\+ _ _ _ _ _ +

No mortal man can slay every dragon

No mortal Dutchman can slay every dragon

No mortal man can slay every animal

No mortal man can decapitate every dragon

Using Natural Logic for Entailment

Entailment determination

Premise

Hypothesis (= thesis to be proven)

Functional view: input an ordered pair (p,h), output a Boolean value, 1 if p entails h, 0 otherwise.

X is a couch X is a sofa	X is a crow X is a bird	X is a fish X is a carp	X is a hippo X is hungry	X is a cat X is a dog

Which notion of entailment?

1. entailment as a two way classification: output labels (entailment, non-entailment) are interpreted as denoting sets of ordered pairs (relations) of declarative expressions (T):

   entailment (def) $\{<p,h> \in Dom_{Tx2}: p \models h\}$

   non-entailment (def) $\{<p,h> \in Dom_{Tx2}: p \not\models h\}$

   X is a crow, X is a bird:

   X is a crow, X is a canary:

   X is a crow, X is hungry:

From MacCartney 2009

Which notion of entailment?

1. entailment as a two way classification: output labels (entailment, non-entailment) are interpreted as denoting sets of ordered pairs (relations) of declarative expressions (T):

   entailment (def) $\{\langle p,h\rangle \in Dom_{T\times 2}: p \models h\}$

   non-entailment (def) $\{\langle p,h\rangle \in Dom_{T\times 2}: p \not\models h\}$

   X is a crow, X is a bird: yes

   X is a crow, X is a canary: no

   X is a crow, X is hungry: no

From MacCartney 2009

2. entailment as a three-way classification: difference between contradiction and compatibility

entailment (def) $\{(p,h) \in Dom_{Tx2}: p \models h\}$

contradiction (def) $\{(p,h) \in Dom_{Tx2}: p \models \neg h\}$

compatibility (def) $\{(p,h) \in Dom_{Tx2}: p \not\models h \wedge p \not\models \neg h\}$

X is a crow, X is a bird:

X is a crow, X is a canary:

X is a crow, X is hungry:

2. entailment as a three-way classification: difference between contradiction and compatibility

entailment (def) $\{(p,h) \in Dom_{Tx2}: p \models h\}$

contradiction (def) $\{(p,h) \in Dom_{Tx2}: p \models \neg h\}$

compatibility (def) $\{(p,h) \in Dom_{Tx2}: p \not\models h \wedge p \not\models \neg h\}$

X is a crow, X is a bird: yes

X is a crow, X is a canary: no

X is a crow, X is hungry: compatible

3. a. entailment as containment (monotonicity); output space

$\equiv$ (def) $\{(p,h) \in Dom_{Tx2}: p \vDash h \wedge h \vDash p\}$

$\sqsubset$ (def) $\{(p,h) \in Dom_{Tx2}: p \vDash h \wedge h \nvDash p\}$

$\sqsupset$ (def) $\{(p,h) \in Dom_{Tx2}: p \nvDash h \wedge h \vDash p\}$

no-containment (def) $\{(p,h) \in Dom_{Tx2}: p \nvDash h \wedge h \nvDash p\}$

X is a crow, X is a bird:

X is a bird, X is a crow:

X is a sofa, X is a coach:

X is a crow, X is a canary:

X is a crow, X is hungry:

From MacCartney 2009

3. a. entailment as containment (monotonicity); output space

≡ (def) $\{(p,h) \in Dom_{Tx2}: p \vDash h \wedge h \vDash p\}$

⊏ (def) $\{(p,h) \in Dom_{Tx2}: p \vDash h \wedge h \nvDash p\}$

⊐ (def) $\{(p,h) \in Dom_{Tx2}: p \nvDash h \wedge h \vDash p\}$

no-containment (def) $\{(p,h) \in Dom_{Tx2}: p \nvDash h \wedge h \nvDash p\}$

X is a crow, X is a bird: ⊏

X is a bird, X is a crow: ⊐

X is a sofa, X is a coach: ≡

X is a crow, X is a canary: no containment

X is a crow, X is hungry: no containment

From MacCartney 2009

3. b. entailment as containment; input space: not just T but also E and mappings

if $x,y \in Dom_T$ then $x \sqsubseteq y$ iff x = false or y = true (material implication)

if $x,y \in Dom_E$ then $x \sqsubseteq y$ iff x = y (remember entities are things like John, Bill, ...)

if $x,y \in Dom_{A \rightarrow B}$ then $x \sqsubseteq y$ iff for all $a \in Dom_A$ $x(a) \sqsubseteq y(a)$

(one function entails another if each of its outputs entails the corresponding output of the other function)

otherwise $x \not\sqsubseteq y$ and $y \not\sqsubseteq x$

From MacCartney 2009

Entailment relations

	X is a couch X is a sofa	X is a crow X is a bird	X is a fish X is a carp	X is a hippo X is hungry	X is a cat X is a dog
2-way RTE1,2,3	**Yes** entailment		**No** non-entailment		
3-way FraCaS, PARC, RTE4	**Yes** entailment		**Unknown** compatibility		**No** contradiction
containment Sánchez-Valencia	**P = Q** equivalence	**P < Q** forward entailment	**P > Q** reverse entailment	**P # Q** non-entailment	

From MacCartney 2011

- Garfield is a cat
- Garfield is a mammal
- Garfield is not a fish
- Garfield is not a carp

Which of these entailments can the monotonicity calculus do?

From premise to hypothesis

premise --> hypothesis/conclusion

what are the inference rules? How do we change the premise(s) into the hypothesis/conclusion

INSertions, DELetions, SUBstitutions

Premise: John has a red convertible.

Conclusion: John has a red car. (SUB)

Conclusion: John has a convertible. (DEL)

Premise: John doesn’t have a car

Conclusion: John doesn’t have a red car. (INS)

What are legitimate SUBs, DELs, INSs?

for each edit:

- determine lexical entailment
- project the lexical entailment upward the semantic composition of the tree
- join atomic entailment relations across the sequence of edits

