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On the assessment of robustness

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Abstract

A framework for assessing robustness is proposed, taking basis in decision analysis theory. Robustness is assessed by computing both direct risk, which is associated with the direct consequences of potential damages to the system, and indirect risk, which corresponds to the increased risk of a damaged system. Indirect risk can be interpreted as risk from consequences disproportionate to the cause of the damage, and so the robustness of a system is indicated by the contribution of these indirect risks to total risk. A framework is presented for measuring robustness in this way, and implications for system modelling and acceptable levels of robustness are discussed. Numerical studies of idealized structural systems are performed using this framework, to demonstrate the use of the proposed robustness index and provide insight into system properties affecting robustness. Considered exposures include the design live load and an extraordinary exposure representing a fire or explosion that causes the loss of one or more system components. The results indicate that properties affecting system reliability, such as number of components or the stochastic properties of the load, also affect robustness. Perhaps more interestingly, it is seen that properties such as failure consequences and time to repair a damaged system also affect this measure of robustness. The assessment framework is applied here to study damage tolerance, but the procedure can be applied as well to other aspects of robustness such as tolerance to human error in design or construction.

Keywords: Robustness; Damage tolerance; Risk assessment; Reliability; Vulnerability; Redundancy

1. Introduction

Robustness has been recognized as a desirable property in structures and systems as a result of several high profile system failures, such as the Ronan Point Apartment Building in 1968, where the consequences were deemed unacceptable relative to the initiating damage. In this paper, robustness is taken to imply tolerance to damage from extreme loads or accidental loads, although the framework here is applicable to other adverse effects such as sensitivity to human error and deterioration.

Probabilistic risk assessment concepts are used here to formulate a new metric for robustness of an engineered system (where *system* is used to refer to both a physical structure as well as its associated inspection,

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maintenance and repair procedures). These concepts have been seen to be effective for assessing the risk and reliability of structures (e.g. [1]). Here, they are extended to the assessment of robustness. The proposed framework incorporates both the probabilities of adverse events and their associated consequences. In addition to quantifying the effect of the physical system's design, it accounts for the effect of inspection, maintenance and repair strategies as well as preparedness for accidental events. Further, decision analysis theory can be used to make decisions regarding acceptable robustness. The focus is on general principles rather than specific numerical techniques used to model damage and failure in a specific system.

Numerical examples are presented to illustrate the use of this framework and investigate the relationship between system performance and the index used to quantify robustness. The results demonstrate the effect of the type of exposure causing damage, parallel versus series system configurations and ductile versus brittle component behavior. Also considered are the effects of load redistribution after component failure, the number of system components, the detection of damage, and the consequences of component failure and system failure. Results from these many analyses illustrate the relationship between system properties and robustness. By identifying relationships between design parameters and the proposed index of robustness, one could identify guidelines for code specifications to increase the robustness of designed structures. Many properties that increase robustness, such as redundancy and ductility, are related to systems reliability. Other considerations, such as the consequences of system failure and the time to repair damage, are also important indicators of robustness, but may not always be given equal attention during typical design of structures.

Robustness definitions used for technical applications vary greatly, as seen in Table 1, which includes definitions from engineering as well as similar concepts from control theory, statistics, linguistics, etc. Many modern building codes refer to the need for robustness in structures [1–5], and an overview of these code provisions is provided by Ellingwood [6]. Many of these codes specify that structures should be robust in the sense that *the consequences of structural failure should not be disproportional to the effect causing the failure*. This description, however, does not provide any criteria for an engineer to use in measuring robustness or determining whether a system's level of robustness is acceptable. A measure of system robustness with the following properties would be helpful for design and analysis applications: (1) it is applicable to general systems (where system refers to both a physical structure and the associated inspection and maintenance strategies over time), (2) it allows for the ranking of alternative system choices, and (3) it provides a criterion for verifying acceptable robustness. The presence of these attributes will be helpful to consider when evaluating robustness-assessment methods.

Several researchers have considered vulnerability of specific classes of structures to specific damage scenarios [7–9]. A relatively well-studied case is the progressive collapse of frame structures. This work is important for characterizing failure probabilities for specific scenarios, but it is often difficult to generalize these findings to other types of systems or other damage.

Other researchers have considered methods of quantifying robustness that are applicable to any engineered system. Information-gap theory has been applied to the problem of robustness [10], and although it can be applied to general systems, challenges remain for using the method to balance robustness improvements with their associated costs. Frangopol and Curley [11] and Fu and Frangopol [12] considered probabilistic indices

Table 1
A selection of proposed properties associated with robustness [23]

The consequences of structural failure are not disproportional to the effect causing the failure [2]
The ability... to react appropriately to abnormal circumstances (i.e. circumstances "outside of specifications"). [A system] may be correct without being robust [24]
The ability of a system to maintain function even with changes in internal structure or external environment [25]
A design principle of natural, engineering, or social systems that have been designed or selected for stability [23]
The degree to which a system is insensitive to effects that are not considered in the design [26]
Insensitivity against small deviations in the assumptions [27]
A robust solution in an optimization problem is one that has the best performance under its worst case (max-min rule) [28]
Instead of a nominal system, we study a family of systems and we say that a certain property (e.g. performance or stability) is robustly satisfied if it is satisfied for all members of the family [29]
The robustness of language... is a measure of the ability of human speakers to communicate despite incomplete information, ambiguity, and the constant element of surprise [30]

to measure structural redundancy, based on the relationship between damage probability and system failure probability. Lind proposed another generic measure of system damage tolerance, based on the increase in failure probability resulting from the occurrence of damage [13,14]. Ellingwood noted that probabilistic risk assessment can be used to assess robustness in a general manner [15]. In the following section, risk assessment concepts will be extended to create a quantitative measure of robustness.

