No mortal man can slay every dragon
No mortal Dutchman can slay every dragon
No mortal man can slay every animal
No mortal man can decapitate every dragon
Using Natural Logic for Entailment
Entailment determination

Premise

Hypothesis (= thesis to be proven)

Functional view: input an ordered pair \((p, h)\), output a Boolean value, 1 if \(p\) entails \(h\), 0 otherwise.
<table>
<thead>
<tr>
<th>X is a couch</th>
<th>X is a crow</th>
<th>X is a fish</th>
<th>X is a hippo</th>
<th>X is a cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>X is a sofa</td>
<td>X is a bird</td>
<td>X is a carp</td>
<td>X is hungry</td>
<td>X is a dog</td>
</tr>
</tbody>
</table>
Which notion of entailment?

1. entailment as a two way classification: output labels (entailment, non-entailment) are interpreted as denoting sets of ordered pairs (relations) of declarative expressions (T):

   entailment (def) \( \{<p,h> \in \text{Dom}_{T_2}: p \models h\} \)

   non-entailment (def) \( \{<p,h> \in \text{Dom}_{T_2}: p \not\models h\} \)

   X is a crow, X is a bird:

   X is a crow, X is a canary:

   X is a crow, X is hungry:

From MacCartney 2009
Which notion of entailment?

1. entailment as a two way classification: output labels (entailment, non-entailment) are interpreted as denoting sets of ordered pairs (relations) of declarative expressions (T):

   - entailment (def) \( \{<p,h> \in \text{Dom}_{T^2}: p \models h\} \)
   - non-entailment (def) \( \{<p,h> \in \text{Dom}_{T^2}: p \not\models h\} \)

X is a crow, X is a bird: yes
X is a crow, X is a canary: no
X is a crow, X is hungry: no

From MacCartney 2009
2. entailment as a three-way classification: difference between contradiction and compatibility

entailment (def) \{((p, h) \in \text{Dom}_{T_{x2}}: p \models h}\}

contradiction (def) \{((p, h) \in \text{Dom}_{T_{x2}}: p \models \neg h}\}

compatibility (def) \{((p, h) \in \text{Dom}_{T_{x2}}: p \not\models h \land p \not\models \neg h}\}

X is a crow, X is a bird:

X is a crow, X is a canary:

X is a crow, X is hungry:
2. Entailment as a three-way classification: difference between contradiction and compatibility

- **Entailment (def)** \( \{(p,h) \in \text{Dom}_{\text{T}x_2}: p \models h\} \)
- **Contradiction (def)** \( \{(p,h) \in \text{Dom}_{\text{T}x_2}: p \models \lnot h\} \)
- **Compatibility (def)** \( \{(p,h) \in \text{Dom}_{\text{T}x_2}: p \not\models h \land p \not\models \lnot h\} \)

X is a crow, X is a bird: yes

X is a crow, X is a canary: no

X is a crow, X is hungry: compatible
3.  a. entailment as containment (monotonicity); output space

\[ \equiv \text{ (def) } \{(p,h) \in \text{Dom}_{T^2_x}: p \models h \land h \models p\} \]

\[ \sqsubseteq \text{ (def) } \{(p,h) \in \text{Dom}_{T^2_x}: p \models h \land h \not\models p\} \]

\[ \sqcap \text{ (def) } \{(p,h) \in \text{Dom}_{T^2_x}: p \not\models h \land h \models p\} \]

no-containment (def) \{(p,h) \in \text{Dom}_{T^2_x}: p \not\models h \land h \not\models p\} \\

X is a crow, X is a bird:

X is a bird, X is a crow:

X is a sofa, X is a coach:

X is a crow, X is a canary:

X is a crow, X is hungry:

From MacCartney 2009
3. a. entailment as containment (monotonicity); output space

≡ (def) \{(p,h) \in \operatorname{Dom}_{T_2}: p \models h \land h \models p\}

□ (def) \{(p,h) \in \operatorname{Dom}_{T_2}: p \models h \land h \not\models p\}

⊬ (def) \{(p,h) \in \operatorname{Dom}_{T_2}: p \not\models h \land h \models p\}

no-containment (def) \{(p,h) \in \operatorname{Dom}_{T_2}: p \not\models h \land h \not\models p\}

