Energy-Efficient Packet Transmission Over a Wireless Link

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***Abstract*—The paper considers the problem of minimizing the energy used to transmit packets over a wireless link via *lazy* schedules that judiciously vary packet transmission times. The problem is motivated by the following observation. With many channel coding schemes, the energy required to transmit a packet can be significantly reduced by lowering transmission power and code rate, and therefore transmitting the packet over a longer period of time. However, information is often time-critical or delay-sensitive and transmission times cannot be made arbitrarily long. We therefore consider packet transmission schedules that minimize energy subject to a deadline or a delay constraint. Specifically, we obtain an optimal offline schedule for a node operating under a deadline constraint. An inspection of the form of this schedule naturally leads us to an online schedule which is shown, through simulations, to perform closely to the optimal offline schedule. Taking the deadline to infinity, we provide an exact probabilistic analysis of our offline scheduling algorithm. The results of this analysis enable us to devise a lazy online algorithm that varies transmission times according to backlog. We show that this lazy schedule is significantly more energy-efficient compared to a deterministic (fixed transmission time) schedule that guarantees queue stability for the same range of arrival rates.**

***Index Terms*—Minimum energy transmission, optimal schedules, power control, wireless LAN.**

I. INTRODUCTION

UBIQUITOUS wireless access to information is gradually becoming a reality. Dedicated-channel voice transmission (as in most existing cellular systems, e.g., GSM, IS-95) has already become a widespread and mature technology. Packet-switched networks are being considered for heterogeneous data (combined voice, web, video, etc.) to efficiently use the resources of the wireless channel. Wireless LANs and personal area networks, where packetization is more suited to the bursty nature of the data, are being developed and deployed. More recently, ad hoc networks and networks of distributed sensors are being designed utilizing the robustness and asynchronous nature of transmissions in packet networks.

A key concern in all of these wireless technologies is energy efficiency. The mobility of a hand-held wireless device is limited by the fact that its battery has to be regularly recharged from a power source. In a sensor network, the sensors may not be charged once their energy is drained, hence the lifetime of the network depends critically on energy. It has therefore been of wide interest to develop low-power signaling and multiaccess schemes, signal processing circuits, and architectures to increase battery life.

There has been a lot of research on transmission power control schemes over the past few years (see, e.g., [4], [8], [11], [12], [14], [19] and [21]). The chief motivation of these schemes, however, has not been to directly conserve energy but rather to mitigate the effect of interference that one user can cause to others. The results ranged from obtaining distributed power control algorithms to determining the information theoretic capacity achievable under interference limitations [2], [13].

Whereas most power control schemes aim at maximizing the amount of information sent for a given average power constraint, a recent study [3] considers minimizing the power subject to a specified amount of information being successfully transmitted. Rather than minimizing power, [5] considers the question of minimizing energy directly; and compares the energy efficiency, defined as the ratio of total amount data delivered and total energy consumed, of several medium access protocols.

In this paper we expand on the work in [17] and consider the problem of minimizing the energy used by a node on a point-to-point link to transmit packetized information within a given amount of time. The setup attempts to model a number of realistic wireless networking scenarios: 1) a node with finite lifetime and finite energy supply such as in a sensor network [16]; 2) a battery operated node with finite-lifetime information; that is, information that must be transmitted before a deadline; and 3) a battery-operated node that is periodically recharged. In this case, minimizing transmission energy ensures that the node does not run out of energy before it is recharged.

To minimize transmission energy, we vary packet transmission times and power levels to find the optimal schedule for transmitting the packets within the given amount of time. The observation that leads to this approach is that transmission energy can be lowered by reducing transmission power and transmitting a packet over a longer period of time. It has been known (see [1], and more recently, [9]) that, with many coding schemes, the energy needed to transmit a given amount of information is strictly decreasing and convex in the transmission duration. The next section provides a few examples in support of this observation.

The above discussion implies that it makes sense to transmit a packet over a longer period of time to conserve energy. However, since all packets must be transmitted within the given amount of time, the transmission time of any one packet cannot be arbitrarily long as this would leave too little a time for the transmis-

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sion of future packets and increase the overall energy spent. The rest of the paper attempts to understand this tradeoff precisely and to exploit it to devise energy-efficient schedules.

A. Outline of the Paper

In Section II, we set up the framework for modeling the minimum energy packet transmission scheduling problem for a node with a finite lifetime T. In Section III, we find the offline optimal-energy transmission schedule for fixed length packets and Section IV extends these results to variable length packets. In Section V, the form of the offline optimal-energy schedule (OOE) suggests a natural online schedule. We show that this online schedule is quite energy efficient—it achieves an average energy that is quite close to the optimal offline algorithm. The comparison is done using simulations since it is hard to conduct analytical comparisons for finite T.

By letting $T \to \infty$ and assuming Poisson arrivals, we are able to conduct an exact analysis of the optimal offline schedule (as outlined in the Appendix). This gives us insight into how to design an energy-efficient online schedule that assigns transmission times according to the backlog in the queue. We call this schedule *Lazy*. Under a queue stability constraint, Lazy is compared with the Deterministic schedule and it is shown to beat the Deterministic schedule significantly for a range of packet arrival rates. This is an interesting comparison because among schedules that are independent of the packet arrival process (and hence are oblivious of backlogs), the deterministic schedule achieves the smallest average delay,[1] which implies that it has the highest transmission times, and hence the lowest energy. The fact that lazy schedules are more energy-efficient than the deterministic schedule, therefore, demonstrates the need to take advantage of lulls in packet arrival times.

Finally, Section VI outlines further work and concludes the paper.

II. Problem Setup

Consider a wireless node whose lifetime is finite, equal to T, say. Suppose that M packets arrive at the node in the time interval $[0, T)$ and must be transmitted to a receiver before T (see Fig. 1). In the figure, the arrival times of packets, t_i, are marked by crosses and interarrival epochs are denoted by d_i. Without loss of generality, we assume that the first packet is received at time 0. The node transmits the packets according to a schedule that determines the beginning and the duration of each packet transmission. We seek an answer to the question: How should the transmission schedule be chosen so that the total *energy* used to transmit the packets is minimized?

Let $\mathcal{E}(q)$ denote the transmission energy per bit for the particular coding scheme that is being used, which has code rate $R = 1/q$ bits/transmission.[2] Hence, q is the number of transmissions per bit. The following are the only assumptions we make about $\mathcal{E}(q)$ in this paper.

[1]By the well-known theorem "determinism minimizes delay" [20].

[2]The word *transmission* in this paper frequently refers to the transmission of an entire packet. The term *bits/transmission* will be used to indicate the number of bits per channel use (also known as *bits/symbol*), i.e., the information theoretic rate, and *transmissions/bit* indicates the reciprocal of this rate.

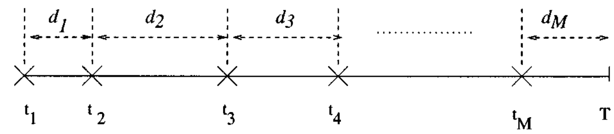

Fig. 1. Packet arrivals in $[0, T)$.

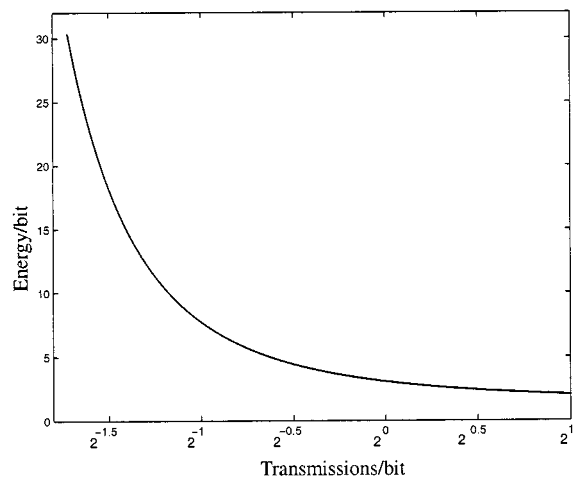

Fig. 2. Energy per bit versus transmission time with optimal coding.

1) $\mathcal{E}(q) \geq 0$.
2) $\mathcal{E}(q)$ is monotonically decreasing in q.
3) $\mathcal{E}(q)$ is strictly convex in q.

Assumption 1) is obvious. We shall now demonstrate that assumptions 2) and 3) hold by considering the energy required to reliably transmit one bit of information over a wireless link. The following two examples assume a discrete time additive white Gaussian noise (AWGN) channel model for the wireless link and consider two different channel coding schemes.

1) Optimal Channel Coding: Consider an AWGN wireless channel with average signal power constraint P and noise power N. As is well known [6], the information theoretically optimal channel coding scheme, which employs randomly generated codes, achieves the channel capacity given by

$$C_1 = \frac{1}{2}\log_2\left(1 + \frac{P}{N}\right) \text{ bits/transmission.} \tag{1}$$

More precisely, given any $0 < \alpha < 1$, information can be reliably transmitted at rate $R = \alpha C_1$. To determine the energy per bit $\mathcal{E}$, note that $q = 1/R$ can be interpreted as the number of transmissions per bit. Substituting into (1), we obtain

$$\mathcal{E} = qP = qN\left(2^{2/\alpha q} - 1\right).$$

It is easy to see that $\mathcal{E}$ is *monotonically decreasing and convex* in q, and that as q approaches infinity the energy required to transmit a bit, $\mathcal{E}_\infty = (2/\alpha)\ln 2 = (1/\alpha)1.3863$. Fig. 2 plots $\mathcal{E}(q)$ versus q for $N = 1$ and $\alpha = 0.99$. The range of q in the plot corresponds to SNR values from 20 dB down to 0.11 dB. This is a typical range of SNR values for a wireless link. In this range, $\mathcal{E}$ can be decreased by a factor of 20 by increasing transmission time and correspondingly decreasing power.