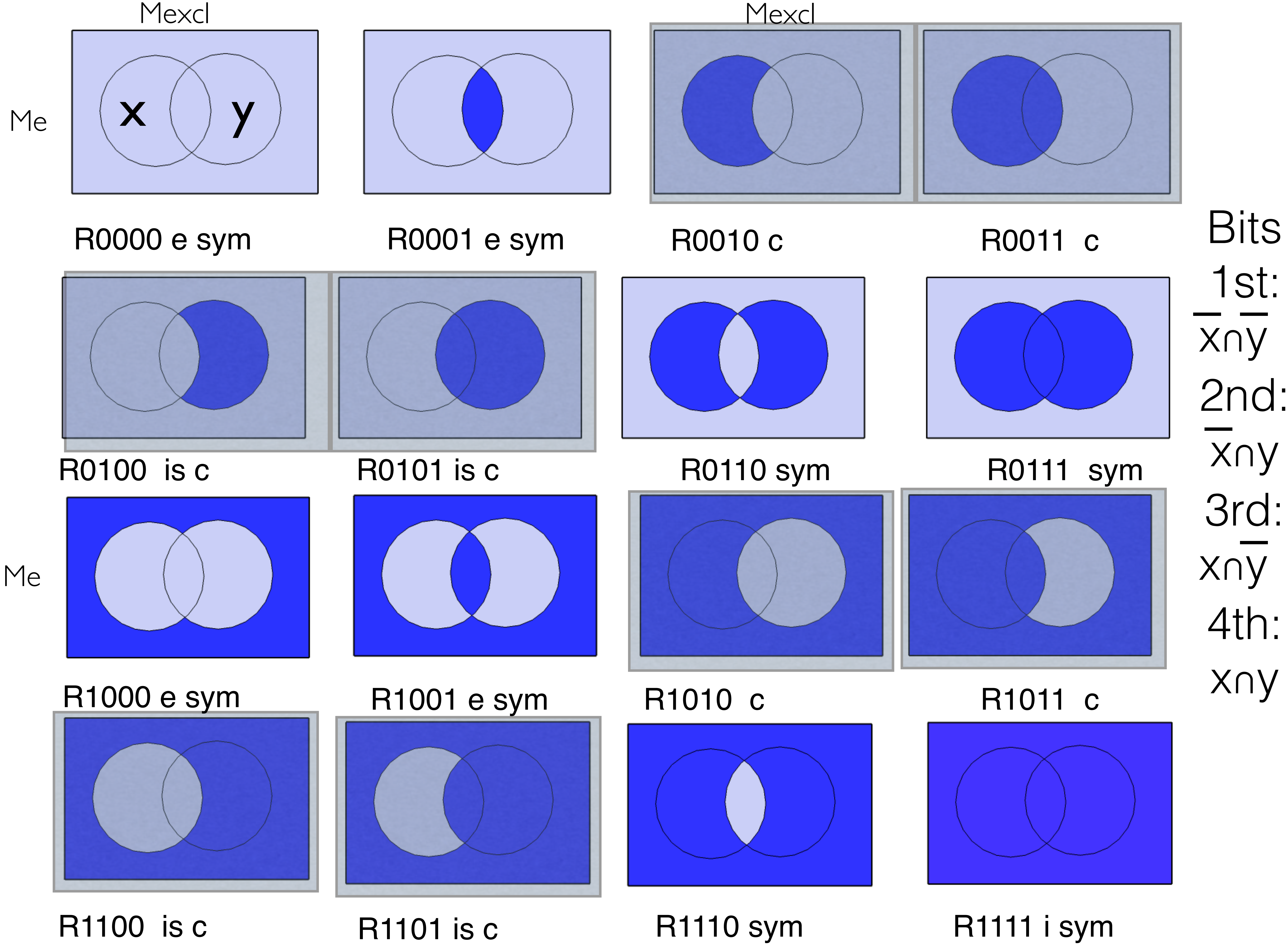
MacCartney's aims

Preserve the semantic containment relations of the monotonicity calculus

Augment them with relations expressing semantic exclusion

Be complete and exhaustive so that each pair of expressions is assigned to some relation and the relations are mutually exclusive

entailment relations


Mexcl
Mexcl
Me
x
y
R0000 e sym
R0001 e sym
R0010 c
R0011 c
R0100 is c
R0101 is c
R0110 sym
R0111 sym
Me
R1000 e sym
R1001 e sym
R1010 c
R1011 c
R1100 is c
R1101 is c
R1110 sym
R1111 i sym
Bits
1st: $\bar{x} \cap \bar{y}$
2nd: $\bar{x} \cap y$
3rd: $x \cap \bar{y}$
4th: $x \cap y$

