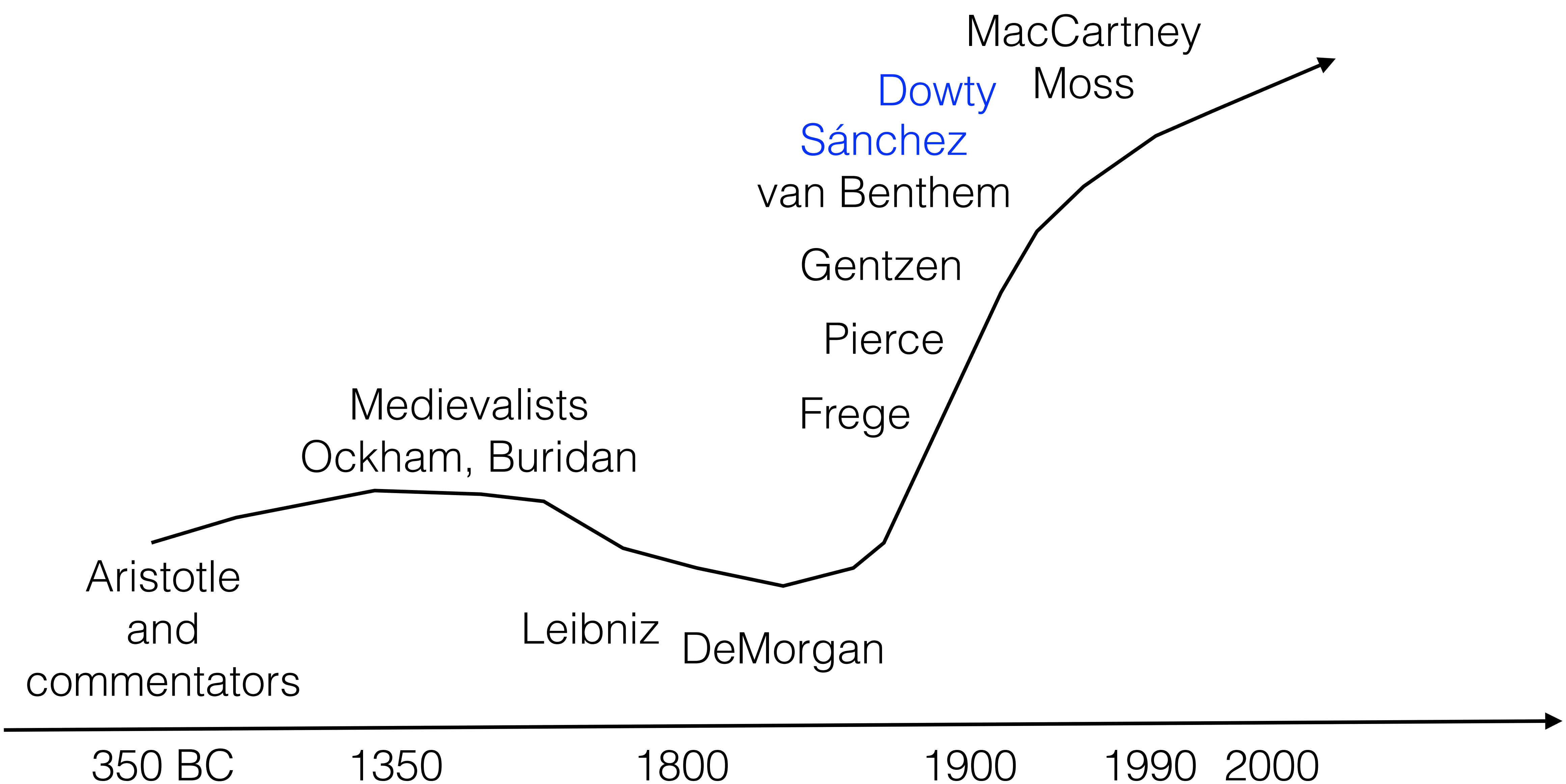
A Brief History of Natural Logic



Natural logic: Sánchez

- Typed lambda calculus
  - Types
  - Lambek terms
  - Lambek Grammar
- Semantics
  - Fregean Universe
  - Ordering of denotations
  - Model
- Monotonicity
- Polarity

Model

A typed model is a pair <D_α, I>, where

(i) $\{D_\alpha\}$ is a Fregean universe.

(ii) I is a function on the set of all constants such as $I(C_\alpha) \in D_\alpha$

Assignments

(i) An assignment is a function f on the set of variables such that $f(X_\alpha) \in D_\alpha$.

(ii) If Y is any variable, f an assignment, then [a/Y]f is the assignment that assigns a to Y.

Denotation Function

The denotation of an expression M of type α with regard to a model $\mathfrak{A}$ and an assignment f, $[\![M]\!]_f$, is defined as follows. $[\![M]\!]_f$ always belongs to D_α.

$[\![M]\!]_f$ is given by the following recursion:

(i) $[\![M]\!]_f = f(M)$ when M is a variable.

(ii) $[\![M]\!]_f = I(M)$ when M is a constant.