X is a crow, X is a bird: □

X is a bird, X is a crow: ⊬

X is a sofa, X is a coach: ≡

X is a crow, X is a canary: no containment

X is a crow, X is hungry: no containment

From MacCartney 2009
3. b. entailment as containment; input space: not just T but also E and mappings

if \( x, y \in \text{Dom}_T \) then \( x \sqsubseteq y \) iff \( x = \text{false} \) or \( y = \text{true} \) (material implication)

if \( x, y \in \text{Dom}_E \) then \( x \sqsubseteq y \) iff \( x = y \) (remember entities are things like John, Bill, ...)

if \( x, y \in \text{Dom}_{A \rightarrow B} \) then \( x \sqsubseteq y \) iff for all \( a \in \text{Dom}_A \) \( x(a) \sqsubseteq y(a) \)

(one function entails another if each of its outputs entails the corresponding output of the other function)

otherwise \( x \not\sqsubseteq y \) and \( y \not\sqsubseteq x \)

From MacCartney 2009
Entailment relations

<table>
<thead>
<tr>
<th></th>
<th>X is a couch</th>
<th>X is a crow</th>
<th>X is a fish</th>
<th>X is a hippo</th>
<th>X is a cat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Yes</strong></td>
<td><strong>entailment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>non-entailment</strong></td>
</tr>
</tbody>
</table>

2-way
RTE1,2,3

<table>
<thead>
<tr>
<th></th>
<th>X is a sofa</th>
<th>X is a bird</th>
<th>X is a carp</th>
<th>X is hungry</th>
<th>X is a dog</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Yes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No</strong></td>
<td><strong>non-entailment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3-way
FraCaS, PARC, RTE4

<table>
<thead>
<tr>
<th></th>
<th>X is a crow</th>
<th>X is a fish</th>
<th>X is a hippo</th>
<th>X is a cat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Yes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Unknown</strong></td>
<td><strong>compatibility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>No</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

containment
Sánchez-Valencia

<table>
<thead>
<tr>
<th></th>
<th>P = Q</th>
<th>P &lt; Q</th>
<th>P &gt; Q</th>
<th>P ≠ Q</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>equivalence</strong></td>
<td><strong>forward entailment</strong></td>
<td><strong>reverse entailment</strong></td>
<td><strong>non-entailment</strong></td>
<td></td>
</tr>
</tbody>
</table>

From MacCartney 2011
• Garfield is a cat
• Garfield is a mammal
• Garfield is not a fish
• Garfield is not a carp

Which of these entailments can the monotonicity calculus do?
From premise to hypothesis

premise --> hypothesis/conclusion

what are the inference rules? How do we change the premise(s) into the hypothesis/conclusion

INSertions, DELetions, SUBstitutions

Premise: John has a red convertible.
Conclusion: John has a red car. (SUB)
Conclusion: John has a convertible. (DEL)
Premise: John doesn't have a car
Conclusion: John doesn’t have a red car. (INS)

What are legitimate SUBs, DELs, INSs?
for each edit:

determine lexical entailment

project the lexical entailment upward the semantic composition of the tree

join atomic entailment relations across the sequence of edits
MacCartney’s aims

Preserve the semantic containment relations of the monotonicity calculus

Augment them with relations expressing semantic exclusion

Be complete and exhaustive so that each pair of expressions is assigned to some relation and the relations are mutually exclusive
entailment relations
R0010
R0011
R1010
R1011
R0100
R0101
R1100
R1101

\( x \subseteq y \)

\( x \subsetneq y \)

\( x \subsetneq y \)
Dual under negation

∀x,y : <x,y> ∈ R ↔ <\bar{x},\bar{y}> ∈ S

R1011 and R1101 (bit strings are reverses), R1001
Leave out cases in which one of the two expressions has a denotation that is either empty or universal.
if \( \neg y \& x \) and \( \neg x \& y \) empty: \( x \equiv y \)
if only \( \neg y \& x \) empty, \( x \sqsubseteq y \)
if there are things that are neither $x$ nor $y$ and things that are not $y$ but $x$ and things that are not $x$ but $y$ but there are no things that are both $x$ and $y$ then $x$ and $y$ are disjoined, they are alternating ($|$)
if \( \neg y \& x \) and \( \neg x \& y \) empty: \( x \equiv y \)

if only \( \neg y \& x \) empty, \( x \sqsubseteq y \)

if there are no things that are neither \( x \) nor \( y \) and no things that are both \( x \) and \( y \) then \( x \) and \( y \) are the negation of each other (\(^\wedge\))
\(\Diamond\): nothing is neither \(x\) or \(y\): the union of \(x\) and \(y\) together is the universe, e.g. animal \(\Diamond\) non-ape
The set $\mathcal{B}$ of 7 basic entailment relations