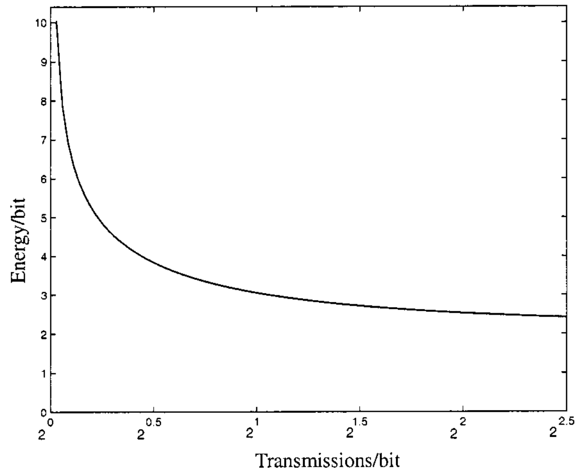

Fig. 3. Energy per bit versus transmission time for the suboptimal coding scheme.

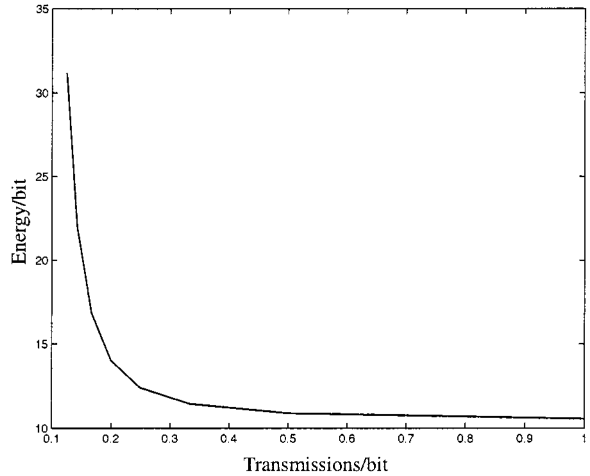

Fig. 4. Energy per bit versus transmission time with uncoded MQAM modulation.

2) A Suboptimal Channel Coding Scheme: Consider a scheme that uses antipodal signaling [18] and binary block error correction coding again over an AWGN wireless link. It can be shown that the minimum error probability per bit using antipodal signaling over an AWGN channel is given by

$$p = Q\left(\sqrt{\frac{P}{N}}\right)$$

where Q is the well-known Gaussian Q-function. Using this signaling scheme, the channel is converted into a binary symmetric channel (BSC) with cross-over probability p. The optimal binary error correction coding scheme achieves the Shannon capacity for the BSC, given by

$$C_2 = 1 - h(p) \text{ bits/transmission}$$

where $h(p)$ is the binary entropy function.

Thus, for any $0 < \alpha < 1$, information can be reliably transmitted at rate $R = \alpha C_2$. Again interpreting $q = 1/R$ to be the number of transmissions per bit, the energy per bit can be computed as a function of q. This is depicted in Fig. 3 for $N = 1$ and $\alpha = 0.99$. Note that $\mathcal{E}$ is again *monotonically decreasing and convex* in q and converges to a limit $\mathcal{E}_\infty = 2.108$, which, as expected, is larger than that using optimal coding. The range of q in the figure corresponds to an SNR between 20 dB to -3.7 dB. In this range, $\mathcal{E}$ drops by over a factor of 8.

3) An Uncoded MQAM Scheme: Here, each symbol can assume one of $M = 2^r$ possible values, hence, one symbol carries r bits of information, i.e., the number of transmissions per bit is $1/r$. This modulation scheme is used in some practical wireless systems, e.g., the IEEE 802.11a wireless LAN standard recommends MQAM with $r = 1, 2, 4, 6$ in each OFDM subcarrier.

Fig. 4 plots the energy per bit as a function of the number of transmissions per bit using MQAM, when the bit error rate (BER) is less than 10^{-4}.

The three examples above support the assumptions made earlier about $\mathcal{E}$. Now, denote by $w(\tau)$ the transmission energy for a packet that takes τ transmissions (i.e., channel uses). If the packet contains B bits, this corresponds to $q = \tau/B$ transmissions/bit, and $w(\tau) = B\mathcal{E}(\tau/B)$. From our earlier assumptions about $\mathcal{E}(q)$, it follows that $w(\tau)$ is a nonnegative, monotonically decreasing and convex function of τ.

III. Optimal Offline Scheduling

In this section, we determine the energy-optimal offline schedule for the above model of a finite number of packets to be transmitted in a given finite-time horizon. This offline optimal schedule provides a lower bound on energy that can be used for evaluating the performance of online algorithms. After briefly introducing the basic setup, a necessary condition for optimality is stated (Lemma 2). This motivates the definition of the specific schedule OOE (Definition 1). OOE is shown to be feasible (Lemma 3), and energy-optimal (Theorem 1).

Suppose that the arrival times $t_i, i = 1, \ldots, M$ of the M packets that arrive in the interval $[0, T)$ are known in advance, i.e., before $t = 0$. Assuming equal length packets each with B bits, the offline scheduling problem is to determine the transmission duration vector $\vec{\tau}$ so as to minimize $w(\vec{\tau}) = \sum_{i=1}^{M} w(\tau_i)$.

The assumption that $w(\tau)$ decreases with τ trivially implies that it is suboptimal to have $\sum_i \tau_i < T$, for we could simply increase the transmission times of one or more packets and reduce $w(\vec{\tau})$. Hence, we only consider "non-idling" transmission schedules where $\sum_i \tau_i = T$. It is also sufficient to consider FIFO schedules where packets are transmitted in the order they arrive. The FIFO and non-idling conditions combined with the causality constraint, i.e., that packet transmission cannot begin before the packet arrives, yield the following feasibility conditions.

Lemma 1: A non-idling FIFO schedule $\vec{\tau}$ is feasible iff

$$\sum_{i=1}^{k} \tau_i \geq \sum_{i=1}^{k} d_i$$

for $k = 1, 2 \ldots, M-1$, and $\sum_{i=1}^{M} \tau_i = \sum_{i=1}^{M} d_i$.

We now state a key observation of this section.

Lemma 2: A necessary condition for optimality is

$$\tau_i \geq \tau_{i+1} \qquad \text{for } i \in \{1, \ldots, M-1\}. \tag{2}$$

Proof: Let $\vec{\tau}$ be a feasible vector such that $\tau_i < \tau_{i+1}$ for some $i \in \{1, \ldots, M-1\}$. Further suppose that it is optimal. Consider the schedule $\vec{\sigma}$ such that $\sigma_i = \sigma_{i+1} = (\tau_i + \tau_{i+1})/2$ and $\sigma_j = \tau_j$ for $j \neq i, i+1$. It is easy to verify that $\vec{\sigma}$ is feasible. Comparing the energies used by $\vec{\tau}$ and $\vec{\sigma}$, we obtain

$$\begin{aligned} w(\vec{\tau}) - w(\vec{\sigma}) &= w(\tau_i) + w(\tau_{i+1}) - w(\sigma_i) - w(\sigma_{i+1}) \\ &= w(\tau_i) + w(\tau_{i+1}) - 2w\left(\frac{\tau_i + \tau_{i+1}}{2}\right) \overset{(a)}{>} 0 \end{aligned}$$

where inequality (a) follows from the strict convexity of $w(\cdot)$. This contradicts the optimality of $\vec{\tau}$ and proves the lemma. ■

The proof of the above lemma suggests the form of the optimal offline schedule: equate the transmission times of each packet, subject to feasibility constraints. We proceed to do just this and define the optimal schedule next.

Given packet interarrival times d_i, $i \in \{1, \ldots, M\}$, let $k_0 = 0$, and define

$$m_1 = \max_{k \in \{1, \ldots, M\}} \left\{ \frac{1}{k} \sum_{i=1}^{k} d_i \right\}$$

and

$$k_1 = \max\left\{ k: \frac{1}{k} \sum_{i=1}^{k} d_i = m_1 \right\}.$$

For $j \geq 1$, let

$$m_{j+1} = \max_{k \in \{1, \ldots, M-k_j\}} \left\{ \frac{1}{k} \sum_{i=1}^{k} d_{k_j+i} \right\}$$

and

$$k_{j+1} = k_j + \max\left\{ k: \frac{\sum_{i=1}^{k} d_{k_j+i}}{k} = m_{j+1} \right\}$$

where k varies between 1 and $M - k_j$. We proceed as above to obtain pairs (m_j, k_j) until $k_j = M$ for the first time.[3] Let $J = \min\{j: k_j = M\}$. The pairs (m_j, k_j), $j = 1, \ldots, J$, are used to define a schedule whose transmission times are denoted by $\vec{\tau}^*$, and Theorem 1 shows that $\vec{\tau}^*$ is the optimal offline schedule.