R0010

R0011

R1010

R1011

$x \sqsupset y$

R0100

R0101

R1100

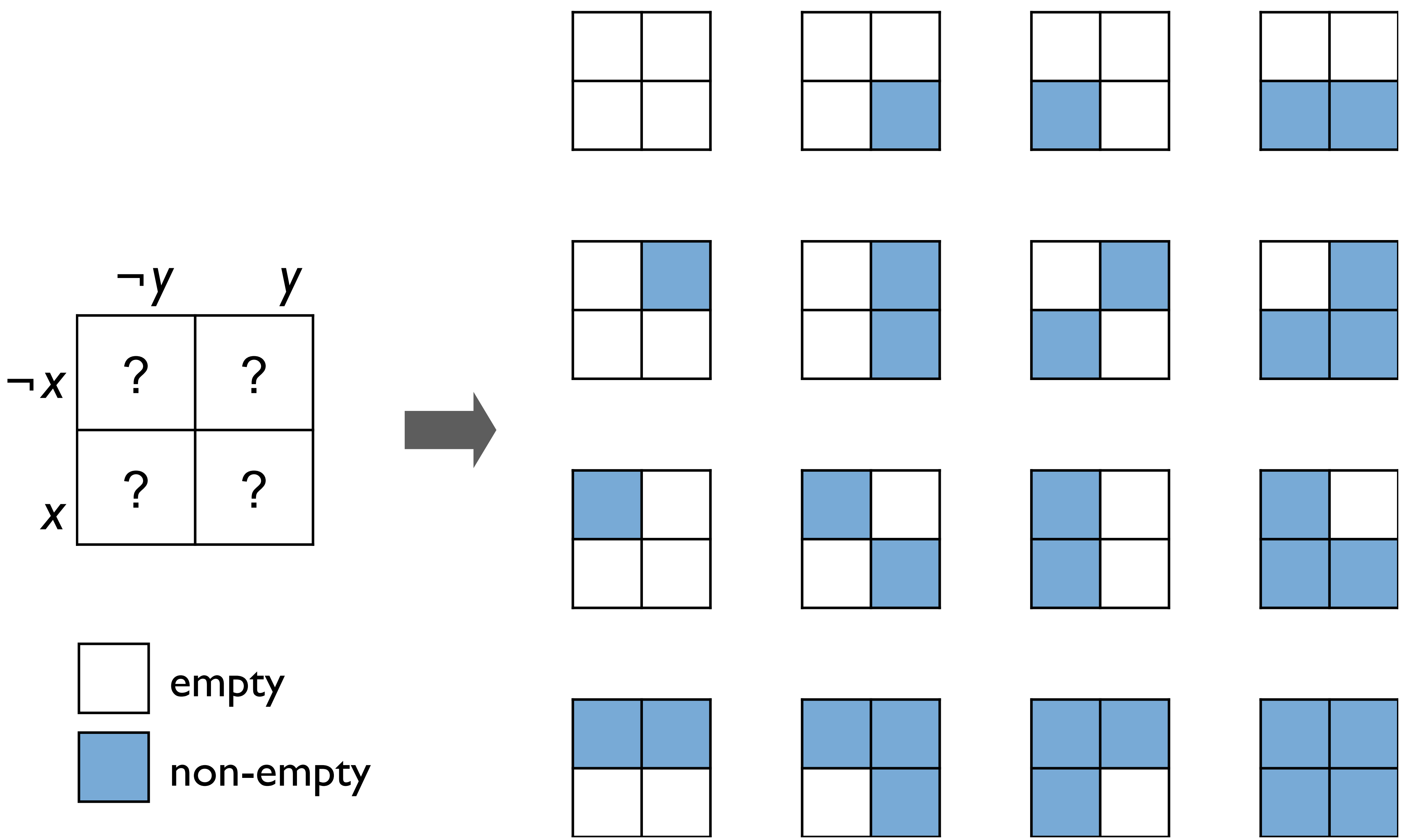
R1101

$x \sqsubset y$

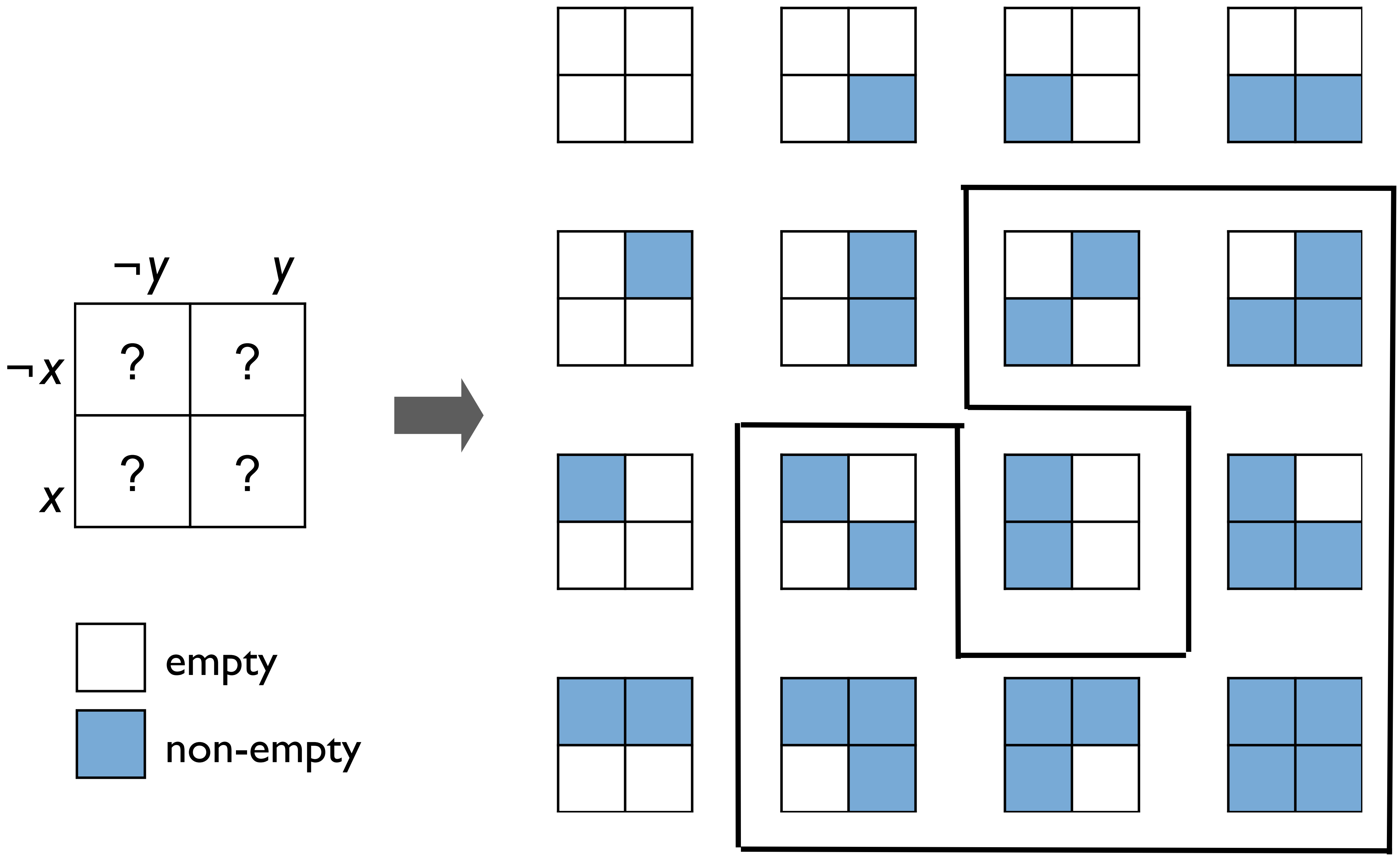
Dual under negation

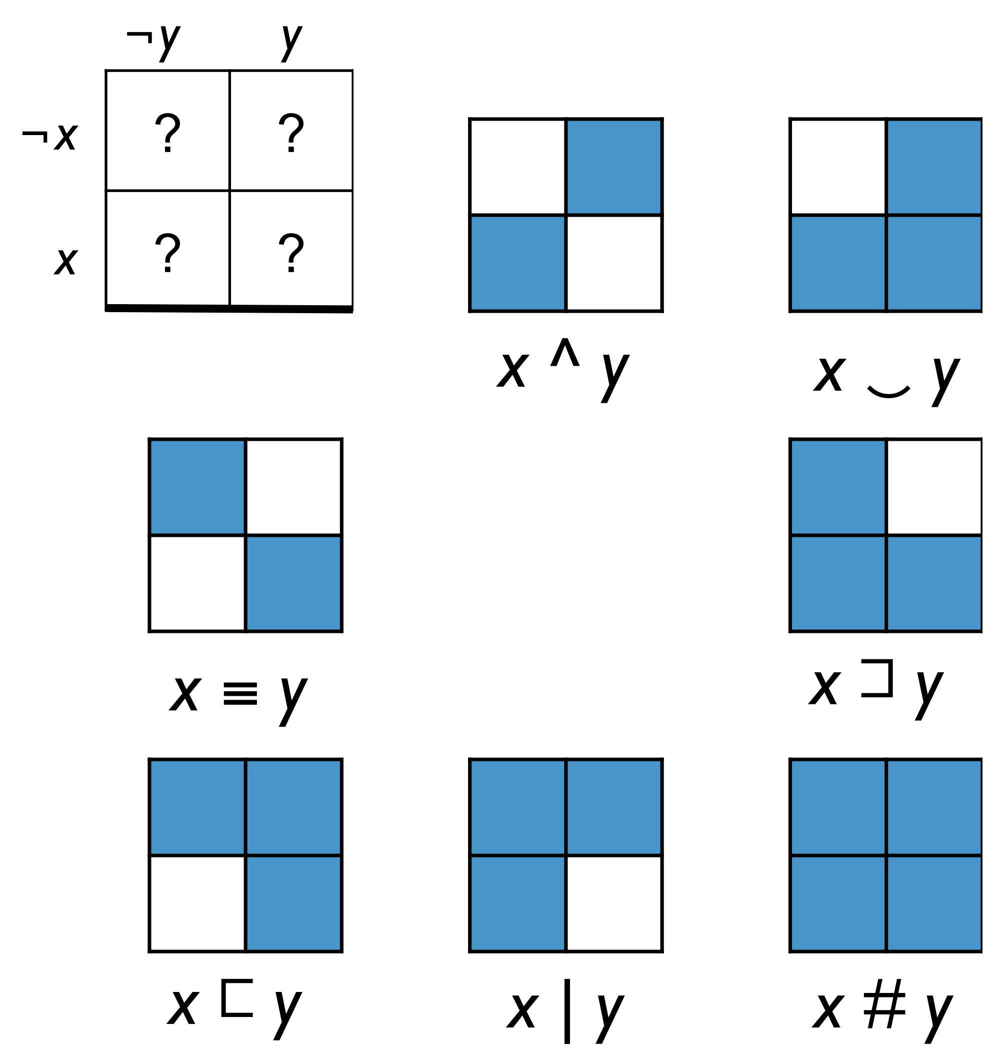
$\forall x,y : \langle x,y \rangle \in R \Leftrightarrow \langle \overline{x},\overline{y} \rangle \in S$