<table>
<thead>
<tr>
<th>Venn</th>
<th>symbol</th>
<th>name</th>
<th>example</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Venn Diagram" /></td>
<td>$x \equiv y$</td>
<td>equivalence</td>
<td>couch \equiv sofa</td>
</tr>
<tr>
<td><img src="image2" alt="Venn Diagram" /></td>
<td>$x \sqsubset y$</td>
<td>forward entailment (strict)</td>
<td>crow \sqsubset bird</td>
</tr>
<tr>
<td><img src="image3" alt="Venn Diagram" /></td>
<td>$x \sqsupset y$</td>
<td>reverse entailment (strict)</td>
<td>European \sqsupset French</td>
</tr>
<tr>
<td><img src="image4" alt="Venn Diagram" /></td>
<td>$x \wedge y$</td>
<td>negation (exhaustive exclusion)</td>
<td>human \wedge nonhuman</td>
</tr>
<tr>
<td><img src="image5" alt="Venn Diagram" /></td>
<td>$x \mid y$</td>
<td>alternation (non-exhaustive exclusion)</td>
<td>cat \mid dog</td>
</tr>
<tr>
<td><img src="image6" alt="Venn Diagram" /></td>
<td>$x \sim y$</td>
<td>cover (exhaustive non-exclusion)</td>
<td>animal \sim nonhuman</td>
</tr>
<tr>
<td><img src="image7" alt="Venn Diagram" /></td>
<td>$x # y$</td>
<td>independence</td>
<td>hungry # hippo</td>
</tr>
</tbody>
</table>

Relations are defined for all semantic types: tiny $\sqsubset$ small, hover $\sqsubset$ fly, kick $\sqsubset$ strike, this morning $\sqsubset$ today, in Beijing $\sqsubset$ in China, everyone $\sqsubset$ someone, all $\sqsubset$ most $\sqsubset$ some
edits
Simple edits
Entailment relations and semantic types

Entailment relation:

any set of ordered pairs where both elements belong to the same semantic type

e→t: common nouns, intransitive verbs, adjectives (predicative)

e→e→t: transitive verbs

(e→t)→(e→t): adverbs

(e→t)→(e→t)→t: generalized quantifier
Lexical open-class SUBs, INSs and DELs

SUBstitutions of open class items belonging the to the same semantic type.

common nouns, adjectives, verbs, ...:

traditional lexicographic relations (stand in for ontological relations): synonyms, hypernyms

happy ≡ glad; forbid≡prohibit

soar ⊏ rise; scalding ⊏ hot

What about antonyms? Are they ^? rise,fall; hot,cold; dead,alive
antonyms: |, rarely
why? rise, fall; hot, cold; dead, alive
rise|fall; hot|cold; dead^alive

More entities in the | category: cat|dog; but also unrelated entities: chalk|battle

What about unrelated adjectives: weak, temporary?
Verbs: skiing|sleeping; skiing#talking, these various categories are not readily available in lexicons.

Proper nouns: USA=United States (of America); JFK|FDR

Kyoto, Japan?
Kyoto ⊆ Japan?

