Characterization of random fields and their impact on the mechanics of geosystems at multiple scales

Graduate Researcher: Andrew Seifried
Faculty Advisor: Jack Baker
Collaborators: Jose Andrade and Qiushi Chen (Northwestern University)

Introduction:

- The goal of this project is to create a computationally efficient framework to simulate and analyze samples of granular media to study the effects of multi-scale fluctuations in material parameters on overall behavior.
- Random field models are used to describe variability of the parameter (porosity in this case).
- Multi-scale techniques are employed to keep fine resolution where it is needed in the analysis while reducing the overall computational burden of large samples.

The primary focus at Stanford is the creation of a simulation program using MATLAB software which will be coupled with a finite element program from collaborators at Northwestern University.

Data Structure:

The basic premise of the multi-scale structure is shown here. Each point in the simulated grid is actually an average value of the finest scale points within the space it occupies. The number of scales required, the resolution of each scale, and which scale is to be considered the “finest” to optimize accuracy and efficiency will be evaluated.

Sequential simulation process:

Samples will be generated as standard normal random variables, then transformed over to the target distribution by:

$$x = F^{-1}(\Phi(z))$$

- $x$: value of porosity
- $\Phi$: standard normal variable
- $F^{-1}$: inverse CDF of target distribution

The field is randomly populated one point at a time by the conditional multivariate normal distribution:

$$\left(\bar{Z}^{(i)}(z) \mid Z^{(\text{sampled})} = z\right) \sim N\left(\sum_{j} \cdot \Sigma_{j}^{-1} \cdot z, 1 - \sum_{j} \cdot \Sigma_{j}^{-1} \cdot \Sigma_{j} \right)$$

$Z$: realization of std. normal RV being generated
$\Sigma$: covariance matrix
$\Phi(A,B)$: implies that $z$ is being generated from a normal distribution with mean $A$ and variance $B$

Example of generated sample:

Areas anticipated to be under high strain and adjacent cells with large gradients in the value of average porosity are then subdivided down to the next scale. The figure to the left is an example of a simulated plate strain compression sample with a single scale subdivision. The diagonal band is the expected failure surface in this example.

Going Forward:

As described previously, this research aims to determine the number of scales required, the resolution of each scale, and which scale is to be considered the “finest” in order to optimize accuracy and efficiency. This research will also:

- Dynamically couple the simulation with the finite element analysis to create a fully automated program.
- Validate results from each end by comparing full scale results to actual tests and verifying parameter distributions to actual samples.

As a final note, it is worth pointing out that once developed, these multi-scale tools will be able to provide insight into other materials and phenomena of interest (such as fracture strength of steel or crushing strength of concrete) beyond their context here.

Obtaining parameters for simulation:

A histogram of porosity data for a sample of granular media reveals a lognormal distribution. Maximum likelihood methods were used to estimate parameters to be used in the simulation.

Spatial correlation in all directions within the sample was determined by matching variogram models (spherical, gaussian, and exponential) to the sample data.

Correlation between local averages is computed by:

$$\rho_{\text{ij}(l,x,y)} = \sum \sum \rho_{\text{point, point}}$$

where the points in the summation are of the finest scale, and $\rho_{\text{point, point}}$ is calculated from the variogram.

Example of generated sample:

Areas anticipated to be under high strain and adjacent cells with large gradients in the value of average porosity are then subdivided down to the next scale. The figure to the left is an example of a simulated plate strain compression sample with a single scale subdivision. The diagonal band is the expected failure surface in this example.

Going Forward:

As described previously, this research aims to determine the number of scales required, the resolution of each scale, and which scale is to be considered the “finest” in order to optimize accuracy and efficiency. This research will also:

- Dynamically couple the simulation with the finite element analysis to create a fully automated program.
- Validate results from each end by comparing full scale results to actual tests and verifying parameter distributions to actual samples.

As a final note, it is worth pointing out that once developed, these multi-scale tools will be able to provide insight into other materials and phenomena of interest (such as fracture strength of steel or crushing strength of concrete) beyond their context here.