R1011 and R1101 (bit strings are reverses), R1001


¬y
y
¬x
x
? ? ? ?
empty
non-empty

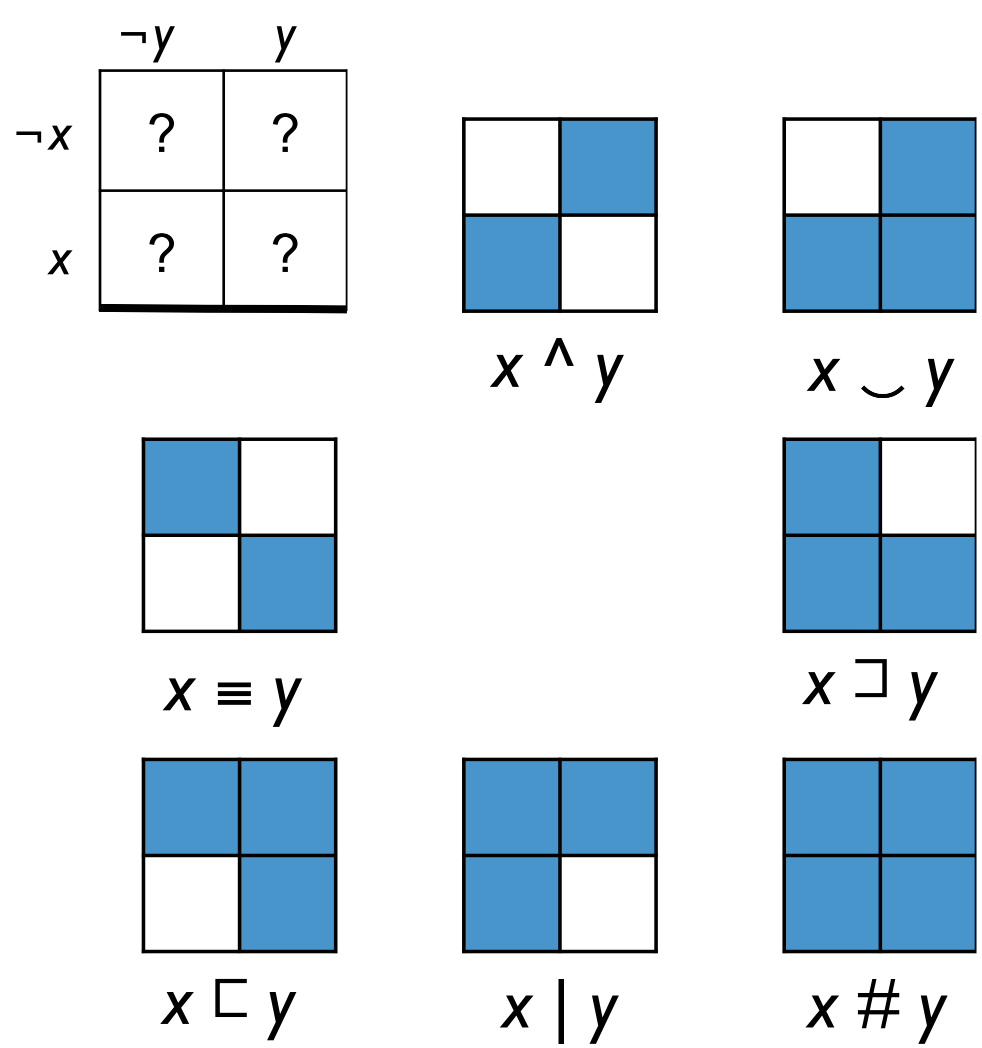
Leave out cases in which one of the two expressions has a denotation that is either empty of universal