Kyoto is a beautiful city; Japan is a beautiful city
What about the ◦ relation? Very rare in lexical pairs: metallic ◦ non-ferrous, mammal ◦ nonhuman

Lots of simplifying assumptions in MacCartney. Some of them come from RTE, e.g. tense is ignored.
Closed-class terms

SUBs

all $\equiv$ every

every $\sqsubseteq$ some (existential import!)

some $\land$ no: some birds talk $\land$ no birds talk

no $\mid$ every: $|$ means $x\cap y = 0$ and $x\cup y = U$ (every student passed $\leftrightarrow$ no student didn’t pass)

four or more $\sqsubseteq$ two or more

exactly four $\mid$ exactly two

four ? two

at most four $\sim$ at least two

most # ten or more
prepositions:

on a plane, in a plane

above the table, under the table
DEL/INS: default for DEL: □; default for INS: ⊐; relies on the assumption that upward monotone contexts are the most prevalent ones when no further context is considered.
DELs and INSs

DEL default: ⊏ red car ⊏ car
INS default: ⊐ sing ⊐ sing off-key
OK for intersective modifiers (adjectives, relatives), conjuncts.
But:

    negation creates a ^ relation :

    sleep ^ didn’t sleep

    fake: |
    former: ?
    alleged: #

We need a better typology of this
projections
Semantic composition of entailment relations

we have \( b(x,y) \)
what is the value of \( b(f(x),f(y)) \)?
if \( f \) is upward monotone then \( b(f(x),(y)) = b(x,y) \)
some parrots talk \( ⊑ \) some birds talk
because \textit{some} is upward monotone in its first argument
if \( f \) is downward monotone then \( ⊑ \) and \( ⊐ \) gets swapped.
if \( f \) is non monotone then \( ⊑ \) and \( ⊐ \) result in #:
most human talks # most animals talk.

we will calculate the effect of these compositions going up
the tree of a grammar parse cf. Sánchez Valencia.
He moved without pants.
He danced without jeans.
pants ⊆ jeans
moved ⊆ danced
without?
Some projections

Negation

happy $\equiv$ glad $\Rightarrow$ not happy $\equiv$ not glad

kiss $\sqsubset$ touch $\Rightarrow$ not kiss $\sqsupset$ not touch

French $|$ German $\Rightarrow$ not French $\cup$ not German

more than 4 $\cup$ less than 6 $\Rightarrow$ not more than 4 $|$ not less than 6
## Projectivity

If $f$ has monotonicity…

<table>
<thead>
<tr>
<th></th>
<th>UP</th>
<th>DOWN</th>
<th>NON</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\equiv$</td>
<td>$\equiv$</td>
<td>$\equiv$</td>
<td>$\equiv$</td>
</tr>
<tr>
<td>$\sqsubseteq$</td>
<td>$\sqsubseteq$</td>
<td>$\sqsubseteq$</td>
<td>$\sqsubseteq$</td>
</tr>
<tr>
<td>$\sqsupset$</td>
<td>$\sqsupset$</td>
<td>$\sqsupset$</td>
<td>$\sqsupset$</td>
</tr>
<tr>
<td>$#$</td>
<td>$#$</td>
<td>$#$</td>
<td>$#$</td>
</tr>
</tbody>
</table>

| $f$ : connective | $\equiv$ | $\sqsubseteq$ | $\sqsupset$ | $^\wedge$ | $|$ | $\sqsubset$ | $#$ |
|-----------------|---------|--------------|-------------|--------|--------|--------|--------|
| negation (not)  | $\equiv$ | $\sqsubseteq$ | $\sqsupset$ | $^\wedge$ | $\sqsubset$ | $|$ | $\#$ |
| conjunction/intersection (and) | $\equiv$ | $\sqsubseteq$ | $\sqsupset$ | $|$ | $|$ | $\#$ | $\#$ |
| disjunction (or) | $\equiv$ | $\sqsubseteq$ | $\sqsupset$ | $\sqsubset$ | $\#$ | $\sqsubset$ | $\#$ |
| conditional antecedent (if) | $\equiv$ | $\sqsubseteq$ | $\sqsupset$ | $\#$ | $\#$ | $\#$ | $\#$ |
| conditional consequent (then) | $\equiv$ | $\sqsubseteq$ | $\sqsupset$ | $|$ | $|$ | $\#$ | $\#$ |
| biconditional (iff) | $\equiv$ | $\#$ | $\#$ | $^\wedge$ | $\#$ | $\#$ | $\#$ | $\#$ |
Conditionals

<table>
<thead>
<tr>
<th>f : connective</th>
<th>≡</th>
<th>⊤</th>
<th>⊥</th>
<th>∧</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>conditional antecedent (if)</td>
<td>≡</td>
<td>⊤</td>
<td>⊥</td>
<td>#</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>conditional consequent (then)</td>
<td>≡</td>
<td>⊤</td>
<td>⊥</td>
<td></td>
<td></td>
<td>#</td>
</tr>
</tbody>
</table>

If he drinks tequila, he feels nauseous; if he drinks liquor he feels nauseous.