2. A framework for robustness assessment

In Fig. 1, events that may damage a system are modeled as follows. First, an exposure occurs which has the potential of damaging components in the system; this is termed the exposure before damage, or EX_{BD}. If no damage occurs ($\bar{D}$), then the analysis is finished. If damage occurs, a variety of damage states (D) can result. For each of these states, there is a probability that system failure (F) results. Consequences are associated with each of the possible damage and failure scenarios, and are classified as either direct (C_{Dir}) or indirect (C_{Ind}). The event tree representation in Fig. 1 is a graphical tool for evaluating event scenarios that could occur to the system, and it also incorporates the associated probabilities of occurrence. This formulation is based on risk assessment methodologies from the Joint Committee on Structural Safety, with particular attention paid here to aspects of robustness.

An exposure is considered to be any event with the potential to cause damage to the system. Damage could come from extreme values of design loads such as snow loads, extraordinary loads such as explosions, or deterioration of the system through environmental processes such as corrosion. Damage refers to reduced performance of system components, and system failure refers to loss of functionality of the entire system. In the case that a design allows for some degree of reduced function (e.g. an allowance for some corrosion), then damage should refer to reduced function beyond the design level. Direct consequences are those associated with the initial damage, while indirect consequences are associated with the subsequent system failure; thus, the damage branch of Fig. 1 is associated with direct consequences, while the failure branch is associated with the direct consequences plus additional indirect consequences.

Consequences typically come in several forms: inconvenience to system users, injuries, fatalities, and/or financial costs. To allow for comparison, these effects can be combined into a scalar measure of consequences, often termed utility (or disutility in the case of negative consequences). Although combining financial costs with other consequences can be difficult and controversial, it is implicitly done whenever one makes decisions about system design, and so combining consequences explicitly makes the decision process more transparent and objective.

2.1. Characterizing probabilities

Current codes already require damage scenarios and consequences to be identified, so that it can be determined whether the two are proportional. The event tree of Fig. 1 requires the additional step of assigning probabilities to the exposures. This may be straightforward for typical design loads, but more difficult for other exposures such as sabotage. Despite this challenge, occurrence probabilities are needed if one wishes to efficiently allocate resources for risk reduction. For example, progressive collapse may be found to occur for some unusual damage scenario, but that scenario might be extremely unlikely and thus no (possibly

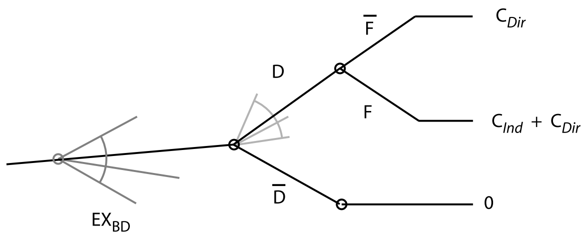

Fig. 1. An event tree for robustness quantification.

expensive) corrective action should be taken. Given that most problems addressed in this framework will have a time-dependent aspect, it is appropriate to assess probabilities in terms of probabilities per annum over the expected lifetime of the system. Many design exposures are characterized in this manner already.

2.2. Calculation of risk

With the event tree defined in Fig. 1, it is possible to compute the system risk due to each possible event scenario. This is done by multiplying the consequence of each scenario by its probability of occurrence, and then integrating over all of the random variables in the event tree. The risk corresponding to each branch is

$$R_{\mathrm{Dir}} = \int_x \int_y C_{\mathrm{Dir}} f_{D|\mathrm{EX_{BD}}}(y|x) f_{\mathrm{EX_{BD}}}(x)\,\mathrm{d}y\,\mathrm{d}x \quad (1)$$

$$R_{\mathrm{Indir}} = \int_x \int_y C_{\mathrm{Indir}} P(F|D=y) f_{D|\mathrm{EX_{BD}}}(y|x) f_{\mathrm{EX_{BD}}}(x)\,\mathrm{d}y\,\mathrm{d}x \quad (2)$$

where $f_Z(z)$ is used to denote the probability density function of a random variable Z. These integrals may be evaluated either through numerical integration or Monte Carlo simulation. For computing damage and failure probabilities, techniques from systems reliability can be used [16]. Note that for brevity and tractability, a Markovian assumption has been made in Eqs. (1) and (2) (i.e. the probability of failure of a system with a given damage state is assumed to be conditionally independent of the exposure causing the damage). This conditional independence can often be achieved by carefully defining, for example, the damage states. It may not always hold, however (e.g. it does not hold for some of the examples below), and in these cases it can be relaxed. For example, the failure probability can be a function of the damage state and the exposure state. In cases where Markovian dependence does hold, it will greatly simplify analysis of the larger decision tree considered later.

3. Index of robustness

To quantify robustness, results from the previous section are used. A robust system is considered to be one where indirect risks do not contribute significantly to the total system risk. With this in mind, the following index of robustness (denoted I_{Rob}) is proposed, which measures the fraction of total system risk resulting from direct consequences

$$I_{\mathrm{Rob}} = \frac{R_{\mathrm{Dir_1}}}{R_{\mathrm{Dir_1}} + R_{\mathrm{Ind_1}}} \quad (3)$$

The index takes values between zero and one depending upon the source of risk. If the system is completely robust and there is no risk due to indirect consequences, then $I_{\mathrm{R}} = 1$. At the other extreme, if all risk is due to indirect consequences, then $I_{\mathrm{R}} = 0$.