(iii) $[\![MN]\!]_f = [\![M]\!]_f ([\![N]\!]_f)$, when M has type $\alpha \rightarrow \beta$ and N has type α.

(iv) When M is of type β and X is of type α then $[\![\lambda X. M]\!]_f$ is that funcion in D such that for all $a \in D_\alpha$: $[\![\lambda X. M]\!]_f(a) = [\![M]\!]_{[a/X]f}$

Monotone Functions

A function $z \in D_{\alpha \to \beta}$ is upward monotone iff for every $x, y \in D_\alpha$, $x \leq_\alpha y$ entails $z(x) \leq_\beta z(y)$

A function $z \in D_{\alpha \to \beta}$ is downward monotone iff for every $x, y \in D_\alpha$, $x \leq_\alpha y$ entails $z(y) \leq_\beta z(x)$

Give some examples!

Monotone Terms

Assume that N'_α is like N_α except for containing an occurrence of M'_β ***whenever*** N_α contains M_β.

N_α is upward monotone in M_β iff for all models and assignments $[\![M]\!]_f \leq_\beta [\![M']\!]_f$ entails $[\![N]\!]_f \leq_\alpha [\![N']\!]_f$.

N_α is downward monotone in M_β iff for all models and assignments $[\![M']\!]_f \leq_\beta [\![M]\!]_f$ entails $[\![N]\!]_f \leq_\alpha [\![N']\!]_f$.

Give some examples!

Polarity of Occurrences

Assume that a language contains constants denoting monotone functions. A specific occurrence of $\mathbf{M}_\beta$ of M_β is called positive (negative) according to the following rules.

(i) **M** is positive in **M**.

(ii) **M** is positive (negative) in PQ if **M** is positive (negative) in P.

(iii) **M** is positive (negative) in PQ if **M** is positive (negative) in Q and P denotes an upward monotone function.

(iv) **M** is negative (positive) in PQ if **M** is positive (negative) in Q and P denotes a downward monotone function.

(v) **M** is positive (negative) in $\lambda X.P$ if **M** is positive (negative) in P and $X \notin FV(\mathbf{M})$.

Polarity $\Vdash$ Monotonicity

A term N is positive (negative) in M iff all the occurrences of N in M are positive (negative).

If N_β is positive (negative) in M_α, then N_β is upward (downward) monotone in M_α.

A Typed Language

Constants	Type
ABELARD, HELOISE	e
LOGICIAN, THEOREM, THING, MAN, MEN, HEAD	p
WANDER, WALK, RUN	e→t
NOT	t→t
FEMALE, MALE, TALL, SMALL	p→p

Constants	Type
PROVE, LOVE, IS	e→e→t *
EVERY	p→(e→t)→t
THAT	(e→t)→(p→p)
OF	((e→t)→t)→p→p
IN, AT, ON, WITH, WITHOUT	((e→t)→t)→(e→t)→(e→t)

* Convention:

e→e→t is an abbreviation for (e→(e→t)).

p→(e→t)→t is the same as p→((e→t)→t) and (p→((e→t)→t))

The outermost parentheses can be left out.

Interpretation of Constants

(1) I(EVERY), I(A), I(NO), I(FEW), I(THE) are those functions in $D_{p \rightarrow (e \rightarrow t) \rightarrow t}$ such that for any $x \in D_p$:

(a) I(EVERY)(x)(y) = 1 iff $x \subseteq y$.

(b) I(A)(x)(y) = 1 iff $x \cap y \neq 0$.

(c) I(NO)(x)(y) = 1 iff $x \cap y = 0$.

(d) I(MOST)(x)(y) = 1 iff $|x \cap y| > |x - y|$.

(e) I(FEW)(x)(y) = 1 iff $|x \cap y| < |x - y|$.

(f) I(THE)(x)(y) = 1 iff $|x| = 1$ and $x \subseteq y$.

(2)

(a) I(THING) = D_p.

(b) I(IS) is that function on $D_{e \to (e \to t)}$ such that for x,y ∈ D_e, I(IS)(x)(y) = 1 iff x = y.

(c) I(THAT) is that function in $D_{(e \to t) \to (p \to p)}$ such that for any x ∈ $D_{e \to t}$, y ∈ D_p I(THAT)(x)(y).

(d) I(NOT) is that function in $D_{t \to t}$ such that for any x in D_t, I(NOT)(x) = 1-x.

(3)

(a) I(FEMALE), I(MALE) are restrictive upward functions on $D_{(e \to t) \to (p \to p)}$.

(b) I(TALL), I(SMALL) are restrictive functions on $D_{(e \to t) \to (p \to p)}$.

Restrictive vs. Upward

Restrictive functions:

I(FEMALE) and I(SMALL) are both restrictive (intersective) functions because female bears and small bears are all bears.

Upward (monotonic) functions:

I(FEMALE) is an upward monotonic function because all female bears are female animals.
I(SMALL) is not an upward function because all small bears need not be small animals.

$$I(\text{BEAR}) \leq_{e\rightarrow t} I(\text{ANIMAL})$$

$$I(\text{FEMALE})(I(\text{BEAR})) \leq_{e\rightarrow t} I(\text{FEMALE})(I(\text{ANIMAL}))$$

(4)

(a) I(IN), I(AT), I(WITH), I(OF) are upward functions on $D_{((e\to t)\to t)\to((e\to t)\to(e\to t))}$ such that I(IN)(x), I(AT)(x), I(WITH)(x) are upward restrictive functions and I(OF)(x) is an upward function.