Definition 1: The vector of transmission times $\vec{\tau}^*$ given by

$$\tau_i^* = m_j \qquad \text{if } k_{j-1} < i \leq k_j \tag{3}$$

is called OOE.

Fig. 5 shows an example run of OOE. The arrivals in the figure have been randomly generated (with exponentially distributed interarrival intervals of mean 1) using a time window of $T = 20$. The heights of the bars are proportional to the magnitudes of the ds and τ^*s.

Lemma 3: The following hold for $\vec{\tau}^*$ of OOE.

i) It is feasible and $\sum_{i=1}^{M} \tau_i^* = T$.

ii) It satisfies the condition stated in Lemma 2.

[3]Note that, by definition, $k_j < k_{j+1}$. Therefore, the k_js are increasing with j and will equal M for some j.

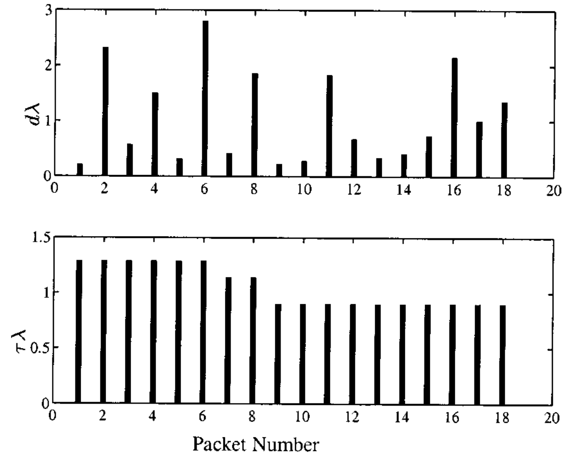

Fig. 5. An example run of ds (top) and τ^*s (bottom).

Proof: We first establish i). For $1 \leq k \leq k_1$,

$$\sum_{i=1}^{k} \tau_i^* = k m_1 \geq k \sum_{i=1}^{k} \frac{d_i}{k} = \sum_{i=1}^{k} d_i,$$

where the inequality follows from the definition of m_1.

Similarly for $k_1 < k \leq k_2$,

$$\begin{aligned} \sum_{i=1}^{k} \tau_i^* &= k_1 m_1 + (k - k_1) m_2 \\ &\geq \sum_{i=1}^{k_1} d_i + (k - k_1) \sum_{i=k_1+1}^{k} \frac{d_i}{k - k_1} = \sum_{i=1}^{k} d_i. \end{aligned}$$

Proceeding thus, we obtain that $\sum_{i=1}^{k} \tau_i^* \geq \sum_{i=1}^{k} d_i$ for all k, $1 \leq k \leq M$.

To finish the proof of i) it only remains to show that $\sum_{i=1}^{M} \tau_i^* = T$. Now

$$\sum_{i=1}^{M} \tau_i^* = \sum_{j=1}^{J} (k_j - k_{j-1}) m_j \tag{4}$$

where $k_0 = 0$ and $k_J = M$. By definition of m_j and k_j, it follows that for each j

$$(k_j - k_{j-1}) m_j = \sum_{k=k_{j-1}+1}^{k_j} d_k.$$

Using this in (4), we get $\sum_{i=1}^{M} \tau_i^* = \sum_{i=1}^{M} d_i = T$. This establishes i).

As for ii), it suffices to show that $m_j > m_{j+1}$ since this implies $\tau_i^* \geq \tau_{i+1}^*$ for each i. We first show that $m_1 > m_2$. For any $k \in [k_1 + 1, k_2]$,

$$\begin{aligned} m_1 &= \frac{d_1 + \cdots + d_{k_1}}{k_1} \\ &\overset{(a)}{>} \frac{d_1 + \cdots + d_{k_1}}{k} + \frac{d_{k_1+1} + \cdots + d_k}{k} \\ &= \frac{k_1}{k} m_1 + \frac{(k - k_1)}{k} \frac{(d_{k_1+1} + \cdots + d_k)}{k - k_1} \end{aligned}$$

where (a) follows from the definition of m_1. Choosing $k = k_2$, we obtain

$$m_1 > \frac{k_1}{k_2} m_1 + \frac{k_2 - k_1}{k_2} m_2$$

from which it follows that $m_1 > m_2$.

In an exactly similar fashion it can be shown that $m_2 > m_3$ and, more generally, that $m_j > m_{j+1}$ for any $j, 1 \le j \le J-1$. This establishes ii) and completes the proof of the lemma. ■

Theorem 1: The schedule OOE of Definition 1 is the optimum offline schedule.

Proof: Consider any other feasible schedule $\vec{\tau}$. Let i be the first index where $\tau_i \neq \tau_i^*$. We show that $w(\vec{\tau}) > w(\vec{\tau}^*)$. There are two possibilities to consider.

Case 1—$\tau_i > \tau_i^*$: Since $\sum_j \tau_j = T$ (otherwise $\vec{\tau}$ would idle for some time, making it suboptimal), there must be at least one $j > i$ for which $\tau_j < \tau_j^*$. Let $r = \min\{j: i < j \le M, \tau_j < \tau_j^*\}$. Consider the schedule $\vec{\sigma}$ defined as follows:

$$\sigma_i = \tau_i - \Delta \tag{5}$$

$$\sigma_r = \tau_r + \Delta \tag{6}$$

$$\sigma_j = \tau_j, \qquad \text{for all } j \neq i, r \tag{7}$$

where $\Delta = \min\{(\tau_i - \tau_i^*), (\tau_r^* - \tau_r)\}$.

Claim 1: The schedule $\vec{\sigma}$ does not idle and is feasible.

Proof of Claim 1: Since $\sum_j \sigma_j = \sum_j \tau_j = T$, it does not idle. By the definition of the indices i and r, and the feasibility of $\vec{\tau}$ and $\vec{\tau}^*$, it follows that

$$\sum_{j=1}^{k} \sigma_j = \sum_{j=1}^{k} \tau_j \ge \sum_{j=1}^{k} d_j, \qquad \text{for } 1 \le k \le i-1 \tag{8}$$

$$\sum_{j=1}^{i} \sigma_j \ge \sum_{j=1}^{i} \tau_j^* \ge \sum_{j=1}^{i} d_j \tag{9}$$

$$\sum_{j=1}^{k} \sigma_j \ge \sum_{j=1}^{k} \tau_j^* \ge \sum_{j=1}^{k} d_j, \qquad \text{for } i < k \le r \tag{10}$$

$$\sum_{j=1}^{k} \sigma_j = \sum_{j=1}^{k} \tau_j \ge \sum_{j=1}^{k} d_j, \qquad \text{for } k > r. \tag{11}$$

This verifies the conditions for feasibility in 1, and Claim 1 is proved.

Claim 2: $w(\vec{\sigma}) < w(\vec{\tau})$.

Proof of Claim 2:

$$\begin{aligned} w(\vec{\tau}) - w(\vec{\sigma}) &= w(\tau_i) + w(\tau_r) - w(\sigma_i) - w(\sigma_r) \\ &= w(\tau_i) - w(\tau_i - \Delta) + w(\tau_r) - w(\tau_r + \Delta) \\ &\overset{(a)}{>} 0 \end{aligned}$$

where inequality (a) follows from two facts: 1) $w(\cdot)$ is strictly convex and decreasing and 2) $\tau_i > \tau_r$. That is, for any real-valued function f that is strictly convex and decreasing, and for any $a, b \in \mathbb{R}$ such that $a < b$, we have $f(b) - f(b-\delta) + f(a) - f(a+\delta) > 0$, where $0 < \delta < b - a$. This proves Claim 2.

Thus, under Case 1, any feasible schedule $\vec{\tau}$ may be modified to obtain a more energy efficient schedule $\vec{\sigma}$. Therefore, schedules which are different from $\vec{\tau}^*$ in the sense of Case 1 are suboptimal.

Case 2—$\tau_i < \tau_i^*$: We shall argue for a contradiction and show that such a $\vec{\tau}$ is infeasible.

From the definition of $\vec{\tau}^*$, we know that $\tau_i^* = m_j$, assuming $k_{j-1} < i \le k_j$. In fact $\tau_l^* = m_j$ for all $k_{j-1} < l \le k_j$.

Since i is the first index where $\vec{\tau}$ and $\vec{\tau}^*$ disagree, $\tau_l = \tau_l^*$ for all $l < i$. Suppose that the schedule $\vec{\tau}$ satisfies the condition of Lemma 2 (else it is suboptimal and we are done). It follows that $\tau_i \ge \cdots \ge \tau_{k_j}$, and we obtain

$$\sum_{l=1}^{k_j} \tau_l^* > \sum_{l=1}^{k_j} \tau_l. \tag{12}$$

But, by definition of $\vec{\tau}^*$,

$$\sum_{l=1}^{k_j} \tau_l^* = \sum_{l=1}^{j} (k_l - k_{l-1}), \qquad m_l = \sum_{l=1}^{k_j} d_l.$$

Equation (12) now gives $\sum_{l=1}^{k_j} \tau_l < \sum_{l=1}^{k_j} d_l$, implying that $\vec{\tau}$ is infeasible.