By examining Fig. 1 and the above equations, several trends between system properties and the robustness index can be identified. First, this index measures only the *relative* risk due to indirect consequences. The acceptability of the direct risk should be determined through other criteria prior to robustness being considered. A system might be deemed robust if its direct risk is extremely large (and thus large relative to its indirect risk), but that system should be rejected on the basis of reliability criteria rather than robustness criteria. Guidelines for evaluating acceptable reliability can be found in existing codes (e.g. [1]).

Second, the index will depend not just upon failure probabilities of damaged systems, but also upon the relative probabilities of the various damage states occurring. Thus, a building could be designed to have a low failure probability after an individual column is removed, but if it is deemed likely that an exposure would cause the loss of two columns and if the building was vulnerable to that damage, then it could still be deemed non-robust.

Third, the index accounts for both the probability of failure of the damaged system *and* the consequences of that failure. For instance, if sensing systems were able to detect damage and signal an evacuation before failure could occur, then robustness could be increased without changing the probabilities of damage or failure. Thus, the possibility of detection and the time between damage and failure can be accounted for in an appropriate manner. The property of robustness depends upon system properties such as redundancy, ductility, load redistribution and damage detection, but it also depends upon failure consequences. This ability to incorporate consequences as well as probabilities is an important new development.

Finally, this index can be easily extended to account for multiple exposures, or more complicated event trees than the one in Fig. 1. The robustness index will still be equal to the sum of direct risk divided by the sum of total risk. This will be shown explicitly below.

A related index of vulnerability can also be defined within the same framework. This index is defined as

$$I_V = \frac{\sum_i R_{\mathrm{Dir}_i}}{\sum_j C_{\mathrm{Dir}_{\mathrm{el}j}}} \tag{4}$$

where R_{Dir_i} are the direct risk terms associated with the i damage scenarios considered. The denominator is the sum of direct consequences associated with the loss of each components of the system. The denominator is a sum over the j system components, rather than the sum over damage states used elsewhere, and so it provides a measure of the total direct consequences associated with the system. This vulnerability index provides an indicator of the risks associated with structural damage, normalized by the direct risk exposure. It will take values smaller than the probability of occurrence of damage (i.e. I_V values less than 10^{-3} or 10^{-4} per year are expected). This index is not associated with robustness properties, but it is easy to calculate within this framework and may provide a helpful indicator of the system's performance.

4. Decision analysis

This robustness framework is particularly valuable when used to guide decisions. Fig. 1 above represents only the most basic system. But by considering other aspects of system performance, as illustrated in Fig. 2, the framework can be used for decision-making. The additional symbols used in Fig. 2 are defined as follows. The decision nodes a_d and a_r represent design and decision actions, respectively. Design actions (a_d) include maintenance, inspection, monitoring and disaster preparedness. Response actions (a_r) could include evacuation, repair, and re-occupation of the system. Response actions can only be taken if damage is indicated (I). A second exposure, EX_{AD}, is included in this tree to represent exposure after damage. By incorporating post-damage exposures, the framework can now account for the increased vulnerability of

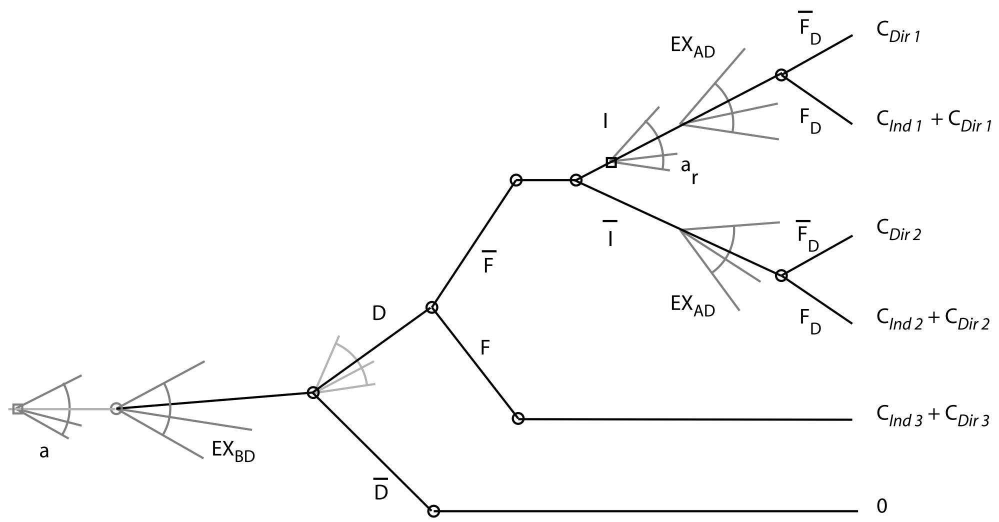

Fig. 2. An event tree that incorporates system choice and post-damage exposures.

the structure in the future. Further, the opportunity to intervene through response actions is now modeled explicitly. These actions are conditional on the indication of a damage (the probability of which is affected by the inspections and monitoring actions which are here assumed to be part of the design decisions). Based on the damage level of the system, and the actions taken as a result of detection, the system has a probability of failure due to post-damage exposures (EX_{AD}).

It is implied that if damage is indicated, then action will be taken either to reduce failure consequences (e.g. by evacuating a structure) or the probability of failure (e.g. through repairs). The choice of post-detection action is part of the definition of the system. The probability of damage detection will be dependent upon actions to inspect the system, and on the type of damage and type of exposure causing damage. For example, damage from explosions will likely be detected, while corrosion of an inaccessible component may not be detected.