(5)

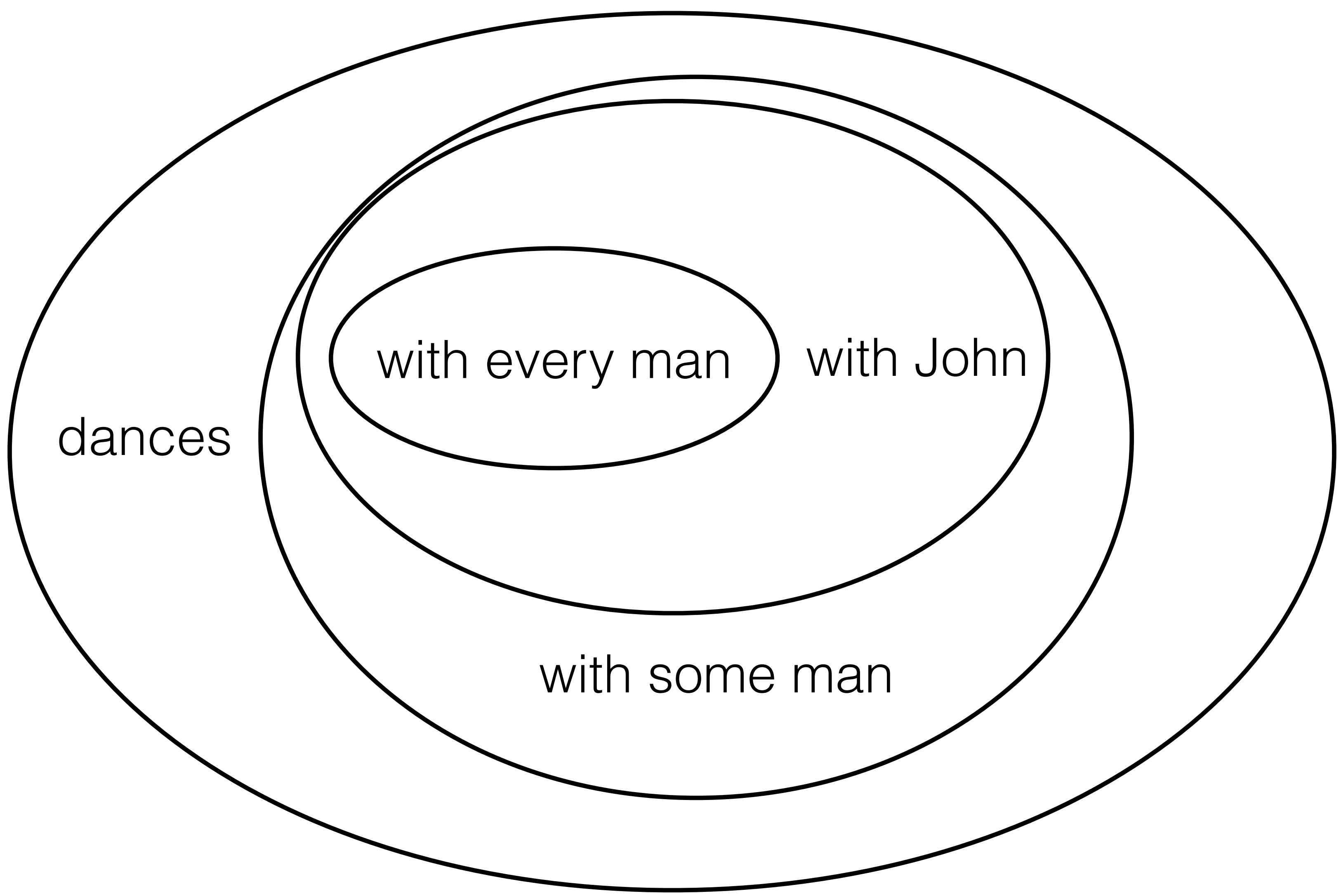
If A_α is a constant not mentioned in (1)-(4) then I(A) is simply a member of D_α.

Question: Expressions of type $(e\to t)\to t$ are NPs such as ***every man***, ***some man***, and *john*$_{NP}$. How are they ordered by $\leq_{(e\to t)\to t}$?

$$[\![\text{every man}]\!] \leq_{(e \to t) \to t} [\![\text{john}]\!] \leq_{(e \to t) \to t} [\![\text{some man}]\!]$$

$$[\![\text{with every man}]\!] \leq_{(e \to t) \to (e \to t)} [\![\text{with john}]\!] \leq_{(e \to t) \to (e \to t)} [\![\text{with some man}]\!]$$

$$[\![\text{dances with every man}]\!] \leq_{e \to t} [\![\text{dances with john}]\!] \leq_{e \to t} [\![\text{dances with some man}]\!]$$