This contradiction concludes Case 2 and the proof of Theorem 1 is complete. ■

Lazy scheduling trades off delay for energy. To do this, it must necessarily buffer packets. The energy savings that come from simply keeping a small buffer is best illustrated by an example. Imagine a scheme that keeps a buffer size of zero (hence transmission times can at most be set equal to interarrival times). For the set of packet arrivals shown in Fig. 5, the optimal offline schedule achieves an energy of 65.445 and the zero-buffer scheme (which, therefore, has no queueing delay) achieves an energy 77.78 × 10^5; five orders of magnitude larger [using an energy function $\tau(2^{6/\tau} - 1)$].

IV. Extension to Optimal Offline Scheduling of Variable-Length Packets

This section extends the results of the previous section to variable-length packets. As the optimal schedule and the arguments that establish its optimality are virtually identical to those of the previous section, for brevity, we shall omit a number of details.

Consider a node at which M packets arrive in $[0, T]$, and suppose that the length of packet i equals l_i bits. Without loss of generality we consider schedules that do not idle. Hence, the feasibility condition in Lemma 1 continues to apply, i.e., $\vec{\tau}$ is feasible if and only if for $1 \le k < M$, $\sum_{i=1}^{k} \tau_i \ge \sum_{i=1}^{k} d_i$.

The arrival times $t_i, i = 1, \ldots, M$ are known at time 0, as are the lengths of the packets, $\vec{l} = [l_1, l_2, \ldots, l_M]$. As before, assume that $t_1 = 0$. Define $w(l, \tau) = l\mathcal{E}(\tau/l)$. The problem is to determine $\vec{\tau}$, the vector of transmission times, so as to minimize $w(\vec{l}, \vec{\tau}) \triangleq \sum_{i=1}^{M} w(l_i, \tau_i)$.

Since it is suboptimal to consider idling policies, we shall only consider schedules $\vec{\tau}$ that satisfy $\sum_i \tau_i = T$.

Lemma 4: A necessary condition for optimality is

$$\frac{\tau_i}{l_i} \ge \frac{\tau_{i+1}}{l_{i+1}}, \qquad \text{for } i \in \{1, \ldots, M-1\}. \tag{13}$$

Proof: Let $\vec{\tau}$ be a feasible vector such that $\tau_i/l_i < \tau_{i+1}/l_{i+1}$ for some $i \in \{1, \ldots, M-1\}$. Further suppose that it is optimal. Consider the schedule $\vec{\sigma}$ such that $\sigma_i/l_i = \sigma_{i+1}/l_{i+1} = (\tau_i + \tau_{i+1})/(l_i + l_{i+1})$ and $\sigma_j = \tau_j$ for $j \neq i, i+1$. It is easy to verify that $\vec{\sigma}$ is feasible (because $\sigma_i > \tau_i$). Comparing the energies used by $\vec{\tau}$ and $\vec{\sigma}$, we obtain

$$\begin{aligned} &w(\vec{l}, \vec{\tau}) - w(\vec{l}, \vec{\sigma}) \\ &= w(l_i, \tau_i) + w(l_{i+1}, \tau_{i+1}) \\ &= l_i\mathcal{E}\left(\frac{\tau_i}{l_i}\right) + l_{i+1}\mathcal{E}\left(\frac{\tau_{i+1}}{l_{i+1}}\right) - l_i\mathcal{E}\left(\frac{\sigma_i}{l_i}\right) - l_{i+1}\mathcal{E}\left(\frac{\sigma_{i+1}}{l_{i+1}}\right) \\ &= l_i\mathcal{E}\left(\frac{\tau_i}{l_i}\right) + l_{i+1}\mathcal{E}\left(\frac{\tau_{i+1}}{l_{i+1}}\right) - (l_i + l_{i+1})\mathcal{E}\left(\frac{\tau_i + \tau_{i+1}}{l_i + l_{i+1}}\right) \\ &\overset{(a)}{>} 0 \end{aligned}$$

where inequality (a) follows from the convexity of $\mathcal{E}(\cdot)$. This contradicts the optimality of $\vec{\tau}$ and the lemma is proved. ■

The proof of the above lemma suggests the principle of the optimal offline schedule: Equate the number of transmissions *per bit* for each packet, subject to feasibility constraints. Note that this principle is similar to the one in the previous section, indeed, as will be the optimal schedule and proofs.

Given packet interarrival times d_i, $i \in \{1, \ldots, M\}$, let $k_0 = 0$, and define

$$\mu_1 = \max_{k \in \{1, \ldots, M\}} \left\{ \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} l_i} \right\}$$

and

$$k_1 = \max\left\{ k: \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} l_i} = \mu_1 \right\}.$$

For $j \geq 1$, let

$$\mu_{j+1} = \max_{k \in \{1, \ldots, M-k_j-1\}} \left\{ \frac{\sum_{i=1}^{k} d_{k_j+i}}{\sum_{i=1}^{k} l_{k_j+i}} \right\}$$

and

$$k_{j+1} = k_j + \max\left\{ k: \frac{\sum_{i=1}^{k} d_{k_j+i}}{\sum_{i=1}^{k} l_{k_j+i}} = \mu_{j+1} \right\}$$

where k varies between 1 and $M - k_j$. We proceed as above to obtain pairs (μ_j, k_j) until $k_j = M$ for the first time. Let $J = \min\{j: k_j = M\}$. The pairs (μ_j, k_j), $j = 1, \ldots, J$ are used to define the general form of OOE (the OOE of the previous section is simply the special case for which $l_i = B$, $\forall i$). As in the previous section, transmission times of OOE are denoted $\vec{\tau}^*$, and Theorem 2 shows that $\vec{\tau}^*$ is the optimal offline schedule for the variable length case.

Definition 2—OOE: The schedule with the vector of transmission times $\vec{\tau}^*$ given by

$$\tau_i^* = l_i\mu_j, \qquad \text{if } k_{j-1} < i \leq k_j \tag{14}$$

is called OOE.

Lemma 5: The following hold for the $\vec{\tau}^*$ of OOE.

1) It is feasible and $\sum_{i=1}^{M} \tau_i^* = T$.
2) It satisfies the condition stated in Lemma 4.

Proof: We first establish 1). For $1 \leq k \leq k_1$,

$$\sum_{i=1}^{k} \tau_i^* = \sum_{i=1}^{k} l_i\mu_1 \geq \sum_{i=1}^{k} l_i \frac{\sum_{j=1}^{k} d_j}{\sum_{j=1}^{k} l_j} = \sum_{i=1}^{k} d_i$$

where the inequality follows from the definition of μ_1.

Similarly, for $k_1 < k \leq k_2$,

$$\sum_{i=1}^{k} \tau_i^* = \sum_{i=1}^{k_1} l_i\mu_1 + \sum_{i=k_1+1}^{k} l_i\mu_2 \geq \sum_{i=1}^{k} d_i.$$

Proceeding thus, we obtain that $\sum_{i=1}^{k} \tau_i^* \geq \sum_{i=1}^{k} d_i$ for all k, $1 \leq k \leq M$.

With similar steps, it can be shown that $\sum_{i=1}^{M} \tau_i^* = T$, and 1) is established.

As for 2), it suffices to show that $\mu_j > \mu_{j+1}$ since this implies $\tau_i^*/l_i \geq \tau_{i+1}^*/l_{i+1}$, for each i. We first show that $\mu_1 > \mu_2$. For any $k \in [k_1 + 1, k_2]$

$$\begin{aligned} \mu_1 &= \frac{d_1 + \cdots + d_{k_1}}{l_1 + \cdots + l_{k_1}} \\ &\overset{(a)}{>} \frac{\sum_{i=1}^{k_1} d_i + \sum_{i=k_1+1}^{k} d_i}{\sum_{i=1}^{k} l_i} = \frac{\sum_{i=1}^{k_1} d_i}{\sum_{i=1}^{k} l_i} + \frac{\sum_{i=k_1+1}^{k} d_i}{\sum_{i=1}^{k} l_i} \end{aligned}$$

where (a) follows from the definition of μ_1. Choosing $k = k_2$, we get

$$\mu_1 > \frac{\sum_{i=1}^{k_1} d_i}{\sum_{i=1}^{k} l_i}\mu_1 + \frac{\sum_{i=k_1+1}^{k} d_i}{\sum_{i=1}^{k} l_i}\mu_2$$

from which it follows that $\mu_1 > \mu_2$.

In a very similar fashion, it can be shown that $\mu_2 > \mu_3$ and, more generally, that $\mu_j > \mu_{j+1}$ for any j, $1 \leq j \leq J-1$. This establishes 2) and completes the proof of the lemma. ■

Theorem 2: The schedule OOE of Definition 2 is the optimum offline schedule.

Proof: The proof is identical to the proof of Theorem 1. Hence, to avoid repetitions, we only present the highlights and not the details.

As before, consider any other feasible schedule $\vec{\tau}$. Let i be the first index where $\tau_i \neq \tau_i^*$. We show that $w(\vec{l}, \vec{\tau}) > w(\vec{l}, \vec{\tau}^*)$. There are the following two possibilities to consider. Case 1 is $\tau_i > \tau_i^*$, and Case 2 is $\tau_i < \tau_i^*$.