The basic choice of design action is now also explicitly included at the beginning of the tree. These actions will include design of the physical structure, maintenance to prevent structural degradation, inspection and monitoring for identifying damages, and disaster preparedness actions. These actions, along with the post-damage response actions, are included here because they will affect the probabilities and consequences associated with the other branches, and so this decision tree can be used as a tool to identify actions which minimize risk and maximize robustness in a system. When alternative systems have varying costs, then these costs should be included in the consequences (and the branch of the tree corresponding to $\bar{D}$ will no longer have zero consequences for some system choices). With this formulation, a pre-posterior analysis can be used to identify systems which minimize total risk.

For a given set of actions, the risks associated with each branch can be computed as before. For example, the indirect risk R_{Ind_2} would now be computed as

$$R_{\text{Ind}_2} = \int_x \int_y \int_z C_{\text{Ind}_2} P(F|D=y,\bar{I},\text{EX}_{\text{AD}}=z) f_{\text{EX}_{\text{AD}}|D,I}(z|y,\bar{I}) P(\bar{I}|D=y) \times P(\bar{F}|D=y) f_{D|\text{EX}_{\text{BD}}}(y|x) f_{\text{EX}_{\text{BD}}}(x)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x \quad (5)$$

The index of robustness can then be calculated using a direct generalization of Eq. (3)

$$I_{\text{Rob}} = \frac{\sum_i R_{\text{Dir}_i}}{\sum_i R_{\text{Dir}_i} + \sum_j R_{\text{Ind}_j}} \quad (6)$$

4.1. Conditional robustness

An additional potentially useful analysis step is the calculation of conditional robustness, given that a structure has a specific level of damage. The event tree for this system starts at the damage branch, as shown in Fig. 3. In this case, all risks are computed conditional on the specified damage. Thus, the R_{Ind_2} term from Eq. (5) would instead become

$$(R_{\text{Ind}_2}|D=y) = \int_z C_{\text{Ind}_2} P(F|D=y,\bar{I},\text{EX}_{\text{AD}}=z) f_{\text{EX}_{\text{AD}}|D,I}(z|y,\bar{I}) P(\bar{I}|D=y) P(\bar{F}|D=y)\,\mathrm{d}z \quad (7)$$

and the conditional index of robustness would become

$$(I_{\text{Rob}}|D=y) = \frac{\sum(R_{\text{Dir}_i}|D=y)}{\sum(R_{\text{Dir}_i}|D=y) + \sum(R_{\text{Ind}_j}|D=y)} \quad (8)$$

where $R_{\text{Ind}}|D=y$ is computed using Eq. (7) for each conditional state i, and $R_{\text{Dir}}|D=y$ is computed in a similar manner.

Computing conditional robustness may be useful for an analyst, because rather than immediately grouping all possible damage scenarios together, individual damage scenarios are examined in order to see which scenarios cause particularly low robustness. Given that the Markovian assumption holds, in accordance with Section 2.2, the conditional risks can then be combined to determine the total (unconditional) robustness. First, the probabilities of each damage scenario occurring can be computed as

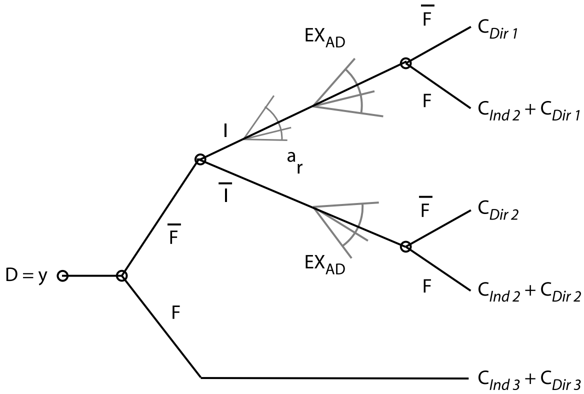

Fig. 3. An event tree, conditional on a level of damage $D = y$.

$$P(D = y) = \int_x f_{D|\mathrm{EX_{BD}}}(y|x) f_{\mathrm{EX_{BD}}}(x)\,\mathrm{d}x \tag{9}$$

Then this result can be combined with the conditional risks (e.g. Eq. (7)), to compute total robustness

$$I_{\mathrm{R}} = \frac{\int_y \sum_i (R_{\mathrm{Dir}_i}|D = y)P(D = y)\,\mathrm{d}y}{\int_y \sum_i (R_{\mathrm{Dir}_i}|D = y)P(D = y)\,\mathrm{d}y + \int_y \sum_j (R_{\mathrm{Ind}_j}|D = y)P(D = y)\,\mathrm{d}y} \tag{10}$$

By assessing conditional robustness first and then later combining the results with the probabilities of damage states occurring, the damage scenarios contributing to low robustness (i.e. those with both high probability of damage occurrence and high conditional risk given the damage state) will be more easily identified.

4.2. Robustness-based design

Building codes typically emphasize design for component loads and component failures. System-level behavior is not considered to the same extent, occasionally leading to systems suffering from cascading system failures associated with a lack of robustness. An ideal system design is the one having minimal risk, achieved by balancing reduction of both direct and indirect risks against the cost of the risk-reducing actions. This approach has practical difficulties, however, and thus has not yet been realized. In the meantime, the following design framework is useful to consider.