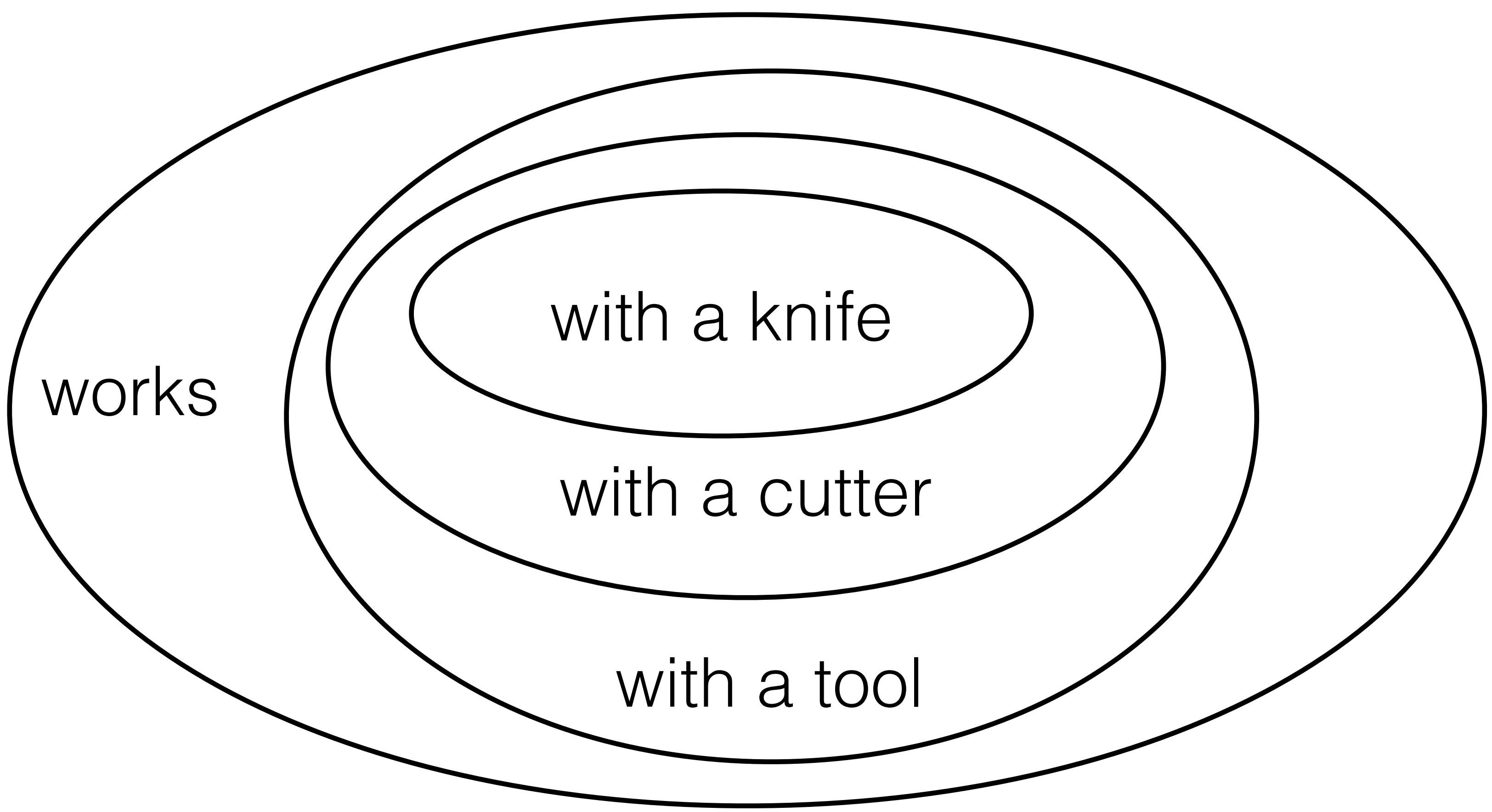
A look ahead

$[\![\text{knife}]\!] \leq_{(e\to t)\to t} [\![\text{cutter}]\!] \leq_{(e\to t)\to t} [\![\text{tool}]\!]$

