \text{Every P VP} \quad \frac{\text{Not every S VP}}{\text{Not every S is a P}} \]
Second figure syllogisms and monotonicity with conversion

Cesare

No P VP,
Every S VP
No S is a P

Every P VP
No P VP
[ is a P ] ⊆ [ VP ]
No P is an S
No S is a P

Conversion rules

Some A VP
No A VP
-------------------C1
Some thing that VP is an A
No thing that VP is an A
-------------------C2
Monotonic extensions of syllogistic logic
• Relative terms

• Every horse is an animal $\rightarrow$ Every tail of a horse is a tail of an animal

• Every man is an animal $\rightarrow$ He who kills a man kills an animal
De Morgan

- De Morgan’s upward monotonicity

Every $X$ is $Y$ $F(X)$

$F(Y)$ provided some of the denotation of $X$ is spoken of in $F(X)$
• De Morgan’s downward monotonicity

Every X is Y  \( F(Y) \)

------------------------------------------

\( F(X) \) provided all of the denotation of Y is spoken of in \( F(Y) \)
De Morgan

• De Morgan’s upward monotonicity

Every X is Y      F(X)
-----------------------------------
F(Y) provided some of the denotation of X
is spoken of in F(X)

Every man is an animal. He who kills a man kills a man.
-------------------------------------------------------------------------
He who kills a man kills an animal
• De Morgan’s upward monotonicity

• Every X is Y  

• → F(Y) provided some of the denotation of X is spoken of in F(X)

• Every man is an animal. He who kills a man kills a man → He who kills a man kills an animal

• Every man is an animal. He who kills a man kills a man → He who kills an animal kills a man.

• No way to handle the difference between a relative clause and a main clause occurrence.
Leibniz

- Jungius (1587-1657) enumerated a number of non-syllogistic valid inference patterns. E.g.
  - Salomon is the son of David --> David is the father of Salomon.
  - Bill is taller than John --> John is less tall than Bill

- Leibniz tried to tackle some of these, unfortunately not very successfully in spite of the fact he was one of the first philosophers to stress the importance of precise formulations.
Leibniz’s view on the relative term problem

- L↑: given the sentence Every S is P, P is substitutable for S in any affirmative sentence in which S occurs as a predicate.

  Example: Every horse is an animal,
  the thing that is a horse,
  the thing that is an animal

- PR: if A is an expression with B as one of its non-subject terms, then B is equivalent to the complex expression ‘thing that is A’. So if we find horse or man or animal in a non subject position we replace it by “thing that is a horse, a man, an animal”
Every tail of a horse is a tail of a horse
-------------------------------------------------------------------------PR
Every tail of a horse is the tail of a thing that is a horse. Every horse is an animal.
-----------------------------------------------------------------------------------------------L↑
Every tail of a horse is a tail of a thing that is an animal
-----------------------------------------------------------------------------------------------PR
Every tail of a horse is a tail of an animal
Every tail of a horse is a tail of a horse
--------------------------------------------------PR
Every tail of a thing that is a horse is the tail of a horse. Every horse is an animal.
-------------------------------------------------------------------------PR
Every tail of a thing that is an animal is a tail of a horse
-------------------------------------------------------------------------PR
Every tail of an animal is a tail of horse
Back to the Middle Ages
An aside

- De Morgan’s laws in a medieval guise
- From the negation of a conjunctive proposition to the disjunction of the negation of its parts, and conversely.
- \( \text{NOT} (P \text{ AND } Q) = (\text{NOT } P) \text{ OR } (\text{NOT } Q) \)
- From the negation of a disjunctive proposition to the conjunction of the negation of its parts and conversely
- \( \text{NOT} (P \text{ OR } Q) = (\text{NOT } P) \text{ AND } (\text{NOT } Q) \)
- Kneale and Kneale p.294
• The main thing to take away from the slides that follow is that the medieval logicians had the notion of quantifier scope. Specifically that existentials with wide and narrow scope were interpreted differently.

• Also: it is assumed that the Latin word order indicates the scope.

• If you are interested in the suppositio theory, an approach that looks at it with a modern logico/linguistic perspective can be found at: http://www.humnet.ucla.edu/humnet/phil/faculty/tparsons/Medieval%20Book/contents.htm
Supposition

• A word supposits for = stands for
  • material supposition (word itself) \textit{Man} is a noun
  • formal supposition
    • simple supposition (concept/form) \textit{Man} (kind) is a species
  • personal supposition (indirect signification)
    • determinata: A \textit{man} is running
    • confusa
      • confusa tantum: every man is an \textit{animal}
      • confusa et distributiva: every \textit{man} is an animal
• suppositio confusa: depends on number of individuals that a term could stand for: either universally quantified or existential in the scope of a universal

• suppositio determinata: stands for a single individual: existential with wide scope
• Every distributive sign (‘all’, ‘no’) gives *suppositio confusa et distributiva* to the term to which it is adjoined; a negative sign does the same for the remote term but an affirmative sign gives *suppositio confusa tantum* to the remote term,

• Inference rules: There is no valid inference from *supposition confusa tantum* to *suppositio confusa et distributiva*. But there is a valid inference from *suppositio confusa et distributiva* to *suppositio determinata*

• No man is an ass --> no man is this ass.