It is assumed here that modern building codes are calibrated to reduce direct risks from component damage to acceptable levels. Thus, direct risks in a code-conforming system can be assumed to be approximately 'optimal.' What remains is to consider robustness issues, in order to also reduce indirect risks to an 'optimal' level. For any system, risk-reducing actions (denoted a) should be undertaken until the marginal cost of additional risk reduction is equal to the marginal benefit in terms of reduced indirect risks, expressed as

$$\frac{\partial C_{\mathrm{R}}(a)}{\partial a} = -\frac{\partial \sum R_{\mathrm{Ind}}(a)}{\partial a} \tag{11}$$

Where $C_R(a)$ is the cost of risk reduction and $\sum R_{\mathrm{Ind}}(a)$ is the total indirect risk, and both are dependent upon a. The actions a may take on binary or discrete values and there may also be boundary limitations on possible actions; these are not formally addressed in Eq. (11), but the important concept of optimality remains. It is here that the decision tree of Fig. 2 will be a useful tool for directly balancing the cost of an action with the risk reduction from that action.

Although risk-reducing actions may include changes to structural detailing, a much broader scope of actions can also be considered. Referring to Eq. (5), one can see that indirect risk might be reduced through actions to reduce exposures, reduce damage probabilities, increase damage identification probabilities, reduce failure probabilities, or reduce consequences. This suggests that possible actions to consider include restricting access to critical components, strengthening components of the system, adding inspection and maintenance activities, or adding alarm systems and egress routes to aid evacuation. Although structural engineers naturally focus on structural improvements because they are knowledgeable and skilled in this area, these other types of actions should be considered as well.

As with component design, it is difficult in practice to find a true optimum system design. The principle of Eq. (11), however, is useful for identifying robustness-based design criteria for building codes, as well as selecting system improvements for individual projects. Robustness calculations of the type shown below will be useful for identifying system features which increase robustness at an acceptable price, and thus come closer to the optimum design specified in Eq. (11). When used in conjunction with existing requirements for design of components, this approach should lead to systems with satisfactory performance in terms of both reliability and robustness.

5. Example calculations

A set of example systems are now considered, to illustrate the information needed in the above computations and to provide insight regarding system properties affecting robustness. The principle structural system considered is a parallel system with n components equally resisting an applied load, as illustrated in Fig. 4. It will be subjected to several types of exposures that have the potential to cause damage.

Systems of up to 10 components are considered in the following. The components are assumed to be either perfectly ductile (rigid/plastic) or brittle with random resistances, and each carries an equal portion of the applied load. When a component's resistance is exceeded, the additional load not carried by that component (in addition to the original load, in the case of brittle failure) is either redistributed equally to the other components (Fig. 4a) or not redistributed (Fig. 4b). Note that this type of system has been a topic of research by other authors [17,18], and the brittle variant is known as a Daniels system.

The robustness assessment utilized requires assessment of component damage and system failure probabilities. In this case, damage is considered to have occurred when the load exceeds the resistance of at least one component. Failure occurs when the resistance of *all* components is exceeded. When extraordinary loads causing the loss of one or more components are considered, damage will represent the loss of a component due to either the extraordinary load or the applied load, and failure will indicate that the remaining components are not able to resist the applied load.

Exposures and resistances are modeled according to the JCCS Probabilistic Model Code [1]. Individual component resistances (denoted R_i) are assumed to be lognormally distributed with a coefficient of variation (CoV) of 7%, which represents the CoV in yield strength of typical structural steel members. Exposures are defined as events which have the ability to cause damage to the system. Several types of exposures will be considered below. The first is the applied load due to environmental effects such as dead and live loads. This is denoted S in Fig. 4. The applied load is assumed to be Weibull distributed. The mean value of the load is chosen to equal one, and various levels of CoV are considered. The mean component resistance is selected so that each element has a specified probability of damage, given the distribution of applied loads; here the annual probability of damage is 0.001 for all components. Design of a component to have a specified damage probability is consistent with current structural design code requirements. The applied loading represents the

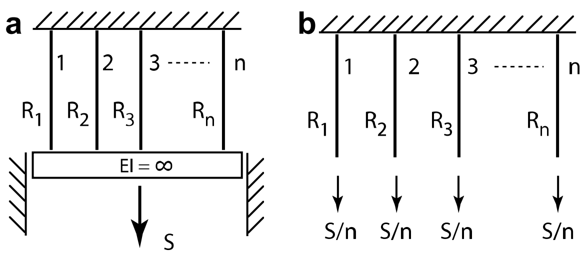

Fig. 4. Schematic illustrations of the systems. (a) A parallel system with load redistribution after component damage. (b) A parallel system with no load redistribution after component damage.

annual maximum value of the time varying environmental loads (dead loads are neglected here). By applying the load in this manner, we can treat the time varying load using a simpler time-invariant approach. In some examples below, the load will be modified to reflect a shorter reference period.

In addition to environmental loads, exposures due to extraordinary events not directly foreseen in the design will also be considered. These events, such as fire, impact or explosion, are assumed to cause the loss of one or more components from the system. Unless otherwise specified, the following system properties will also be used: ductile component failure (i.e. ideal plastic), uncorrelated component resistances, uniform redistribution of excess loads after component damage, and system failure consequences equal to 100 times the component damage consequences.

In order to evaluate the robustness index, it is necessary to compute component damage probabilities and system failure probabilities. These tasks relate to computation of system reliability [16]. In general, analytic solutions are not available for these computations. Here, Monte Carlo simulation was adopted, in order to obtain results for a variety of stochastic loading and resistance scenarios. These computations could be performed in practice by using software which links structural finite element analysis with reliability calculations [19].

6. Design loads

Robustness calculations for a variety of systems subjected to design loads are summarized in this section, in order to provided insight into the relationship between system properties and the proposed robustness index.