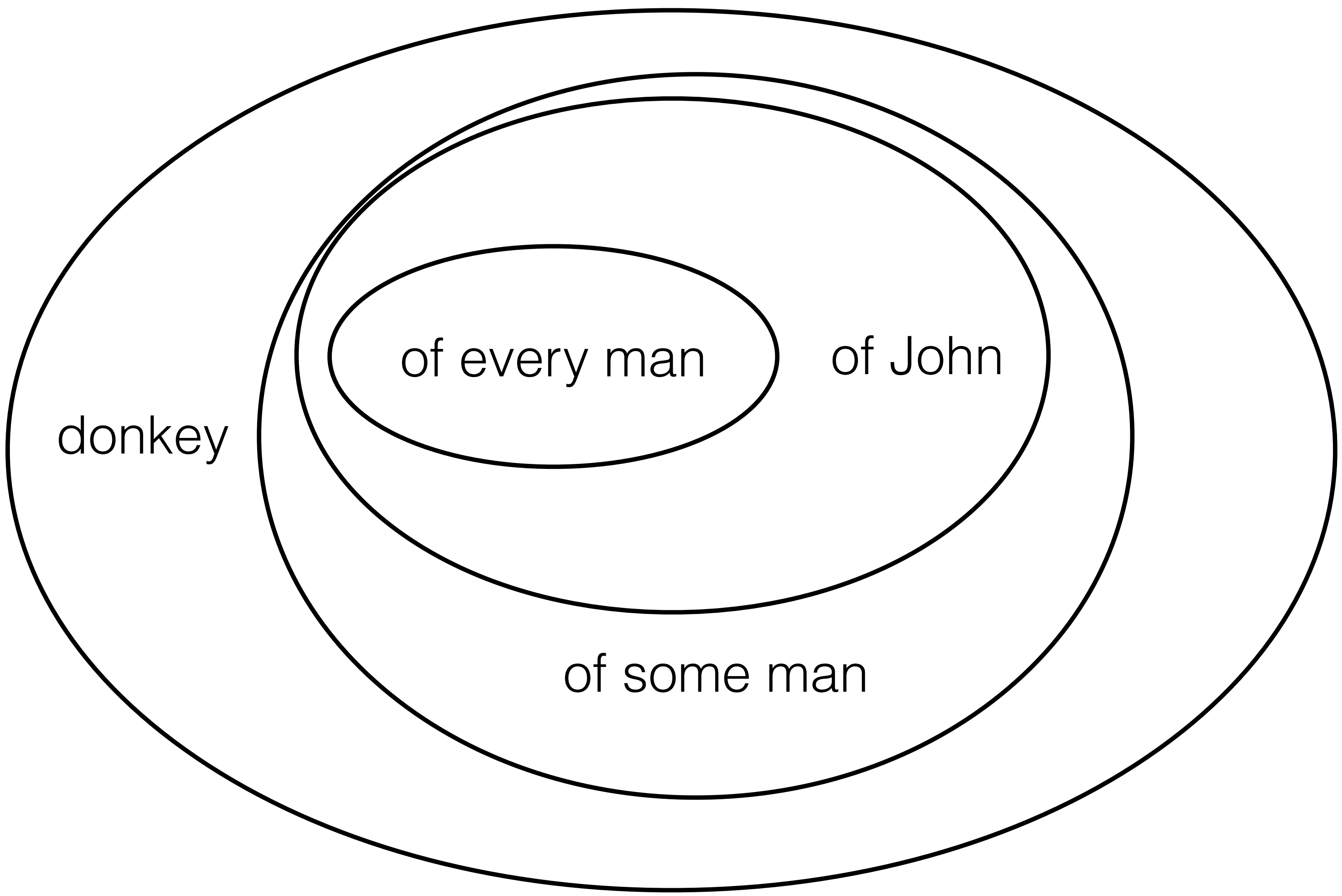
$[\![\text{a knife}]\!] \leq_{(e\to t)\to t} [\![\text{a cutter}]\!] \leq_{(e\to t)\to t} [\![\text{a tool}]\!]$

$[\![\text{with a knife}]\!] \leq_{(e\to t)\to t} [\![\text{with a cutter}]\!] \leq_{(e\to t)\to t} [\![\text{with a tool}]\!]$

$[\![\text{works with a knife}]\!] \leq_{(e\to t)\to t} [\![\text{works with a cutter}]\!] \leq_{(e\to t)\to t} [\![\text{works with a tool}]\!]$



Why is I(OF)(x) not restrictive?



(4)