• *Every man is an animal --> every man is this animal.*

• Socrates does not see a man --> A man is not seen by Socrates

• *Every man sees a man --> a man is seen by every man*
There is no valid inference from suppositio determinata to suppositio confusa et distributiva but only to suppositio confusa tantum

• A man is seen by every man --> every man sees a man

• *A man is not seen by Socrates --> Socrates does not see a man
Summarizing supposition for the various moods

- **Mood**
  - A = all S are P (S distributed, P not)
  - I = some S are P (S and P undistributed)
  - E = no S are P (S and P distributed)
  - O = some S are not P (P distributed S not distributed)

- distributed: refers to everything it signifies
Scope of existentials

- Asinum omnis homo videt (An ass, every man sees): suppositio determinata

- Omnis homo videt asinum (Every man sees an ass): suppositio confusa tantum

  the first sentence implies the second but not vice versa
De Morgan

• De Morgan’s upward monotonicity

Every X is Y \quad F(X)
\hline
F(Y) provided some of the denotation of X is spoken of in F(X)
Revising De Morgan in the light of medieval suppositio theory

• De Morgan’s upward monotonicity

Every X is Y      F(X)
--------------------------
F(Y) provided some of the denotation of X is spoken of in F(X)

Every X is Y      F(X)
--------------------------
F(Y) provided X occurs in F(X) with suppositio non-distributiva
• De Morgan’s downward monotonicity

Every X is Y  F(Y)
------------------------------------------------
F(X) provided all of the denotation of Y is spoken of in F(Y)

Every X is Y  F(Y)
------------------------------------------------
F(X) provided Y occurs in F(Y) with supposition confusa et distributiva
Every horse is a animal. Every animal sees a man
-------------------------------------------------------------------
   Every horse sees a man

Every horse is a animal. A horse sees a man
-------------------------------------------------------------------
   An animal sees a man
And the tail of the horse?

• Suppositio was restricted to subjects and objects as a whole: in ‘tail of a horse’, ‘tail’ and ‘horse’ do not have any suppositio.

• No good derivations but also no false conclusions.

• But there is the insight that terms have to be marked depending on where they occur in the sentence.
Supposition

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    • simple supposition (concept/form) $\text{Man(\text{kind})}$ is a species
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    • determinata: A $\text{man}$ is running
    • confusa
      • confusa tantum: every man is an $\text{animal}$
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• suppositio confusa: depends on number of individuals that a term could stand for: either universally quantified or existential in the scope of a universal

• suppositio determinata: stands for a single individual: existential with wide scope
• A man is seeing every horse
• Brunellus is a horse
• → A man is seeing Brunellus
• Every man is an animal
• Some one is a killer of a man
• → Some one is a killer of an animal
Consequentiae

From any proposition implying formal contraction follows every other proposition as a formal consequence.
From an impossible proposition there follows every other proposition as a material consequence good simple.
From any proposition a necessary proposition follows good simple.
From any false proposition follows every other proposition as material consequence good as things are now.
Every true proposition follows from any other proposition as a material consequence good as things are now.
Abelard: dici de omni et nullo

- If something A is predicated of something else B universally and a third thing C places the subject B under it universally, then the same thing C also places the predicate A under it with the same mode, namely universally. (If all B A and all C B, then all C A)

- If something A is removed from something else B universally and a third thing C places the subject B under it universally, then the first predicate A is removed from the second subject C universally. (If no B A and all C B, then no C A)

- (more rules for the other figures)
Medieval principles

- Dictum De Omni: whatever is true of every X is true of what is X
- A term is distributed in a sentence if the sentence is true about all of the predicate
- de re de dicto
- temporal interval logic
- modal logic

Every B is necessarily A
Every B is necessarily not A
Every B is possible A
Every B is possibly not A
Some B is necessarily A
Some B is necessarily not A
Some B is possibly A
Some B is possibly not A
Every B is necessarily A

Every B is necessarily not A

Every B is possible A

Every B is possibly not A

Some B is necessarily A

Some B is necessarily not A

Some B is possibly A

Some B is possibly not A

Every B is possibly A  Some A is possibly B

Some B is possibly A = Some A is possibly B

Evert B is necessarily not A = Every A is necessarily not B

B is contingently A = B is contingently not A
Signification

- Concept
  - Form
    - Word
      - Indirectly signifies
    - Is of
      - Is shared by
    - Individual

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