Under Case 1, we use the schedule $\vec{\tau}$ to define another schedule $\vec{\sigma}$ as before and establish the following two claims.

Claim 1: The schedule $\vec{\sigma}$ does not idle and is feasible.

Claim 2: $w(\vec{l}, \vec{\sigma}) < w(\vec{l}, \vec{\tau})$.

Hence, we conclude that any feasible schedule $\vec{\tau}$ differing from $\vec{\tau}^*$ in the sense of Case 1 may be modified to obtain a strictly more energy efficient schedule $\vec{\sigma}$. This concludes Case 1.

Under Case 2, we shall argue for a contradiction and show that the schedule $\vec{\tau}$ must be infeasible exactly as in the proof of Theorem 1.

This completes the proof of Theorem 2. ■

V. Online Scheduling

In this section, we develop and evaluate energy-efficient online scheduling algorithms based on the optimal offline algorithm discussed in Section III. Henceforth, we shall assume that the packets are of the same length.

In order to design online algorithms that are energy-efficient on *average*, one needs the statistics of the arrival process. Whilst our approach is general, for concreteness and tractability, we mainly assume Poisson arrivals in this paper. We note that Poisson arrivals are unrealistic in the wireless LAN environment, where arrivals tend to be more bursty. In fact, we have observed that when arrivals are bursty, lazy scheduling performs even better than in the Poisson case, for one can take advantage of a small queueing delay and greatly reduce transmission energy.

We proceed by first formulating the offline algorithm OOE in a manner that is suited for online use (Section V-A). Based on this formulation, we propose an online algorithm (Section V-B) and, using simulations, show that on the average it is almost as energy-efficient as the optimal offline schedule (Section V-C).

We then investigate the important special case of $T \to \infty$. In this case, we are able to analyze the optimal offline schedule exactly (in the Appendix), obtain an online lazy schedule as a result of this analysis, and perform comparisons of the energy efficiency of the lazy schedule and a fixed-transmission time online algorithm (Section V-D).

A. Online Formulation of OOE

Consider the time interval $[0, T)$ and as before assume that a packet arrives at time 0. Suppose also that packets arrive as a Poisson process of rate λ. Conditioned on there being $M-1$ arrivals in $(0, T)$, let the interarrival times be denoted by D_i. Let the optimal offline schedule, OOE, assign transmission times $\vec{\tau}^*$ to these M packets. The time at which the jth packet starts transmitting is

$$T_j^* = \sum_{i=1}^{j-1} \tau_i^*.$$

The quantity b_j given by

$$b_j = \max\left\{k\colon \sum_{i=1}^{k-1} D_i < T_j^*\right\} - j$$

is the *backlog* in the queue when the jth packet starts transmitting. Observe that this backlog does not include the jth packet; that is, if $b_j = 1$, then there is precisely one packet [namely, the $(j+1)$th] in the queue when the jth packet starts transmitting. Finally, let C_i, $i \in \{1, \ldots, M-j-b_j\}$ be the interarrival times between packets that arrive *after* T_j^*. Thus, when the jth packet starts transmitting the situation is this: 1) the "time to go" equals $T - T_j^*$; 2) there are b_j packets currently backlogged; 3) $M-j-b_j$ packets are yet to arrive and the first of these will arrive in C_1 units of time, the second will arrive in $C_1 + C_2$ units of time, etc.

With this notation and some algebra, it can be shown that τ_j^* is also given by

$$\tau_j^* = \max_{k \in \{1, \ldots, M-(j+b_j)\}} \left\{ \frac{1}{k+b_j} \sum_{i=i}^{k} C_i \right\}. \tag{15}$$

This formula is just an alternative representation of OOE and gives exactly the same schedule. It schedules packets one at a time, taking into account the current backlog, future arrivals, and the time to go.

B. Online Algorithm

The alternative form of OOE in (15) strongly suggests the following online algorithm. The transmission time of a packet that starts being transmitted at time $t < T$ when there is a backlog of b packets can be set equal to the *expected value* of the random variable

$$\tau^*(b, t) = \max_{k \in \{1, \ldots, M\}} \left\{ \frac{1}{k+b} \sum_{i=1}^{k} D_i \right\} \tag{16}$$

where b is the current backlog and D_i are the interarrival times of the (random number) M of packets that will arrive in (t, T).

In the following, schedules based on $\mathrm{E}(\tau^*(b, t))$ will be used. At the moment, we do not know that this will produce the optimal online schedule, nor do we believe that it should. However, it is an online schedule and its performance can be compared to that of the optimal offline schedule. We proceed to do this in the next section and evaluate $\mathrm{E}(\tau^*(b, t))$ numerically when T is finite.

C. Simulations: Finite-Time Horizon

Using simulations, we compare the energies expended by the online algorithm defined above and the optimal offline algorithm. The setup is as follows. A finite-time horizon $T = 10$ s is chosen. We assume a packet length of $B = 10$ kB and a maximum rate of 6 b/transmission, with a link speed of 10^6 transmissions/s. (Hence, the minimum transmission duration for a packet is 10/6 ms, which we shall call a *time unit*.) Within the time period T, we assume that packets arrive according to a Poisson process at a loading factor of $\lambda = 0.7$ arrivals per time unit. Since it is possible for packets to arrive arbitrarily close to the finish time T, if we insist that these very late arrivals also be transmitted before the deadline T, then *any* algorithm, including the optimal offline algorithm, incurs a huge energy cost. This makes comparisons of performance difficult and meaningless. We therefore use a "guard band" g around the finish time and

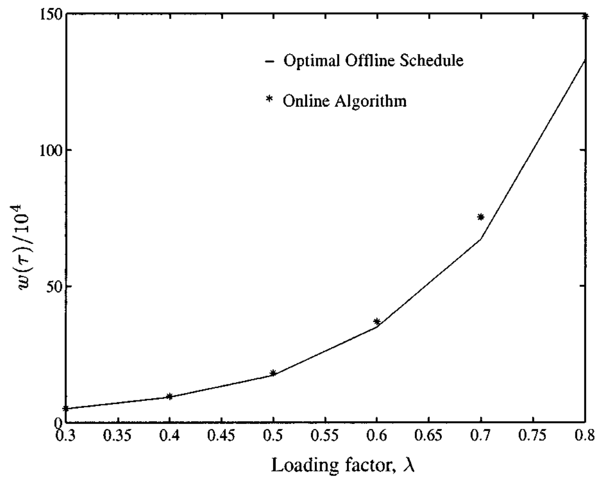

Fig. 6. A comparison of the online algorithm with the optimal offline algorithm.

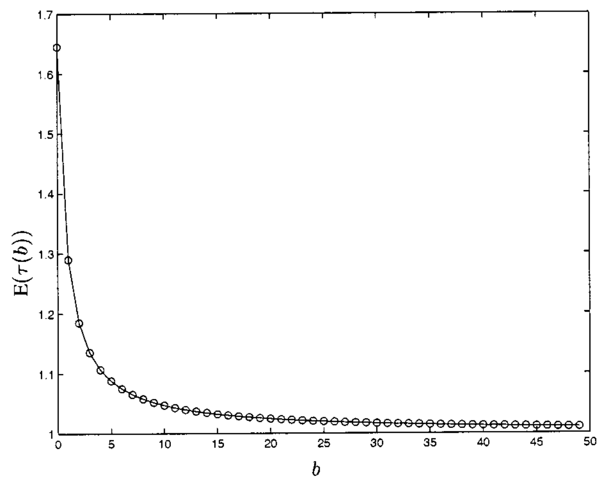

Fig. 7. A plot of $\mathrm{E}(\tau(b))$ versus b for $\lambda = 1$.

disallow packets from arriving after time $T - g$. For the comparison, we use $g = 0.1$ s and the following formula[4] for the packet transmission energy w as a function of packet transmission time τ in seconds,

$$w(\tau) = 10^6 \tau \left(2^{0.02/\tau} - 1\right). \tag{17}$$

Fig. 6, which plots the energy per packet against transmission time, shows that the online algorithm is almost as energy-efficient as the optimal offline algorithm.

D. Infinite-Time Horizon: Formulation and Simulations

The algorithm presented above was directly motivated by the optimal offline algorithm. It is of interest to let $T \to \infty$ and look at how the lazy schedule performs in terms of energy and delay. Defining $\mathrm{E}(\tau^*(b)) \triangleq \mathrm{E}(\lim_{t\to\infty} \tau^*(b, t))$, it is shown in the Appendix[5] that $\mathrm{E}(\tau^*(b)) = ((1 + b)/\lambda)((\pi^2/6) - \sum_{k=1}^{b}(1/k^2))$.

Fig. 7 plots $\mathrm{E}(\tau^*(b))$ as a function of the backlog b when the arrivals are a rate 1 Poisson process. As can be seen, the average transmission time of the offline schedule decreases with the backlog, approaching $1/\lambda$ as the backlog, b, approaches infinity. This exact analysis of the offline algorithm not only provides us with insight into the manner in which transmission times should depend on backlog, but also suggests a specific online schedule.