6.1. Effect of number of components and load variability

The results in Fig. 5 illustrate the effect of varying coefficients of variation of the loading. In Fig. 5a, results are shown for ductile systems with a varying number of components, and in Fig. 5b the same result is shown for brittle systems. It is apparent that increasing the number of components increases a system's robustness. Note also that the system with one component has nearly zero robustness, because the damage to one component immediately causes system failure. Increasing the coefficient of variation of loading has the same effect as reducing the number of components. This result occurs because systems with fewer components or a higher CoV of loading are more likely to experience system failure, given that damage to a component has occurred. The robustness of brittle systems is nearly zero in all cases; this is because brittle failure of a component is very likely to trigger cascading system failure when the component's load is redistributed. For these cases, the default choice of uncorrelated component resistances was used. Increasing the correlation among resistances has the same effect as reducing the number of components in the system.

For many of the systems shown in Fig. 5 and later figures, the index of robustness (I_{Rob}) takes values that are very near to 0. This implies that the risk to a system primarily comes from indirect risks due to system failure. Perhaps this is true even for systems that are considered robust, because the (typically) much larger consequences of system failure outweigh their low probability of occurrence relative to more common component damage states. Further calibration is needed to identify the values for this index that should be interpreted to indicate a robust system.

6.2. Effect of failure consequences

The robustness index used here is believed to be unique in that it accounts for both the probabilities and consequences of failure. Thus, one might choose to increase robustness by reducing consequences rather than reducing the failure probability. To maintain generality, here the direct (failure) consequences are defined as a multiple of the indirect (damage) consequences. In Fig. 6, robustness values are reported for the default system with a range of failure consequence values. It is seen that systems with higher failure consequences have relatively lower robustness.

This result can be applied to real systems with intuitive results. Consider a building with alarm systems and effective egress routes to aid evacuation after a fire or explosion. These systems do not decrease the probability of failure, but they increase robustness by allowing the possibility of evacuation and thus lowering

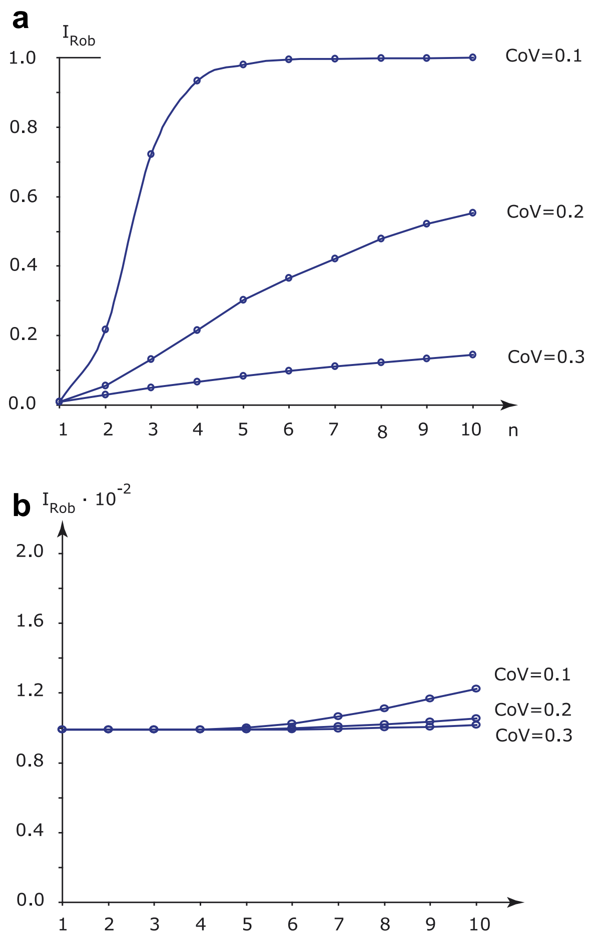

Fig. 5. Index of robustness versus number of system components (n), for three values of the load's coefficient of variation. (a) Ideal ductile parallel system. (b) Ideal brittle parallel system. Note the change of y-axis scale.

consequences (fatalities) if a failure were to occur. Conversely, seven Swiss fire workers were killed by the collapse of a fire-damaged parking garage in 2004 [20]. If there are no people in a structure to rescue, perhaps the decision to decrease the potential failure consequences by keeping the fire workers out of the structure is more robust than the decision to fight the fire, even if the decision increases the probability of structural failure. The ability of this index to account for both probabilities and consequences allows it to account for these effects.

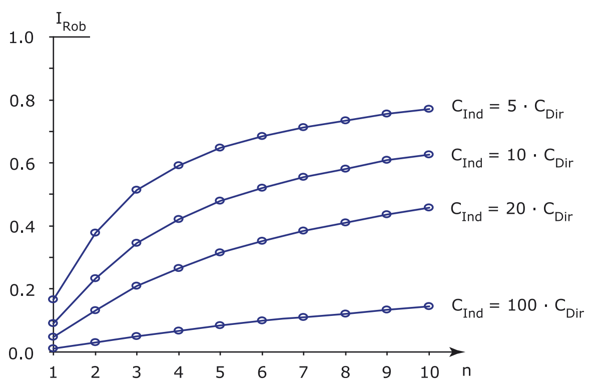

Fig. 6. Index of robustness versus number of system components (n), illustrating the influence of indirect (failure) consequences. Results are shown for a ductile system with load redistribution and a load CoV of 0.3.

Fig. 6 also illustrates the non-intuitive result that for fixed indirect consequences, robustness is *increased* if the direct (damage) consequences increase. This demonstrates that robustness does not address all aspects of system performance, and thus cannot be considered in isolation. For a system with high consequences from component damage, other building code requirements should ensure that those components be designed using a greater importance factor or some similar modification. That is, problems associated with component damage consequences should be treated using other code requirements, because robustness criteria address only system effects and indirect consequences. Current design codes already address component damage probabilities and consequences, and so this measure of robustness provides a useful supplement to those provisions.