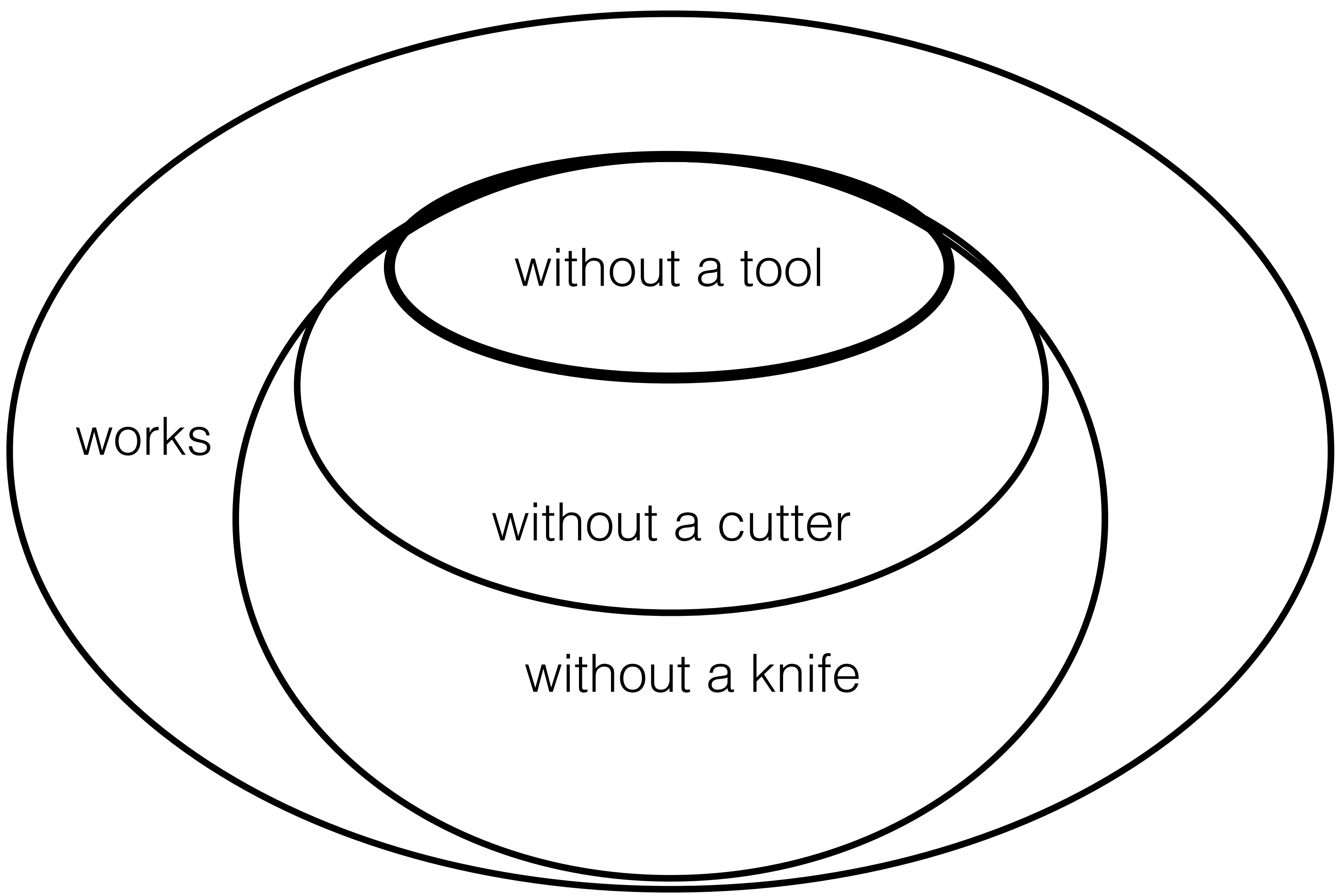
(b) I(WITHOUT) is a downward function in $D_{((e\to t)\to t)\to((e\to t)\to(e\to t))}$ such that I(WITHOUT)(x) is an upward restrictive function.

$[\![\text{knife}]\!] \leq_{(e\to t)\to t} [\![\text{cutter}]\!] \leq_{(e\to t)\to t} [\![\text{tool}]\!]$

$[\![\text{a knife}]\!] \leq_{(e\to t)\to t} [\![\text{a cutter}]\!] \leq_{(e\to t)\to t} [\![\text{a tool}]\!]$

$[\![\text{with a knife}]\!] \leq_{(e\to t)\to(e\to t)} [\![\text{with a cutter}]\!] \leq_{(e\to t)\to(e\to t)} [\![\text{with a tool}]\!]$

$[\![\text{without a tool}]\!] \leq_{(e\to t)\to(e\to t)} [\![\text{without a cutter}]\!] \leq_{(e\to t)\to(e\to t)} [\![\text{without a knife}]\!]$



Sanchez' Natural Logic

Three components:

(1) Lexical monotonicity marking of functors

(2) External polarity marking of the nodes of a derivation.

(3) Computation of the final node polarities that may reverse the initial polarity assignments.

Phrases that end up with a + are upward monotone, phrases with a - are downward monotone, unmarked phrases are neither.

Comment:

It is a complex system and a bit confusing because the + and - signs are used for marking monotonicity in step (1) and for marking polarity in steps (2) and (3).
David Dowty's paper proposes an incrementally better simpler version.
Larry Moss' work is a recasting of the whole enterprise.

Lexical Monotonicity

The main idea: introduce notation that allows the constraints on the interpretation and the assignments in the model to be expressed in the derivations of Lambek Grammars.

In addition to the unmarked functors of type $\alpha \to \beta$, there are two new types: $\alpha^{+} \to \beta$ and $\alpha^{-} \to \beta$.

If A is an expression assigned to the category $\alpha^{+} \to \beta$, then A denotes an upward monotone function in $D_{\alpha \to \beta}$.

If A is an expression assigned to the category $\alpha^{-} \to \beta$, then A denotes an downward monotone function in $D_{\alpha \to \beta}$.