Unlike the finite T case where online schedules can be compared solely on the basis of their energy expenditure, when $T = \infty$ packet delays (or queue size, stability, etc.) must be taken into consideration. Otherwise, energy comparisons become meaningless since we can simply let transmission times be arbitrarily long and obtain the minimum possible transmission energy per packet whereas the delay can become infinite.

1) Online Scheduling Under a Stability Guarantee: As above, packets arrive according to a rate λ process at a transmission node with infinite queue capacity. The node transmits a packet p for a duration $\tau(b)$ when the backlog in the queue, excluding packet p, is b. The arrival rate λ is not known at the transmitter, but it is known that $\lambda \le \lambda_{\max}$.

The transmitter needs to be designed to ensure stability, and since $\lambda_{\max}$ is a worst case estimate of the arrival rate, stability will be ensured if the rate of transmission is higher than $\lambda_{\max}$. Since a lazy schedule varies transmission times depending on the backlog according to the function $\tau(b)$, for stability it suffices that $\tau(b) < 1/\lambda_{\max}$ for all b large enough.

Poisson Arrivals: We now compare the specific lazy schedule, Lazy_1, that sets $\tau_{\mathrm{Lazy}}(b) = \alpha((1 + b)/\lambda_{\max})((\pi^2/6) - \sum_{k=1}^{b}(1/k^2))$ to a deterministic schedule with $\tau_{\mathrm{Det}}(b) = \alpha/\lambda_{\max}$. The arrival process is a rate λ Poisson process.

Note that as long as $\alpha < 1$, both scheduling algorithms ensure stability for arrival rates less than $\lambda_{\max}$. We performed simulations using both scheduling algorithms for $\alpha = 0.95, \lambda_{\max} = 1$, varying λ from 0.3 to 0.9. To allow energy and delay to come close to equilibrium, each simulation was performed for 100 000 arrivals. The results are given in Table I.

TABLE I
AVERAGE ENERGY/PACKET AND AVERAGE DELAY/PACKET FOR LAZY_1 AND DETERMINISTIC OVER AN INFINITE TIME HORIZON. DELAY VALUES ARE IN MILLISECONDS (HIGH SNR)

λ	Poisson arrivals			
	Lazy_1		Deterministic	
	E/pkt $\times 10^{-4}$	Dly/pkt	E/pkt $\times 10^{-4}$	Dly/pkt
.3	91.6	3.19	1004.6	1.89
.4	118.8	3.56	1004.6	2.07
.5	159.4	4.01	1004.6	2.30
.6	218.4	4.60	1004.6	2.64
.7	308.6	5.51	1004.6	3.23
.8	435.1	6.92	1004.6	4.23
.9	623.7	9.58	1004.6	6.61

The energy/packet values in Table I are dimensionless due to the normalization with noise PSD [see (17)]. The energy values correspond to an average SNR per packet of approximately 25–34 dB for Lazy_1 and 36 dB for Deterministic.

[4]The formula is obtained using the information theoretic capacity formula in (1) for the AWGN channel with noise power $N = 1$ for the transmission of 10-kB packets for a duration τ s at symbol rate 10^6 transmissions/s.

[5]The analysis in the Appendix leads to some side results about the running averages of exponential random variables, seemingly of independent interest.

TABLE II
AVERAGE ENERGY/PACKET AND AVERAGE DELAY/PACKET FOR LAZY_1 AND DETERMINISTIC OVER AN INFINITE TIME HORIZON. DELAY VALUES ARE IN MILLISECONDS (LOW SNR)

λ	Poisson arrivals			
	Lazy_1		Deterministic	
	E/pkt $\times 10^{-4}$	Dly/pkt	E/pkt $\times 10^{-4}$	Dly/pkt
.3	4.27	9.56	8.32	5.700
.4	4.48	10.66	8.32	6.219
.5	4.76	12.00	8.32	6.935
.6	5.14	13.76	8.32	7.984
.7	5.62	16.28	8.32	9.622
.8	6.22	20.27	8.32	12.567
.9	6.97	27.81	8.32	19.453

TABLE III
AVERAGE ENERGY/PACKET AND AVERAGE DELAY/PACKET FOR LAZY_1 AND DETERMINISTIC OVER AN INFINITE TIME HORIZON. DELAY VALUES ARE IN MILLISECONDS

λ	Bursty arrivals			
	Lazy_1		Deterministic	
	E/pkt $\times 10^{-4}$	Dly/pkt	E/pkt $\times 10^{-4}$	Dly/pkt
.3	66.972	2.473	1004.6	$\simeq$ 1.583
.4	95.746	3.731	1004.6	$\simeq$1.583
.5	212.769	4.453	1004.6	$\simeq$ 1.583
.6	326.072	5.642	1004.6	2.480
.7	431.995	6.996	1004.6	4.233
.8	552.195	8.598	1004.6	6.263
.9	729.285	11.801	1004.6	10.607

In order to give a fuller picture, let us also consider lower SNR values. We do this by considering lower rates. In the rest of this section, the maximum rate is set to 2 bits/transmission, while the symbol rate is kept the same as before.[6] Table II shows how the energy per packet ranges for Lazy_1 and Deterministic. For Lazy, the SNR goes from 7 to 11 dB.

Bursty Arrivals: We have just seen that the schedule Lazy_1 is more energy-efficient compared to a deterministic schedule when the arrivals are Poisson. The schedule Lazy_1 was developed by conducting an asymptotic analysis of $\tau^*(b, t)$ (see the Appendix), where $\tau^*(b, t)$ is defined in (16). The asymptotic analysis for Poisson arrivals assumes that the interarrival times D_i in (16) are i.i.d. exponentials. Thus, Lazy_1 is "tuned" to Poisson arrivals.

It is therefore interesting to ask just how well Lazy_1 will perform under non-Poisson input processes. To this end, we consider the following "bursty" arrival process. The interarrival times D_i are i.i.d. with $\Pr(D_i = a_1) = \beta = 1 - \Pr(D_i = a_2)$, where a_1, a_2 and β are parameters. When a_1 is small and β is large arrivals tend to be bursty with a high probability.

First, we run Lazy_1 on the bursty arrival process with $a_2/a_1 = 9$, and $\lambda_{\max} = 1$. The results are summarized in Table III. Comparing the energy/packet values in the last three rows of Tables I and III, we see that Lazy_1 is indeed better tuned for Poisson arrivals. A second conclusion from the tables is that, at low values of λ, lazy scheduling works better on the bursty arrival process than on the Poisson arrival process.

Now we develop another algorithm, called Lazy_2, which is derived from the bursty arrival process, and hence potentially better tuned to it. Consider an infinite time horizon as above. Recall that, for a backlog of b, $\tau(b) = \sup_{n\geq 1}\{(1/(n+b))\sum_{i=1}^{n} D_i\}$. In order to obtain a bound on $\mathrm{E}(\tau(b))$, we consider

$$\begin{aligned}\Pr(\tau(b) < r) &= \Pr\left(\sup_{n\geq 1}\left\{\frac{1}{n+b}\sum_{i=1}^{n} D_i\right\} < r\right) \\ &= \Pr\left(\sum_{i=1}^{n}(D_i - r) < br,\ \forall\, n \geq 1\right).\end{aligned}$$

Define $Y_i^{(r)} \triangleq D_i - r$. For a given r, $Y_i^{(r)}$ are i.i.d. random variables of mean $\mathrm{E}(D) - r$. Define $\tilde{S}_n^{(r)} \triangleq \sum_{i=1}^{n} Y_i^{(r)}$. When $\mathrm{E}(D) - r < 0$, $\tilde{S}_n^{(r)}$ is a random walk with a negative drift. It is known (see [10, Ch. 7], especially Problem 7.12) that the following bound holds:

$$\Pr\left(\tilde{S}_N^{(r)} \geq br\right) \leq e^{-s^* br} \tag{18}$$

where s^* is the solution of the equation

$$\mathrm{E}\left(e^{sY_i^{(r)}}\right) = 1.$$

In our case, the above equation reduces to

$$\beta e^{s^*(a_1 - r)} + (1-\beta)e^{s^*(a_2 - r)} = 1. \tag{19}$$

Using the above definitions and results, $\Pr(\tau(b) \geq r) = \Pr(\tilde{S}_N \geq br) \leq e^{-s^* br}$, provided $r > \mathrm{E}(D)$. Now we are ready to bound $\mathrm{E}(\tau(b))$

$$\begin{aligned}\mathrm{E}(\tau(b)) &= \int_0^\infty \Pr(\tau(b) \geq r)\, dr \\ &\leq \int_0^{\mathrm{E}(D)} \Pr(\tau(b) \geq r)\, dr + \int_{\mathrm{E}(D)}^\infty e^{-s^* br}\, dr \\ &\leq \mathrm{E}(D) + \int_{\mathrm{E}(D)}^\infty e^{-s^* br}\, dr \\ &\triangleq B(b).\end{aligned}$$

This suggests an online lazy schedule $\sigma(b) = \alpha * B(b)$, where $\alpha < 1$ is there to ensure stability. We will call this schedule Lazy_2.

The schedule Lazy_2, where $\sigma(b)$ is calculated as described above for $a_2/a_1 = 9, 1 - \beta = 1/9$, and $\lambda_{\max} = 1$ is plotted in Fig. 8 (for $\alpha = 1$). Note that as b grows, $\sigma(b)$ asymptotes to $1/\lambda_{\max}$ in the figure, and in general, it asymptotes to $\alpha/\lambda_{\max}$.