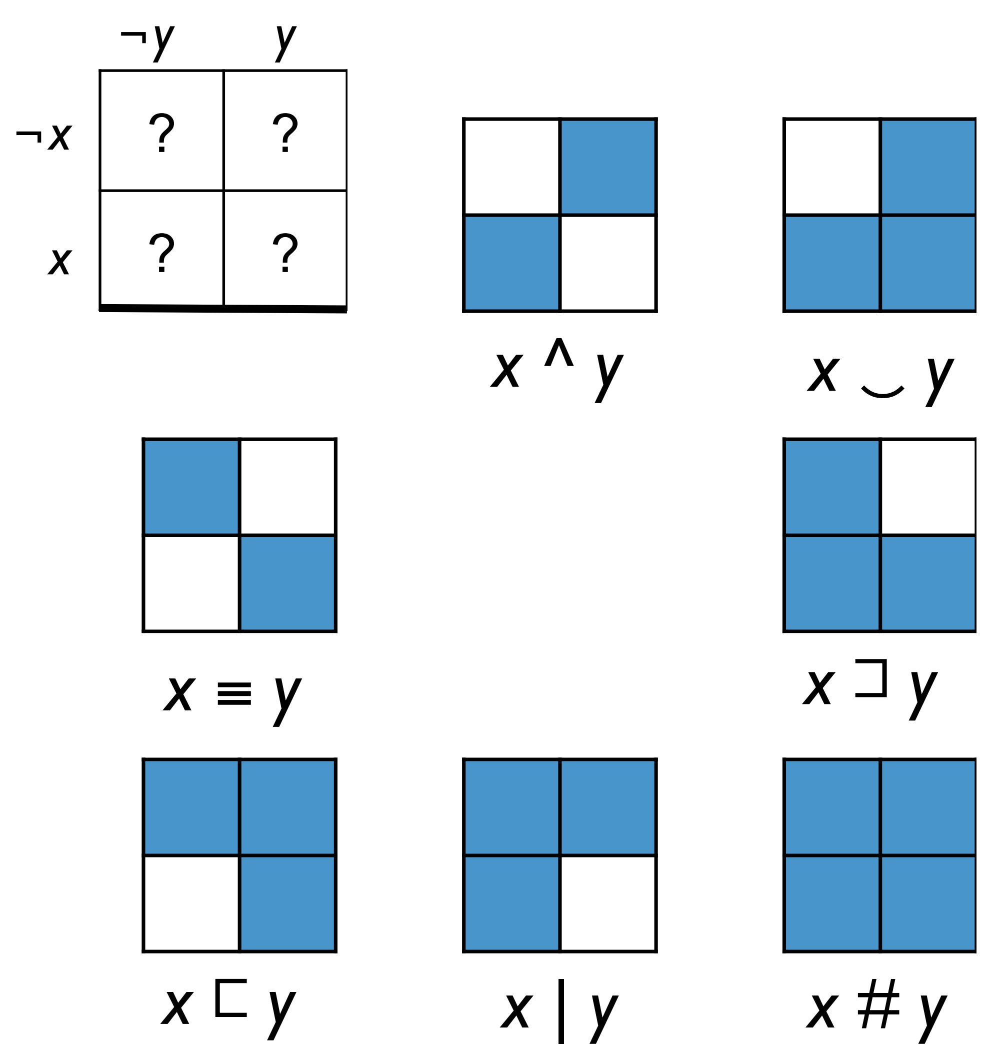

¬y
y
¬x
x
?
?
?
?
x ^ y
x ⌣ y
x ≡ y
x ⊐ y
x ⊏ y
x | y
x # y

if ¬y&x and ¬x&y empty: x≡y

if only ¬y&x empty, x⊏y

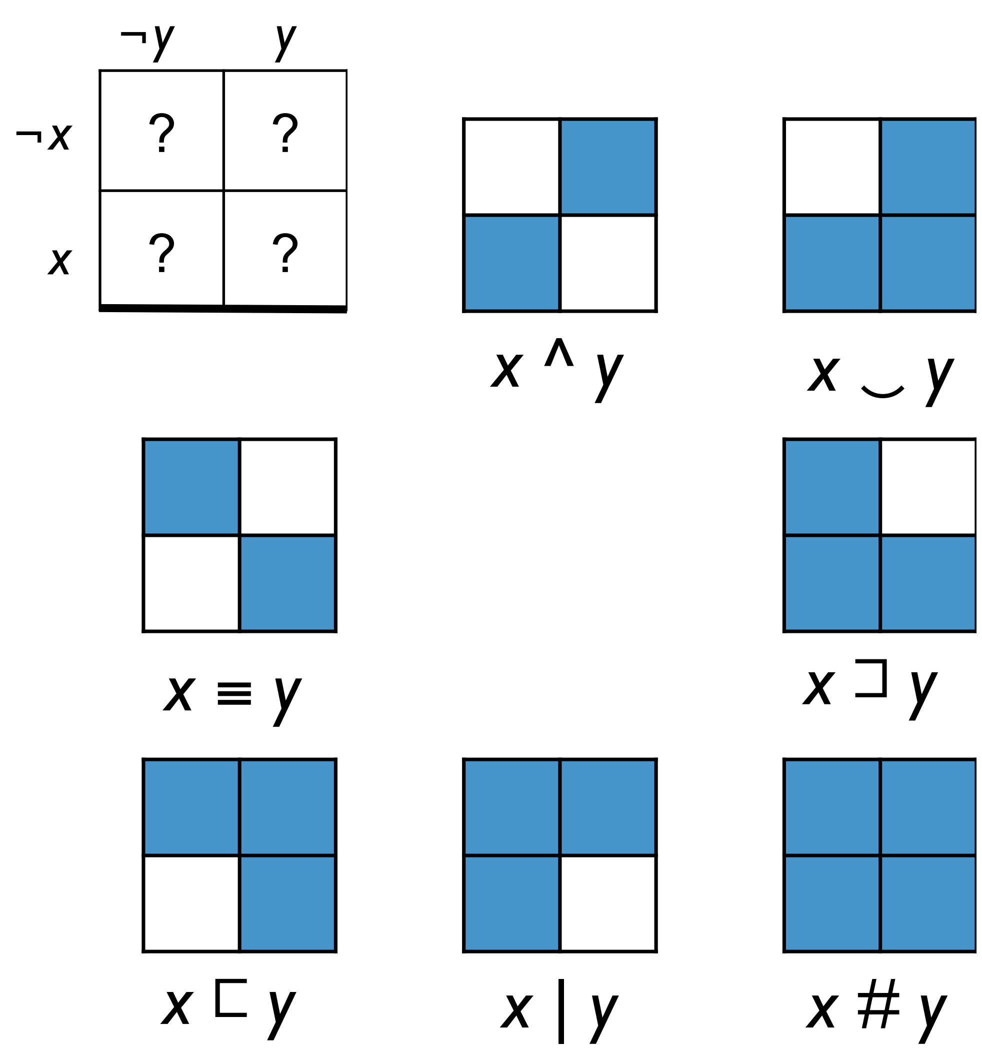


if there are things that are neither x nor y and things that are not y but x and things that are not x but y but there are no things that are both x and y then x and y are disjoined, they are alternating (|)

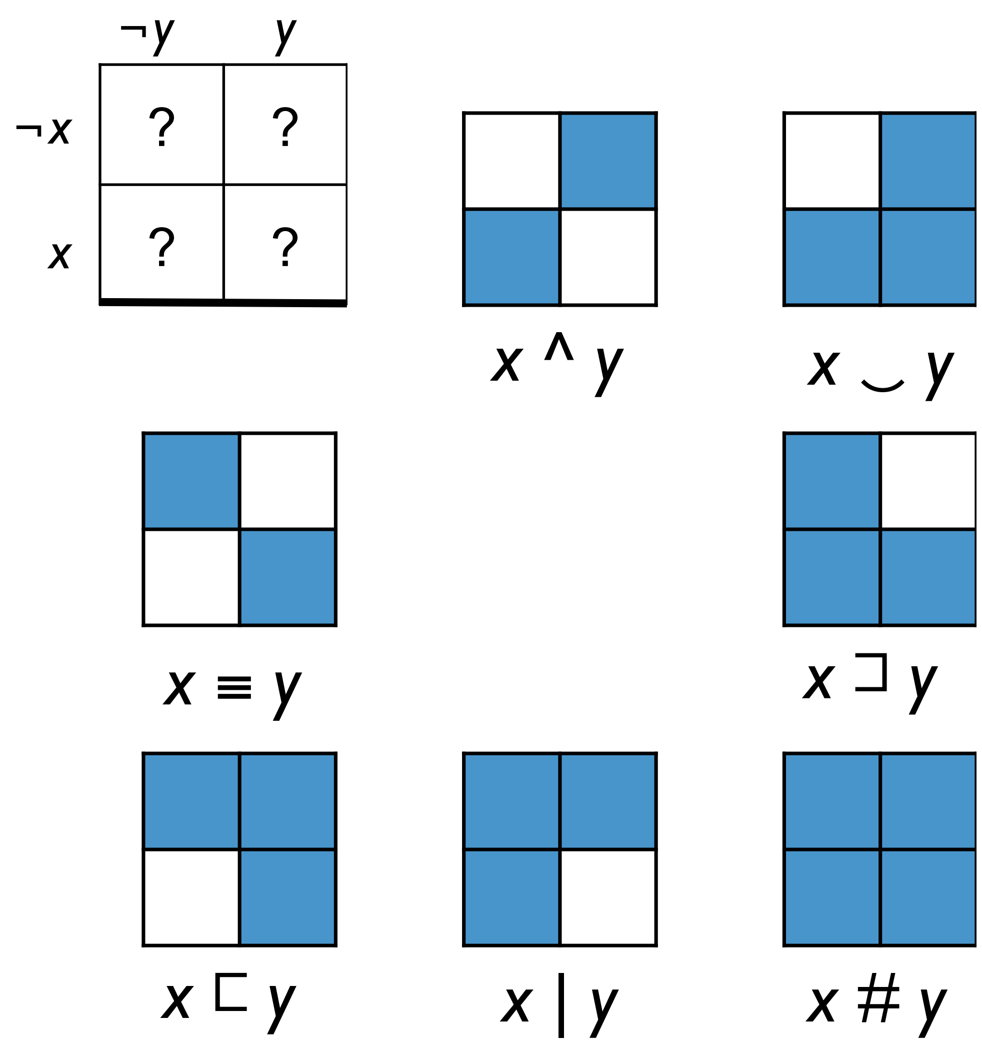


if ¬y&x and ¬x&y empty: x≡y
if only ¬y&x empty, x⊏y

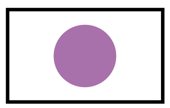
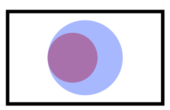
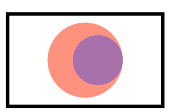
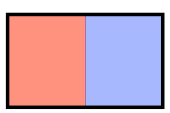
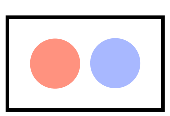
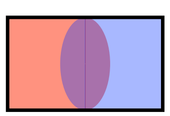
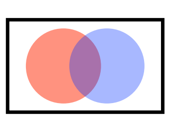
if there are no things that are neither x nor y and no things that are both x and y then x and y are the negation of each other (^)