Examples

Constants	Type
EVERY, ALL	$p^{-} \to ((e \to t)^{+} \to t)$
SOME, A	$p^{+} \to ((e \to t)^{+} \to t)$
NO	$p^{-} \to ((e \to t)^{-} \to t)$
NOT	$t^{-} \to t$
MOST	$p \to ((e \to t)^{+} \to t)$

Constants	Type
FEW	$p \to ((e \to t)^- \to t)$
THE	$p \to ((e \to t)^+ \to t)$
FEMALE, MALE	$p^+ \to p$
IN, AT, WITH	$((e \to t) \to t)^+ \to (e \to t)^+ \to (e \to t)$
WITHOUT	$((e \to t) \to t)^- \to (e \to t)^+ \to (e \to t)$
OF	$((e \to t) \to t)^+ \to p^+ \to p$
THAT	$(e \to t)^+ \to (p^+ \to p)$

Polarity Marking in Derivations

The major premiss in a Modus Ponens application (Elimination) is positive.

$$\frac{\alpha \underset{+}{\to} \beta \qquad \alpha}{\beta}$$

The minor premiss in a Modus Ponens application is positive if the major is in the category $\alpha^{+}\to\beta$.

$$\frac{\alpha^{+} \underset{+}{\to} \beta \qquad \underset{+}{\alpha}}{\beta}$$

The minor premiss in a Modus Ponens application is negative if the major is in the category $\alpha^{-}\to\beta$.

$$\frac{\alpha^{-} \underset{+}{\to} \beta \qquad \underset{-}{\alpha}}{\beta}$$

The discharching of a numerical assumption leaves the previous marking unchanged. This will be indicated by putting a + symbol below the last but one node.

$$
\begin{array}{c}
[\alpha^{i}] \\
D \\
\beta \\
+ \\
\end{array}
$$

$$
(i)\ \dfrac{\beta}{\alpha \to \beta} \to I
$$

Final Polarities

Assume that D is a derivation with conclusion α.

(i) A node γ has polarity iff all the nodes in the path from α are marked.

(ii) A node γ is positive if (a) γ has polarity and (b) the number of nodes marked by '-' is even.

(iii) A node γ is negative if (a) γ has polarity and (b) the number of nodes marked by '-' is odd.