6.3. Effect of load redistribution

An important consideration in robustness is how loads will be carried by the structure in the event of damage. Building codes often specify that a structure should be 'tied together' so that if, for example, a column in a building is damaged, the surrounding columns will be able to carry the load [2]. This requirement removes the possibility of local collapses of a building, but in some situations it may be preferable to compartmentalize the damage rather than tying the structure together and increasing the probability of global collapse. For example, a portion of the Murrah Federal Building in Oklahoma City collapsed after a large bomb was detonated there in 1995 [21], but if the structure had been further tied together, the entire structure might have collapsed. These possibilities can be considered using parallel systems with either complete load redistribution or zero load redistribution after component failures (e.g. Fig. 4).

For a system with no load redistribution, indirect consequences can occur even if the entire system has not failed. This is because when, for example, a building column fails without load redistribution, this triggers a local (subsystem) collapse and causes indirect consequences that would not have occurred if the building components were tied together. For this example, consequences were assigned as follows: one unit of direct consequences for a component failure in either system, 100 units of indirect consequences for total system failure in either system, and $100/n$ units of indirect consequences for a local failure in the system with no load redistribution (where n is the number of components). Two parallel systems with five components each are considered in Fig. 7, and subjected to applied loads having several values of coefficient of variation. The system with no load redistribution has constant robustness, because its components are calibrated to have a fixed probability of failure and component failures do not affect the other system components. For the system with load redistribution, high load CoVs lead to decreased robustness because, although individual components have a fixed probability of failure, extreme loads capable of causing cascading failures are more likely. Thus it is observed that the choice of which load redistribution system is robust depends upon the properties of the

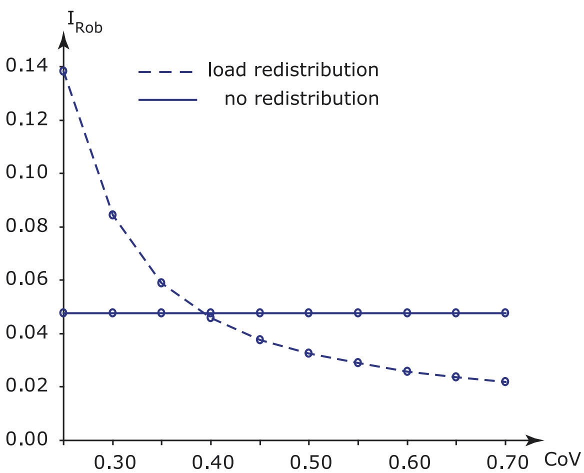

Fig. 7. Index of robustness versus coefficient of variation of applied load, for five-component parallel systems with and without load redistribution after component damage.

system and applied load. The robustness assessment used here can help identify the proper strategy to use for a given situation.

Simple load redistribution models were used here to illustrate this concept, but in real structures the behavior is more complex. Multistory buildings must redistribute loads somewhere after a column failure – either to surrounding columns or to the slab on the floor below after a collapse. Nonlinear and dynamic effects may need to be considered as well. Nonetheless, compartmentalization of failure is a design strategy that may increase the robustness of some systems. Calibration with real structures will be helpful, and this example illustrates the potential of the proposed framework to address the problem rigorously.

7. Extraordinary loads

Previous examples focused on damage and failures from extreme values of design loads. Extraordinary loads such as fires or explosions are also of great interest for robustness assessment, however. Here an extraordinary load is considered which results in the loss of either one or two components of the system. The 'instantaneous' live loads which might be acting on the structure at the time of component loss are also applied (i.e. a weekly maximum live load is applied rather than a yearly maximum, recognizing that the structure may not remain in the damaged state for an entire year). The above calculations can be repeated when considering this exposure instead of the design load; rather than repeating those analyses, this exposure is used to illustrate several other aspects of the framework.

7.1. Effect of subsequent loading and repair

An exposure which causes damage to a system has a possibility of immediate system failure as discussed above, but also a possibility of later failure due to subsequent loading. Both factors can be accounted for in robustness calculations when the expanded tree from Fig. 2 is used.

Here the default parallel system is subjected to an initial exposure consisting of an extraordinary load, which causes the loss of one component, plus an instantaneous live load. This exposure causes damage as well as the possibility of system failure. After the initial exposure, two possibilities are considered. In the first case, it is assumed that the damage is detected and repairs occur after one week; thus the damaged system is only subjected to a weekly maximum live load. Repairs are assumed to restore the system to its original state. In the second case, the damage is not detected and thus the yearly maximum live load is applied as the secondary exposure to measure the probability of subsequent failure of the damaged structure. The structure is more robust when the damage is detected and repair is performed quickly, because the damaged structure has less opportunity to fail from subsequent exposures, as seen in Fig. 8. This demonstrates that robustness is affected not only by the initial design choice, but also by later actions such as monitoring and repair. The analysis can be easily generalized to include features such as imperfect damage detection, partial repairs and random repair times.

7.2. Conditional robustness

Extraordinary loads can also be used to compute conditional robustness, as defined in Eq. (8). Example results from this computation are given in Fig. 9a, which shows conditional robustness of the default system, conditioned upon the loss of either one or two components due to an extraordinary load. Once probabilities of occurrence for these two damage states (as well as others that are deemed important) are available, then the conditional robustness values can easily be combined with these probabilities to compute the marginal robustness value, as shown in Eq. (10). In Fig. 9b, conditional robustness is shown for systems with one component removed, for systems with varying design-load CoVs and load redistribution properties. Again it is seen that compartmentalizing the damage may be more robust than tying the system together in some situations.