If he drinks tequila, he feels nauseous; if he drinks tequila he feels sick.

If it is sunny we surf, if it is not sunny we surf

If it is sunny we surf, if it is sunny we do not surf
Conditionals

<table>
<thead>
<tr>
<th>f : connective</th>
<th>≡</th>
<th>⊤</th>
<th>⊥</th>
<th>^</th>
<th></th>
<th>⊓</th>
<th>#</th>
</tr>
</thead>
<tbody>
<tr>
<td>conditional antecedent (if)</td>
<td>≡</td>
<td>⊥</td>
<td>⊤</td>
<td>#</td>
<td>#</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>conditional consequent (then)</td>
<td>≡</td>
<td>⊤</td>
<td>⊥</td>
<td></td>
<td></td>
<td></td>
<td>#</td>
</tr>
</tbody>
</table>

If he drinks tequila, he feels nauseous; if he drinks liquor he feels nauseous.

If he drinks tequila, he feels nauseous; if he drinks tequila he feels sick.

If it is sunny we surf # if it is not sunny we surf

If it is sunny we surf | if it is sunny we do not surf

Note this is not the same as the material implication.
Conditionals

| $f$ : connective | $\equiv$ | $\Box$ | $\Box$ | $\wedge$ | $|$ | $\vee$ | $#$ |
|------------------|---------|-------|-------|--------|----|-------|----|
| conditional antecedent (if) | $\equiv$ | $\Box$ | $\Box$ | $#$ | $#$ | $#$ | $#$ |
| conditional consequent (then) | $\equiv$ | $\Box$ | $\Box$ | $|$ | $|$ | $#$ | $#$ |

If he drinks tequila, he feels nauseous $\Box$ if he drinks liquor he feels nauseous.

If he drinks a tiny bit of tequila, he feels nauseous $\Box$ if he drinks a lot of tequila he feels nauseous.
<table>
<thead>
<tr>
<th>quantifier</th>
<th>1st argument</th>
<th>2nd argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>some</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>no</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>every</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>not every</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>at least two</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>most</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>exactly one</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>all but one</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
<td>≡ ⊂ ⊃ ∨ ⊤ ✓  #  ◯  #</td>
</tr>
<tr>
<td>quantifier</td>
<td>1st</td>
<td>2nd</td>
</tr>
<tr>
<td>----------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>some</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>no</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>every</td>
<td>#</td>
<td>_</td>
</tr>
<tr>
<td>not every</td>
<td>#</td>
<td>_</td>
</tr>
<tr>
<td>at least two</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>most</td>
<td>#</td>
<td>_</td>
</tr>
<tr>
<td>exactly one</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>all but one</td>
<td>#</td>
<td>#</td>
</tr>
</tbody>
</table>

first argument: bird|fish: no fish talk # no birds talk
early|late
everyone was early|everyone was late
most people were early|most people were late
Verbs

Most verbs are upward-monotone, \(^, | \) and \(\sim\) get projected as \#

cats|dogs; eat cats \# eat dogs

but:
German ^ non-German
is married to a German| is married to a non-German
is married to a German| is married to an Italian

What assumptions about marriage and nationality are made here?
some ignored cases

car ⊏ vehicle
red car ⊏ red vehicle
mouse ⊏ animal
big mouse ? big animal

human ~ non-human
brown human | brown non-human
joins
Up to now: calculating results of one edit. But we will typically have more than one in a sentence. How do they interact?
Joins

dog ⊆ mammal ; mammal ⊆ animal
→ dog ⊆ animal

dog ^ non-dog ; non-dog ^ dog
→ dog = dog
Joins

\[ R \bowtie S \overset{\text{def}}{=} \{ \langle x, z \rangle : \exists y (\langle x, y \rangle \in R \land \langle y, z \rangle \in S) \} \]

dog \bowtie \text{mammal}
\text{mammal} \bowtie \text{animal}

\[ \rightarrow \text{dog} \bowtie \text{animal} \]