Example

With lexical monotonicity markings

every $p^{-} \to ((e \to t)^{+} \to t)$ logician p

$(e \to t)^{+} \to t$ wanders $e \to t$

---

t

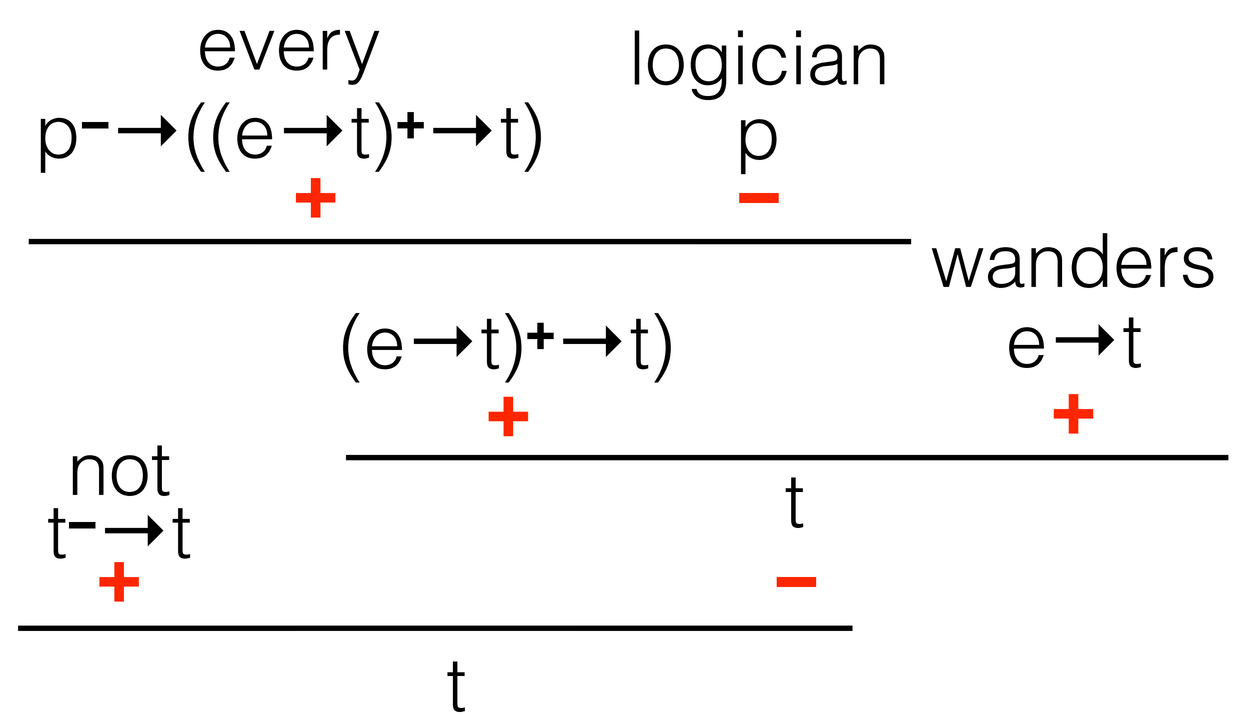
not $t^{-} \to t$

---

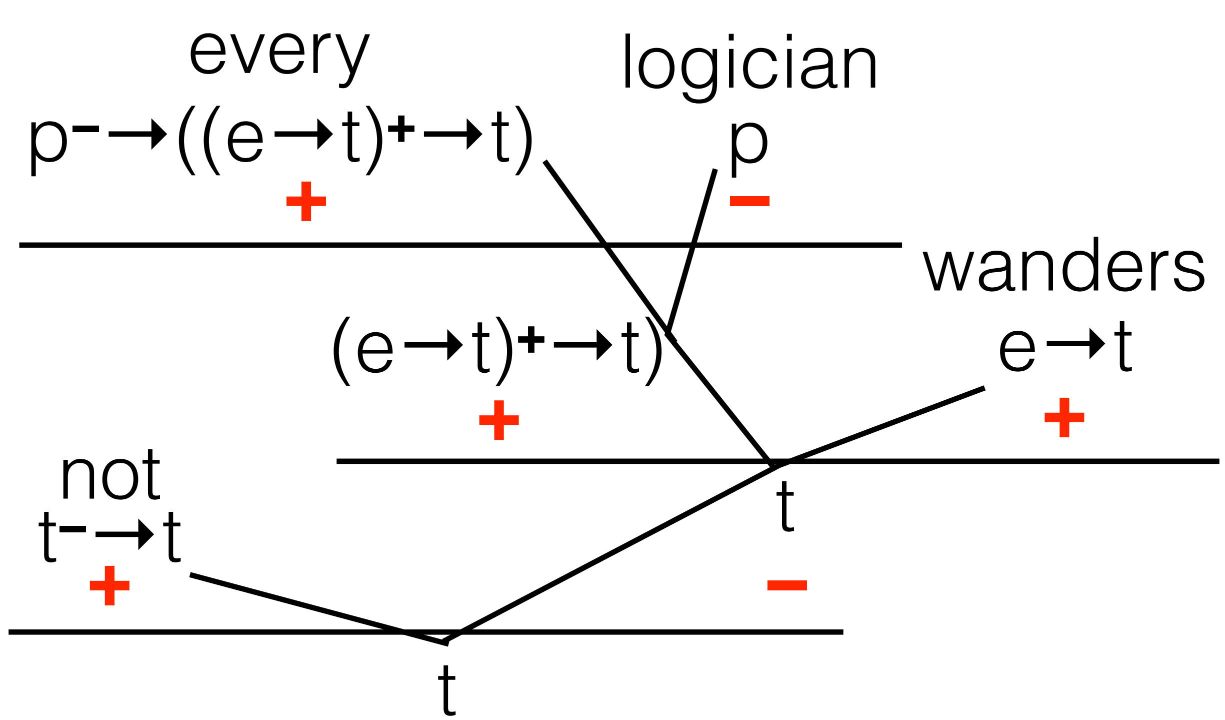
t

not((every logician) wanders)

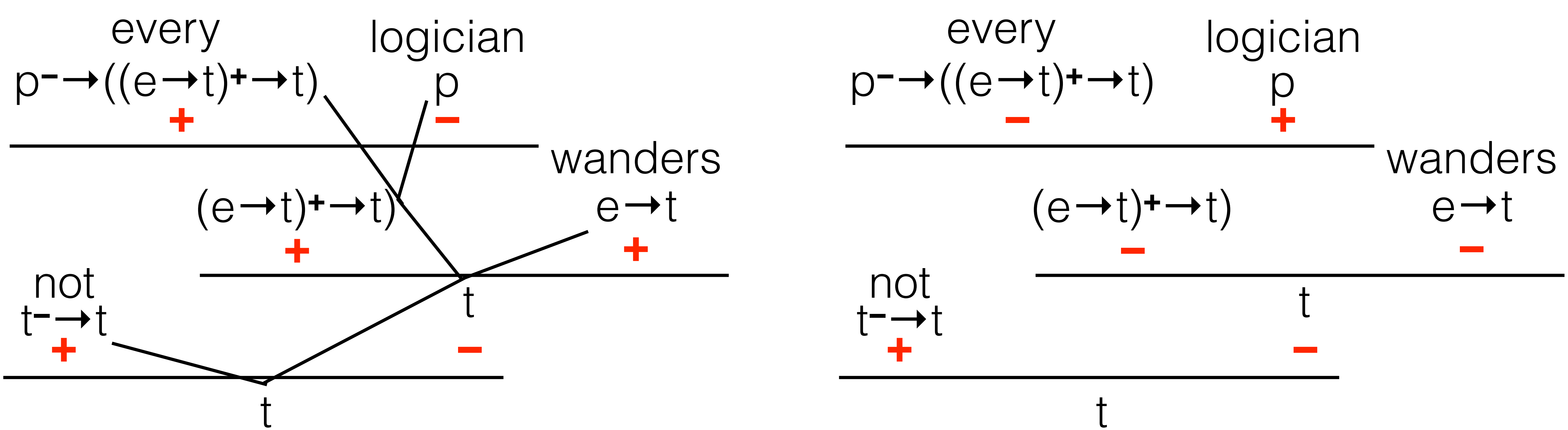
Augmented with external polarity markings in the derivation



Tracing paths from the root of the derivaton to the intermediate nodes



Reading off the final result.



$\text{not}^{+}((\text{every}^{-}\ \text{logician}^{+})^{-}\ \text{wanders}^{-})^{-}$

$\text{not}{\uparrow}((\text{every}{\downarrow}\ \text{logician}{\uparrow}){\downarrow}\ \text{wanders}{\downarrow}){\downarrow}$