These brief examples using extraordinary loads demonstrate that the framework can be used to assess robustness from terrorist attacks and other extraordinary events of concern. Further, rather than completely removing a component, one could weaken the component to represent the effect of corrosion or other slow

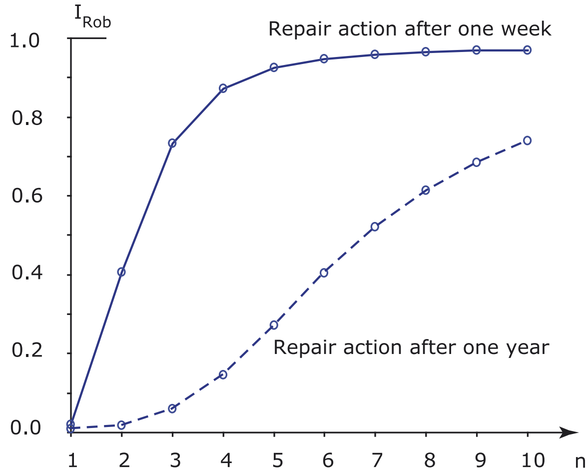

Fig. 8. Robustness of a system exposed to an extraordinary load, with repairs performed after one week or one year. Results are shown for a ductile system with load redistribution and a CoV for the annual maximum load of 0.3 (revised appropriately in the case of one week loading).

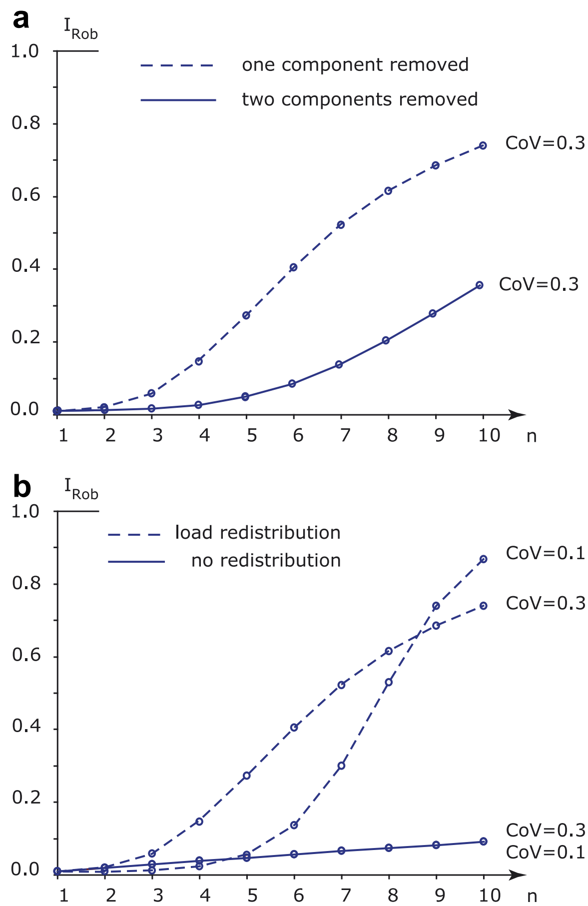

Fig. 9. (a) Conditional robustness of the system for two damage states: loss of one component and loss of two components due to extraordinary loads. (b) Conditional robustness with one component removed, for four systems with varying load CoV and load redistribution properties (the two plots for no load redistribution lie on top of each other).

deterioration process. When used with the decision tree in Fig. 2, decisions about inspections and repairs of corroding structures are a straightforward extension of the calculations shown here.

8. Conclusions

A framework has been presented for assessment of system robustness for systems subject to structural damage, and illustrated with numerical results for a variety of idealized structural systems. The framework is founded in probabilistic risk analysis, which forms a basis for rational decision making. This allows the robustness of different systems to be compared, and aids in determining optimal levels of robustness for a given system. The proposed index measures the additional risk to the system due to indirect consequences of damage. If indirect consequences contribute significantly to risk, either because there is a high probability of damage triggering a secondary system failure or because the consequences of secondary failure are high, then the consequences of damage are likely disproportionate to the cause of damage, and the resulting robustness index will be low. Here the robustness framework is used to investigate damage tolerance, but the assessment procedure can be applied as well to other aspects of robustness such as tolerance to human error in design or construction.

Example calculations illustrate the possibility of using the proposed index to quantify robustness of real systems. Idealized systems are considered in order to clearly illustrate the effects of various system parameters. Results indicate that systems with few redundant components, with highly correlated components, and with high load variability relative to resistance variability typically display low robustness. Further, it is observed that ductile systems are more robust than brittle systems. These results are consistent with past observations regarding systems reliability, and may be intuitively clear to many engineers. Results which may be less obvious include the observation that reducing failure consequences can increase robustness even if the probability of failure is unchanged. This indicates that the robustness index can account for the effects of, for example, improved warning and evacuation systems which lower the number of fatalities resulting from a failure. Also, system inspection and repair is seen to increase the robustness of systems by lowering the probability that a damaged system will fail in the future.

Ongoing work is focusing on computation of robustness for real systems. This will help address a range of important issues for system design. An index-of-robustness value that indicates acceptable robustness in systems will be identified. Design decisions will be considered, to identify efficient actions that should be taken to increase robustness. Case studies will be examined to illustrate the trade-offs between compartmentalizing damage and tying a structure together to prevent local collapses. These insights should be helpful for formulating building design codes which are able to more fully address design guidelines for robustness. For example, analogous studies of system reliability have facilitated the development of 'redundancy factors' for code-based design that account for system reliability in a simplified manner (e.g. [22]). Results of the type presented here thus offer hope for identification of 'robustness factors' that may be used in code specifications.

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