Table IV summarizes results of the comparison of Lazy_2 with Deterministic on a bursty arrival process. Comparing Tables III and IV shows that Lazy_2 is a better schedule for the bursty arrivals process than is Lazy_1, as ought to be the case.

[6]Since the symbol rate is 10^6 transmissions/s, the minimum transmit time of a 10^4 bit packet (i.e., unit time) is now 10/2 ms as opposed to the previous 10/6. Note that λ is arrivals/unit time, hence for the same λ, the actual number of packet arrivals/s is lower than before.

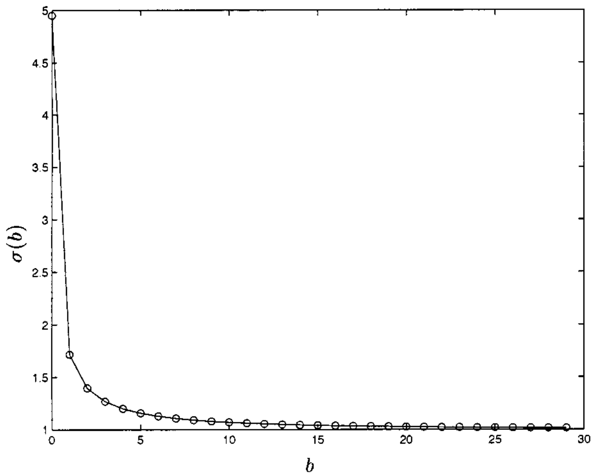

Fig. 8. A plot of $\sigma(b)$ versus b for a lazy schedule designed for $a_2/a_1 = 9$, $\lambda_{\max} = 1$, and with $\alpha = 1$.

TABLE IV
BURSTY ARRIVALS: AVERAGE ENERGY/PACKET AND AVERAGE DELAY/PACKET FOR LAZY$_2$ AND DETERMINISTIC OVER AN INFINITE TIME HORIZON. DELAY VALUES ARE IN MILLISECONDS

λ	Bursty arrivals			
	Lazy$_2$		Deterministic	
	E/pkt$\times 10^{-4}$	Dly/pkt	E/pkt$\times 10^{-4}$	Dly/pkt
.3	51.192	7.580	1004.6	$\simeq$1.583
.4	107.495	7.786	1004.6	$\simeq$1.583
.5	209.923	9.110	1004.6	$\simeq$1.583
.6	293.675	10.033	1004.6	2.480
.7	389.735	11.159	1004.6	4.233
.8	513.605	12.959	1004.6	6.263
.9	692.246	16.492	1004.6	10.607

The simulation results demonstrate that lazy schedules achieve significantly lower energy than the deterministic schedule with a moderate increase in average delay. This comparison with the deterministic schedule is important since, for a given mean service time, the deterministic schedule achieves the smallest average delay among all schedules that are independent of the arrival process and hence oblivious to backlogs [20]. In turn, this implies that the deterministic schedule has the largest transmission times and hence the lowest energy among backlog-oblivious schedules. The fact that our suboptimal lazy schedule is more energy efficient than the deterministic schedule demonstrates the advantage of lazy scheduling.

VI. CONCLUSION

Conservation of energy is a key concern in the design of wireless networks. Most of the research to date has focused on transmission power control schemes for interference mitigation and only indirectly address energy conservation. In this paper, we put forth the idea of conserving energy by lazy scheduling of packet transmissions. This is motivated by the observation that in many channel coding schemes the energy required to transmit a packet over a wireless link can be significantly reduced by lowering transmission power and transmitting the packet over a longer period of time. However, information is often time-critical or delay-sensitive, hence transmission times cannot be arbitrarily long. We therefore considered packet transmission schedules that minimize energy subject to a deadline or a delay constraint. Specifically, we obtained an optimal offline schedule for a node operating under a deadline constraint. An inspection of the form of this schedule naturally lead us to an online schedule, which was shown, through simulations, to be quite energy-efficient. We then relaxed the deadline constraint and provided an exact probabilistic analysis of our offline scheduling algorithm. We then devised an online algorithm, which varies transmission times according to backlog and showed that it is more energy efficient than a deterministic schedule with the same stability region and similar delay.

Several important problems remain open. The most obvious is that of finding the optimal online schedule in the finite and infinite T cases. The question of how much energy can be saved by using lazy scheduling in practice has not been addressed in the paper. The theoretical and simulation results presented here encourage further investigation into the use of lazy scheduling in real-world wireless networks.

APPENDIX

Consider a transmitter which, at time 0, has b packets in the queue. Suppose that M packets arrive at this node in $[0, T)$, with the first of these arriving at time 0. This situation can be modeled as $M + b$ packets arriving in $[0, T)$ with $d_1 = \cdots = d_b = 0$ and $\sum_{i=1}^{M+b} d_i = T$. Then, as we have seen in Section V-A, the optimal offline schedule will transmit the first packet for an amount of time, say $\tau_M(b)$, which is given by

$$\tau_M(b) = \max_{k\in\{1,\ldots,M+b\}} \left\{ \frac{1}{k} \sum_{i=i}^{k} d_i \right\} \tag{20}$$

$$= \max_{k\in\{1,\ldots,M\}} \left\{ \frac{1}{k+b} \sum_{i=b}^{k} d_i \right\}. \tag{21}$$

Here we analyze the optimal offline schedule by allowing T to approach infinity. Thus, suppose that the arrivals in $[0, T)$ occur as a rate λ Poisson process and let T go to infinity to yield

$$\tau(b) = \sup_{\{k\geq 1\}} \left\{ \frac{1}{k+b} \sum_{i=1}^{k} D_i \right\} \tag{22}$$

where the D_i are i.i.d. mean $1/\lambda$ exponential random variables.

To evaluate the distribution of $\tau(b)$, consider

$$\Pr(\tau(b) < r) = \Pr\left(\sup_{k\geq 1} \left\{ \frac{\sum_{i=1}^{k} D_i}{k+b} \right\} < r \right) \tag{23}$$

$$= \Pr\left(\sum_{i=1}^{k} (D_i - r) < br \; \forall\, k \geq 1 \right)$$

$$\triangleq \Pr\left(\sum_{i=1}^{k} Y_i < br \; \forall\, k \geq 1 \right). \tag{24}$$

Since the D_i are assumed to be i.i.d., mean $1/\lambda$, exponential random variables,

$$\lim_{k\to\infty} \frac{\sum_{i=1}^{k} D_i}{k+b} = \frac{1}{\lambda}.$$

Therefore, from (23) we at once see that $\Pr(\tau(b) < r) = 0$ for all $r < 1/\lambda$.

Hence, suppose $r > 1/\lambda$. Then the $Y_i = D_i - r$ are i.i.d. random variables with a negative mean and $\sum_{i=1}^{k} Y_i$ is a random walk with a negative drift. Evaluating the probability at (24) is therefore equivalent to determining the probability that a random walk with a negative drift never exceeds a positive threshold of br.

For notational convenience, define $S_0 = 0$ and $S_n = \sum_{i=1}^{n} Y_i$. Define the associated exponential martingale $Z_n = e^{\alpha S_n}$, where $\alpha > 0$ is yet to be determined. Also define the stopping time $X \triangleq \inf\{n: S_n \geq br\}$ and observe that $\Pr(X = \infty) = \Pr(\tau(b) < r)$.

We shall consider the stopped exponential martingale Z_X and use the optional stopping theorem [7] to determine $\Pr(X < \infty)$ and hence $\Pr(\tau(b) < r)$. The details follow.

First, in order that Z_n be a martingale, we need to choose an α such that $\mathrm{E}(Z_n) = \mathrm{E}(Z_0) = \mathrm{E}(e^{\alpha Y_0}) = 1$ for every n. In particular, α should be such that

$$\mathrm{E}(Z_1) = \mathrm{E}(e^{\alpha(D_1 - r)}) = 1 \tag{25}$$

$$\Rightarrow \mathrm{E}(e^{\alpha D_1}) = e^{\alpha r}. \tag{26}$$

But, D_1 is exponentially distributed with mean $1/\lambda$. Using this in (26), we see that α is the solution to the equation

$$1 - \frac{\alpha}{\lambda} = e^{-\alpha r}. \tag{27}$$

It is not hard to see that, for a given λ and $r > 1/\lambda$, there is a unique α that satisfies (27).