⌣: nothing is neither x or y:
the union of x and y
together is the universe, e.g.
animal ⌣ non-ape



The set $\mathfrak{B}$ of 7 basic entailment relations

Venn	symbol	name	example
	x ≡ y	equivalence	*couch* ≡ *sofa*
	x ⊏ y	forward entailment (strict)	*crow* ⊏ *bird*
	x ⊐ y	reverse entailment (strict)	*European* ⊐ *French*
	x ^ y	negation (exhaustive exclusion)	*human* ^ *nonhuman*
	x | y	alternation (non-exhaustive exclusion)	*cat* | *dog*
	x ‿ y	cover (exhaustive non-exclusion)	*animal* ‿ *nonhuman*
	x # y	independence	*hungry* # *hippo*

Relations are defined for all semantic types: *tiny* ⊏ *small*, *hover* ⊏ *fly*, *kick* ⊏ *strike*, *this morning* ⊏ *today*, *in Beijing* ⊏ *in China*, *everyone* ⊏ *someone*, *all* ⊏ *most* ⊏ *some* edits

Simple edits

Entailment relations and semantic types

Entailment relation:

any set of ordered pairs where both elements belong to the same semantic type

$e \to t$: common nouns, intransitive verbs, adjectives (predicative)

$e \to e \to t$: transitive verbs

$(e \to t) \to (e \to t)$: adverbs

$(e \to t) \to (e \to t) \to t$: generalized quantifier

Lexical open-class SUBs, INSs and DELs

SUBstitutions of open class items belonging the to the same semantic type.

common nouns, adjectives, verbs, ...:

traditional lexicographic relations (stand in for ontological relations): synonyms, hypernyms

happy ≡ glad; forbid≡prohibit

soar ⊏ rise; scalding ⊏ hot

What about antonyms? Are they ^? rise,fall; hot,cold; dead,alive antonyms: |,rarely ^

why? rise,fall; hot,cold; dead,alive

rise|fall; hot|cold; dead^alive

More entities in the | category: cat|dog; but also unrelated entities: chalk|battle

What about unrelated adjectives:

weak, temporary?

weak#temporary

Verbs: skiing|sleeping; skiing#talking, these various categories are not readily available in lexicons.

Proper nouns: USA=United States (of America); JFK|FDR

Kyoto, Japan?

Kyoto$\sqsubset$Japan?

Kyoto is a beautiful city; Japan is a beautiful city in Kyoto$\sqsubset$in Japan

Kyoto|Japan

What about the $\smile$ relation? Very rare in lexical pairs: metallic$\smile$non-ferrous, mammal$\smile$nonhuman

Lots of simplifying assumptions in MacCartney. Some of them come from RTE, e.g. tense is ignored.

Closed-class terms SUBs

all $\equiv$ every

every $\sqsubset$ some (existential import!)

some ^ no: some birds talk^no birds talk

no | every: | means $x \cap y = 0$ and $x \cup y = U$ (every student passed <--> no student didn't pass)

four or more $\sqsubset$ two or more

exactly four | exactly two

four ? two

at most four $\smile$ at least two

most # ten or more

prepositions:

on a plane, in a plane

above the table, under the table

DEL/INS: default for DEL: $\sqsubset$; default for INS: $\sqsupset$; relies on the assumption that upward monotone contexts are the most prevalent ones when no further context is considered.

DELs and INSs

DEL default: ⊏ red car ⊏ car

INS default: ⊐ sing ⊐ sing off-key

OK for intersective modifiers (adjectives, relatives), conjuncts.

But:

negation creates a ^ relation :

sleep ^ didn’t sleep

fake: |

former: ?

alleged: #

We need a better typology of this

projections

Semantic composition of entailment relations

we have b(x,y)

what is the value of b(f(x),f(y))?

if f is upward monotone then b(f(x),(y)) = b(x,y)

some parrots talk ⊏ some birds talk

because *some* is upward monotone in its first argument

if f is downward monotone then ⊏ and ⊐ gets swapped.

if f is non monotone then ⊏ and ⊐ result in #:

most human talks # most animals talk.

we will calculate the effect of these compositions going up the tree of a grammar parse cf. Sánchez Valencia.

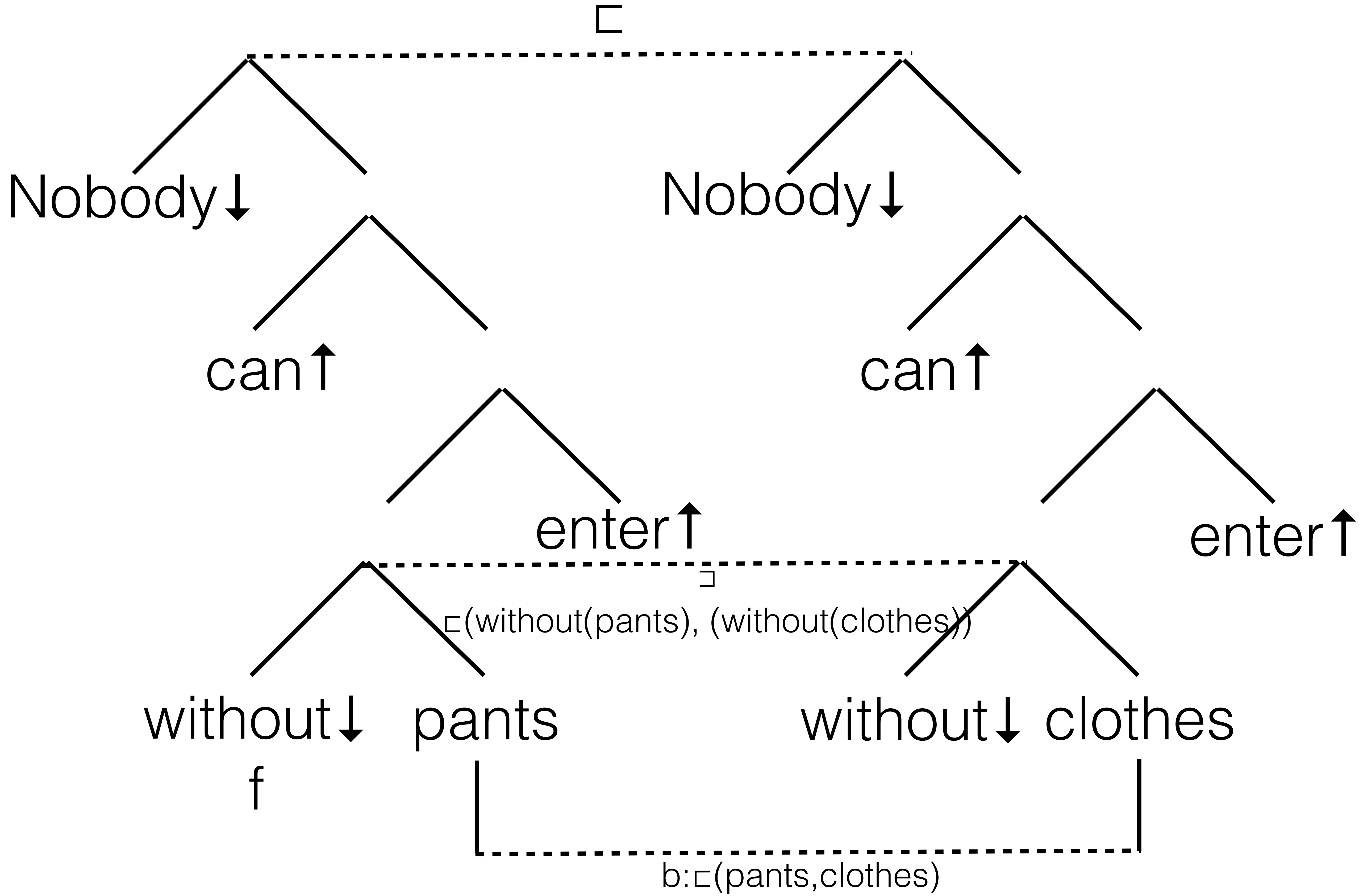
He moved without pants.