dog \bowtie \text{non-dog}
\text{non-dog} \bowtie \text{dog}

\[ \rightarrow \text{dog} = \text{dog} \]
Non commutative joins

<table>
<thead>
<tr>
<th>logistic expression</th>
<th>semantic meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ni \land \land \rightarrow \sqcap$</td>
<td>fish</td>
</tr>
<tr>
<td>$\land \ni \lor \rightarrow \sqcap$</td>
<td>human $\land$ nonhuman</td>
</tr>
<tr>
<td>$\land \ni \lor \rightarrow \sqcap$</td>
<td>fish $\sqsubseteq$ nonhuman</td>
</tr>
<tr>
<td>$\land \ni # \rightarrow #$</td>
<td></td>
</tr>
<tr>
<td>$\land \ni \land \rightarrow #$</td>
<td></td>
</tr>
</tbody>
</table>
Non unique joins
(leaving out combinations with #)

<table>
<thead>
<tr>
<th>⊏ ⊑ ⊐</th>
<th>⊏ ⊑ ⊐</th>
</tr>
</thead>
<tbody>
<tr>
<td>⊏ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
<td>⊏ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
</tr>
<tr>
<td>⊏ ⊑ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
<td>⊏ ⊑ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
</tr>
<tr>
<td>⊏ ⊑ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
<td>⊏ ⊑ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
</tr>
<tr>
<td>⊏ ⊑ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
<td>⊏ ⊑ ⊑ ⊑ ⊐ → ⊑ ⊐ ⊐ ⊐</td>
</tr>
</tbody>
</table>
gasoline \equiv petrol
pistol \subseteq gun
woman \# Eskimo
Calculation: consider all ordered triples of a universe \( U \). We get 256 equivalent classes...

\[
R \Join S \overset{\text{def}}{=} \{ \langle x, z \rangle : \exists y \ (\langle x, y \rangle \in R \land \langle y, z \rangle \in S) \} \]
32 cases where there is a unique answer, 17 where there is a union relation
For practical purposes we may see all the ones that contain # as the same, namely #
In a certain sense a sad result, would have been nicer if we had found ?, c (entailment) or ^,| (contradiction) or even c,?, # (compatibility)
Example

Stimpy is a cat
Stimpy is not a poodle
SUB(cat,dog)
Stimpy is a dog
INS(not)
Stimpy is not a dog
SUB(dog,poodle)
Stimpy is not a poodle
Example: lexical relations from edit

Stimpy is a cat
Stimpy is not a poodle
SUB(cat,dog) |
Stimpy is a dog
INS(not) ^
Stimpy is not a dog
SUB(dog,poodle) ⊓
Stimpy is not a poodle
Example: projection

Stimpy is a cat
Stimpy is not a poodle
SUB(cat,dog)            |     |
Stimpy is a dog
INS(not)                    ^    ^
Stimpy is not a dog
SUB(dog,poodle)       ⊐     ⊏
Stimpy is not a poodle
Example: joins

Stimpy is a cat
Stimpy is not a poodle

\text{SUB}({\text{cat, dog}}) \rightarrow \top \top \top

Stimpy is a dog

INS({\text{not}}) \rightarrow \top \top \top

Stimpy is not a dog

\text{SUB}({\text{dog, poodle}}) \rightarrow \bot \bot \bot \bot

Stimpy is not a poodle
Stimpy can run fast without a leash.
Stimpy can move.
DEL(without a leash)
DEL(fast)
DEL(fast)
SUB(run,move)
Stimpy can run fast without a leash.
Stimpy can move.
DEL(without a leash) □
DEL(fast) □
SUB(run, move) □
Stimpy can run fast without a leash.
Stimpy can move.
DEL(without a leash) ⊏ ⊏ ⊏
DEL(fast) ⊏ ⊏ ⊏ ⊏
SUB(run, move) ⊏ ⊏ ⊏
Stimpy can’t run fast with a leash
Stimpy can’t run fast. NO
DEL(with a leash) □ □ □
Jimmy Dean moved without blue jeans
James Dean danced without pants

SUB(Jimmy, James) ⊑ ⊑ ⊑ ⊑
SUB(blue jeans, pants) ⊒ ☐ ☐ ☐
SUB(move, dance) ☐ ☐ ☐ ☐