He danced without jeans.

pants⊐jeans

moved⊐danced

without?


⊏
Nobody↓
Nobody↓
can↑
can↑
enter↑
enter↑
⊐
⊏(without(pants), (without(clothes))
without↓
pants
without↓
clothes
f
b:⊏(pants,clothes)

Some projections

Negation

happy $\equiv$ glad $\Rightarrow$ not happy $\equiv$ not glad

kiss $\sqsubset$ touch $\Rightarrow$ not kiss $\sqsupset$ not touch

French | German $\Rightarrow$ not French $\smile$ not German

more than 4 $\smile$ less than 6 $\Rightarrow$ not more than 4 | not less than 6

Projectivity

If *f* has monotonicity…	UP			DOWN			NON		
	$\equiv$	$\rightarrow$	$\equiv$	$\equiv$	$\rightarrow$	$\equiv$	$\equiv$	$\rightarrow$	$\equiv$
	$\sqsubset$	$\rightarrow$	$\sqsubset$	$\sqsubset$	$\rightarrow$	$\sqsupset$	$\sqsubset$	$\rightarrow$	#
	$\sqsupset$	$\rightarrow$	$\sqsupset$	$\sqsupset$	$\rightarrow$	$\sqsubset$	$\sqsupset$	$\rightarrow$	#
	#	$\rightarrow$	#	#	$\rightarrow$	#	#	$\rightarrow$	#

f : connective	$\equiv$	$\sqsubset$	$\sqsupset$	^	|	$\smile$	#
negation (not)	$\equiv$	$\sqsupset$	$\sqsubset$	^	$\smile$	|	#
conjunction/intersection (and)	$\equiv$	$\sqsubset$	$\sqsupset$	|	|	#	#
disjunction (or)	$\equiv$	$\sqsubset$	$\sqsupset$	$\smile$	#	$\smile$	#
conditional antecedent (if)	$\equiv$	$\sqsupset$	$\sqsubset$	#	#	#	#
conditional consequent (then)	$\equiv$	$\sqsubset$	$\sqsupset$	|	|	#	#
biconditional (iff)	$\equiv$	#	#	^	#	#	#

Conditionals

f : connective	≡	⊏	⊐	^	|	◡	#
conditional antecedent (if)	≡	⊐	⊏	#	#	#	#
conditional consequent (then)	≡	⊏	⊐	|	|	#	#

If he drinks tequila, he feels nauseous; if he drinks liquor he feels nauseous.

If he drinks tequila, he feels nauseous; if he drinks tequila he feels sick.

If it is sunny we surf, if it is not sunny we surf

If it is sunny we surf, if it is sunny we do not surf

Conditionals

f : connective	≡	⊏	⊐	^	|	◡	#
conditional antecedent (if)	≡	⊐	⊏	#	#	#	#
conditional consequent (then)	≡	⊏	⊐	|	|	#	#

If he drinks tequila, he feels nauseous; if he drinks liquor he feels nauseous.

If he drinks tequila, he feels nauseous; if he drinks tequila he feels sick.

If it is sunny we surf # if it is not sunny we surf

If it is sunny we surf | if it is sunny we do not surf

Note this is not the same as the material implication.

Conditionals

f : connective	≡	⊏	⊐	^	|	◡	#
conditional antecedent (if)	≡	⊐	⊏	#	#	#	#
conditional consequent (then)	≡	⊏	⊐	|	|	#	#

If he drinks tequila, he feels nauseous ⊐ if he drinks liquor he feels nauseous.

If he drinks a tiny bit of tequila, he feels nauseous⊐ if he drinks a lot of tequila he feels nauseous.

quantifier	1st argument							2nd argument						
	≡	⊂	⊃	^	|	‿	#	≡	⊂	⊃	^	|	‿	#
some	≡	⊂	⊃	‿	#	°	#	≡	⊂	⊃	‿	#	‿	#
no	≡	⊃	⊂	|	#	|	#	≡	⊃	⊂	|	#	|	#
every	≡	⊃	⊂	|	#	|	#	≡	⊂	⊃	|	|	#	#
not every	≡	⊂	⊃	‿	#	‿	#	≡	⊃	⊂	‿	‿	#	#
at least two	≡	⊂	⊃	#	#	#	#	≡	⊂	⊃	#	#	#	#
most	≡	#	#	#	#	#	#	≡	⊂	⊃	|	|	#	#
exactly one	≡	#	#	#	#	#	#	≡	#	#	#	#	#	#
all but one	≡	#	#	#	#	#	#	≡	#	#	#	#	#	#

‿

<table>
<tr><th>quantifier</th><th>1st
|</th><th>2nd
|</th></tr>
<tr><td>some</td><td>#</td><td>#</td></tr>
<tr><td>no</td><td>#</td><td>#</td></tr>
<tr><td>every</td><td>#</td><td>|</td></tr>
<tr><td>not every</td><td>#</td><td>◡</td></tr>
<tr><td>at least two</td><td>#</td><td>#</td></tr>
<tr><td>most</td><td>#</td><td>|</td></tr>
<tr><td>exactly one</td><td>#</td><td>#</td></tr>
<tr><td>all but one</td><td>#</td><td>#</td></tr>
</table>

assume existential import for every

first argument: bird|fish: no fish talk # no birds talk

early|late

everyone was early|everyone was late

most people were early|most people were late

Verbs

Most verbs are upward-monotone, ^, | and ⌣ get projected as #

cats|dogs; eat cats # eat dogs
but:
German^non-German
is married to a German| is married to a non-German
is married to a German| is married to an Italian

What assumptions about marriage and nationality are made here?

some ignored cases

car $\sqsubset$ vehicle

red car $\sqsubset$ red vehicle

mouse $\sqsubset$ animal

big mouse ? big animal

human $\smile$ non-human

brown human | brown non-human

joins

Up to now: calculating results of one edit. But we will typically have more than one in a sentence. How do they interact?

Joins

dog⊏mammal ; mammal⊏animal
→ dog⊏animal

dog^non-dog ; non-dog^dog
→ dog=dog

Joins

$$R \bowtie S \stackrel{\text{def}}{\equiv} \{\langle x, z\rangle : \exists y \ (\langle x, y\rangle \in R \wedge \langle y, z\rangle \in S)\}$$

⊏	⋈	⊏	→	⊏
⊐	⋈	⊐	→	⊐
^	⋈	^	→	≡
≡	⋈	≡	→	≡
R	⋈	≡	→	R
≡	⋈	R	→	R

dog⊏mammal
mammal⊏animal
→ dog⊏animal

dog^non-dog
non-dog^dog
→ dog=dog

Non commutative joins

$\mid \bowtie \wedge \rightarrow \sqsubset$	$\wedge \bowtie \mid \rightarrow \sqsupset$
$\wedge \bowtie \smile \rightarrow \sqsubset$	$\smile \bowtie \wedge \rightarrow \sqsupset$
$\mid \bowtie \smile \rightarrow \sqsubset$	$\smile \bowtie \mid \rightarrow \sqsupset$
$\wedge \bowtie \# \rightarrow \#$	$\# \bowtie \wedge \rightarrow \#$

fish | human

human ^ nonhuman

fish ⊏ nonhuman

Non unique joins
(leaving out combinations with #)

⊏ ⋈ ⊐ ➞ ≣⊏⊐|#	⊏ ⋈ ◡ ⟞ ⊏^|◡#
⊐ ⋈ ⊏ ➞ ≣⊏⊐◡#	⊐ ⋈ | ⟞ ⊐^|◡#
| ⋈ ⊏ ➞ ⊏^|◡#	| ⋈ | ⟞ ≣⊏⊐|#
◡ ⋈ ⊐ ➞ ⊐^|◡#	◡ ⋈ ◡ ⟞ ≣⊏⊐◡#

gasoline|water	water|petrol	gasoline $\equiv$ petrol
pistol|knife	knife|gun	pistol $\sqsubset$ gun
woman|frog	frog|Eskimo	woman#Eskimo

$$R \bowtie S \stackrel{\text{def}}{=} \{\langle x, z\rangle : \exists y\ (\langle x, y\rangle \in R \wedge \langle y, z\rangle \in S)\}$$

Calculation: consider all ordered triples of a universe U. We get 256 equivalent classes...

<table>
<tr><th>⋈</th><th>≡</th><th>⊏</th><th>⊐</th><th>^</th><th>|</th><th>⌣</th><th>#</th></tr>
<tr><td>≡</td><td>≡</td><td>⊏</td><td>⊐</td><td>^</td><td>|</td><td>⌣</td><td>#</td></tr>
<tr><td>⊏</td><td>⊏</td><td>⊏</td><td>≡⊏⊐|#</td><td>|</td><td>|</td><td>⊏^|⌣#</td><td>⊏|#</td></tr>
<tr><td>⊐</td><td>⊐</td><td>≡⊏⊐⌣#</td><td>⊐</td><td>⌣</td><td>⊐^|⌣#</td><td>⌣</td><td>⊐⌣#</td></tr>
<tr><td>^</td><td>^</td><td>⌣</td><td>|</td><td>≡</td><td>⊐</td><td>⊏</td><td>#</td></tr>
<tr><td>|</td><td>|</td><td>⊏^|⌣#</td><td>|</td><td>⊏</td><td>≡⊏⊐|#</td><td>⊏</td><td>⊏|#</td></tr>
<tr><td>⌣</td><td>⌣</td><td>⌣</td><td>⊐^|⌣#</td><td>⊐</td><td>⊐</td><td>≡⊏⊐⌣#</td><td>⊐⌣#</td></tr>
<tr><td>#</td><td>#</td><td>⊏⌣#</td><td>⊐|#</td><td>#</td><td>⊐|#</td><td>⊏⌣#</td><td>≡⊏⊐^|⌣#</td></tr>
</table>

32 cases where there is a unique answer, 17 where there is a union relation
For practical purposes we may see all the ones that contain # as the same, namely #
In a certain sense a sad result, would have been nicer if we had found ≡, ⊏ (entailment) or ^,| (contradiction) or even ⊐,⌣, # (compatibility)

How it works

Example

Stimpy is a cat

Stimpy is not a poodle

SUB(cat,dog)

Stimpy is a dog

INS(not)

Stimpy is not a dog

SUB(dog,poodle)

Stimpy is not a poodle

Example:lexical relations from edit

Stimpy is a cat

Stimpy is not a poodle

SUB(cat,dog) $|$

Stimpy is a dog

INS(not) $\wedge$

Stimpy is not a dog

SUB(dog,poodle) $\sqsupset$

Stimpy is not a poodle

Example:projection

Stimpy is a cat

Stimpy is not a poodle

SUB(cat,dog) $| \rightarrow |$

Stimpy is a dog

INS(not) $\wedge \rightarrow \wedge$

Stimpy is not a dog

SUB(dog,poodle) $\sqsupset \rightarrow \sqsubset$

Stimpy is not a poodle

Example: joins

Stimpy is a cat			
Stimpy is not a poodle			
SUB(cat,dog)	$\mid$	$\mid$	$\mid$
Stimpy is a dog			
INS(not)	$\wedge$	$\wedge$	$\sqsubset$
Stimpy is not a dog			
SUB(dog,poodle)	$\sqsupset$	$\sqsubset$	$\sqsubset$
Stimpy is not a poodle			

Stimpy can run fast without a leash.

Stimpy can move.

DEL(without a leash)

DEL(fast)

SUB(run,move)

Stimpy can run fast without a leash.

Stimpy can move.

DEL(without a leash) $\sqsubset$

DEL(fast) $\sqsubset$

SUB(run,move) $\sqsubset$

Stimpy can run fast without a leash.

Stimpy can move.

DEL(without a leash) ⊏ ⊏ ⊏

DEL(fast) ⊏ ⊏ ⊏

SUB(run,move) ⊏ ⊏ ⊏

Stimpy can’t run fast with a leash

Stimpy can’t run fast. NO

DEL(with a leash) ⊏ ⊐ ⊐

Jimmy Dean moved without blue jeans

James Dean danced without pants

SUB(Jimmy,James)	$\equiv$	$\equiv$	$\equiv$
SUB(blue jeans,pants)	$\sqsubset$	$\sqsupset$	$\sqsupset$
SUB(move,dance)	$\sqsupset$	$\sqsupset$	$\sqsupset$

http://www.stanford.edu/~icard/logic&language/

http://www.stanford.edu/~icard/logic&language/djalali-potts-natlog.pdf

http://modestconsequences.wordpress.com/