Continuing with the determination of $\Pr(\tau(b) < r)$, consider $\mathrm{E}(Z_X)$:

$$\mathrm{E}(Z_X) = \mathrm{E}(Z_X | X < \infty)\Pr(X < \infty) + \mathrm{E}(Z_X | X = \infty)\Pr(X = \infty) \tag{28}$$

$$\stackrel{(a)}{=} \mathrm{E}(Z_X | X < \infty)\Pr(X < \infty) \tag{29}$$

$$= \mathrm{E}(e^{\alpha S_X} | X < \infty)\Pr(X < \infty) \tag{30}$$

where (a) holds because $S_X = -\infty$ on the set $\{X = \infty\}$. By the optional sampling theorem (see [15, Proposition IV-4-19]), we obtain

$$1 = \mathrm{E}(Z_X) = \mathrm{E}(e^{\alpha S_X} | X < \infty)\Pr(X < \infty). \tag{31}$$

Therefore, $\Pr(X < \infty) = (\mathrm{E}(e^{\alpha S_X} | X < \infty))^{-1}$. To evaluate $\mathrm{E}(e^{\alpha S_X} | X < \infty)$ we determine $\Pr(S_X > t | X < \infty)$ as follows. For $t < br$, by definition of the stopping time X, $\Pr((S_X > t | X < \infty) = 1$. For $t \geq br$,

$$\begin{aligned}
&\Pr(S_X > t | X < \infty) \\
&\quad = \Pr(Y_X > t + s | Y_X > s) \\
&\qquad \text{where } s = br - S_{X-1} \\
&\quad = \Pr(D_X > t + r + s | D_X > r + s) \\
&\quad \stackrel{(a)}{=} \Pr(D_X > t) \\
&\quad = e^{-\lambda(t - br)}, \qquad \text{since } t \geq br.
\end{aligned}$$

Equality (a) is due the memoryless property of the exponential random variables D_i. Using this to evaluate $\mathrm{E}(e^{\alpha S_X} | X < \infty)$, we get

$$\mathrm{E}(e^{\alpha S_X} | X < \infty) = \int_{br}^{\infty} e^{\alpha t} e^{-\lambda(t - br)}\, dt = \frac{e^{\alpha br}}{\lambda - \alpha}.$$

Using this in (31), we get that

$$\Pr(X < \infty) = (\lambda - \alpha)e^{-\alpha br}.$$

Hence, we finally obtain

$$\Pr(\tau(b) < r) = \Pr(X = \infty) = 1 - (\lambda - \alpha)e^{-\alpha br} \tag{32}$$

where α solves (27).

Given the distribution of $\tau(b)$, one could numerically evaluate $\mathrm{E}(\tau(b))$. The approach of the next section allows us to express $\mathrm{E}(\tau(b))$ explicitly.

A. An Alternative Analysis

Define $S_i = \sum_{j=1}^{i} D_j$ and $\tau_n(b) = \max_{\{1\leq i\leq n\}}\{S_i/(i + b)\}$.

Lemma 6:

$$\mathrm{E}(\tau_n(b)) = \frac{1+b}{\lambda} \sum_{k=1}^{n} \frac{1}{(k+b)^2}. \tag{33}$$

Proof: We start by expressing the distribution function of $\tau_n(b)$ as

$$\Pr(\tau_n(b) < t) = \Pr\left(\max_{\{1\leq i\leq n\}}\left\{\frac{S_i}{i+b}\right\} < t\right) \tag{34}$$

$$= \Pr\left(\frac{S_i}{i+b} < t,\ \forall i{:}\ 1 \leq i \leq n\right) \tag{35}$$

$$= \int_0^{t(1+b)} \int_{s_1}^{t(2+b)} \cdots \tag{36}$$

$$\int_{s_{n-1}}^{t(n+b)} f_{S_1,\ldots S_n}(s_1, \ldots s_n)\, ds_n \cdots ds_1. \tag{37}$$

Note that $f_{D_1,\ldots,D_n}(d_1,\ldots,d_n) = \lambda^n \exp(-\lambda \sum_{i=1}^{n} d_i)$ (recall the independence assumption). The Jacobian of the transformation $S_i = \sum_{j=1}^{i} D_i,\ \forall i$ is 1. Therefore, $f_{S_1,S_2,\ldots,S_n}(s_1, s_2, \ldots, s_n) = \lambda^n \exp(-\lambda s_n)$, and (37) can be written as

$$\begin{aligned}
&\Pr(\tau_n(b) < t) \\
&\quad = \int_0^{t(1+b)} \int_{s_1}^{t(2+b)} \cdots \\
&\qquad \int_{s_{n-1}}^{t(n+b)} \lambda^n \exp(-\lambda s_n)\, ds_n \cdots ds_1
\end{aligned} \tag{38}$$

$$= \lambda^{n-1} \int_0^{t(1+b)} \int_{s_1}^{t(2+b)} \cdots \int_{s_{n-2}}^{t(n-1+b)} e^{-\lambda s_{n-1}} - e^{-\lambda t(n+b)}\, ds_{n-1} \cdots ds_1$$

$$= \Pr(\tau_{n-1}(b) < t) - \lambda^{n-1} \int_0^{t(1+b)} \cdots \int_{s_{n-2}}^{t(n-1+b)} e^{-\lambda t(n+b)}\, ds_{n-1} \cdots ds_1. \quad (39)$$

Using the identity $\mathrm{E}(Y) = \int_0^\infty \Pr(Y > t)\, dt$ for any positive random variable Y, we obtain from (39)

$$\mathrm{E}(\tau_n(b)) = \mathrm{E}(\tau_{n-1}(b)) + \lambda^{n-1} \int_0^\infty t^{n-1} e^{-\lambda t(n+b)} \int_0^{(1+b)} \cdots \int_{u_{n-2}}^{(n-1+b)} du_{n-1} \cdots du_1$$

by the normalization $u_i = s_i/t$. The $(n-1)$-dimensional volume

$$\int_0^{(1+b)} \int_{u_1}^{(2+b)} \cdots \int_{u_{n-2}}^{(n-1+b)} du_{n-1} \cdots du_1$$

can be shown (by an induction argument) to equal

$$(1+b)\, \frac{(n+b)^{(n-2)}}{(n-1)!}.$$

Substituting this into the above equation and integrating with respect to t yields

$$\mathrm{E}(\tau_n(b)) = \mathrm{E}(\tau_{n-1}(b)) + \frac{(1+b)}{\lambda(n+b)^2}. \quad (40)$$

Since $\mathrm{E}(\tau_1(b)) = 1/\lambda(1+b)$, Lemma 6 follows. ■

Corollary 1: Define $\tau(b) = \sup_{k \geq 1}(S_i/i)$ and recall the definition $\mathrm{E}(\tau(b)) \triangleq \mathrm{E}(\lim_{n\to\infty} \tau_n(b))$. Then,

$$\mathrm{E}(\tau(b)) = \frac{(1+b)}{\lambda} \left(\frac{\pi^2}{6} - \sum_{m=1}^{b} \frac{1}{m^2} \right). \quad (41)$$

Proof: The proof follows from monotone convergence. ■

Out of the proof of Lemma 6, we get the following interesting results about i.i.d. exponential random variables (of mean $1/\lambda$) and the convergence of their sample average to $1/\lambda$. To our knowledge, these explicit results are not found in the literature.

Corollary 2: Define $Z_n = \max_{\{1 \leq i \leq n\}}(S_i/i)$, and $I_n = Z_n - Z_{n-1}$. The following hold:

1) $\mathrm{E}(I_n) = 1/\lambda n^2$
2) $\mathrm{E}(Z_n) = (1/\lambda)\sum_{i=1}^n (1/i^2)$
3) $\Pr((S_n/n) > Z_{n-1}) = 1/n$
4) $\mathrm{E}((S_n/n) - Z_{n-1} | (S_n/n) > Z_{n-1}) = 1/\lambda n$
5) $\sup_{\{n \geq 1\}} Z_n = \lim_{\{n \geq 1\}} Z_n = \pi^2/6\lambda$.

Proof: Part 1) follows by taking $b = 0$ in (40). Part 2) follows by setting $b = 0$ in Lemma 6.

We now show parts 3)–5). For notational convenience, we set $\lambda = 1$ for the time being; the results trivially scale by $1/\lambda$, as will be clear in the calculations below.

To establish part 3), we write

$$\Pr\left(\frac{S_n}{n} > Z_{n-1}\right) = \Pr\left(\frac{S_n}{n} > \frac{S_i}{i},\ \forall 1 \leq i < n\right) = \Pr\left(\frac{S_n}{n} > S_1, \ldots, \frac{S_n}{n} > \frac{S_{n-1}}{n-1}\right) = \int_{z=0}^{\infty} A_{n-1}(z) f_{S_n}(nz) n\, dz \quad (42)$$

where $A_{n-1}(z) \triangleq \Pr(S_1 < z, S_2 < 2z, \ldots, S_{n-1} < (n-1)z | S_n = nz)$. Recall that S_i are arrival epochs in a Poisson process. The condition $S_n = nz$ is the same as saying that $n-1$ arrivals occurred in $[0, nz)$, and it is well known that under this condition $S_1, \ldots, S_{n-1}$ are distributed as order statistics [10], i.e.,

$$f_{(S_1, \ldots, S_{n-1}|S_n)}(s_1, \ldots, s_{n-1} | s_n = nz) = \frac{(n-1)!}{(nz)^{n-1}}. \quad (43)$$

Therefore,

$$A_{n-1}(z) = \int_0^z \cdots \int_{s_{n-2}}^{(n-1)z} \frac{(n-1)!}{(nz)^{n-1}}\, d_{s_{n-1}} \cdots d_{s_1}$$

$$= n^{-(n-1)}(n-1)! \int_0^1 \cdots \int_{u_{n-2}}^{(n-1)} d_{u_{n-1}} \cdots d_{u_1}$$

$$= n^{-(n-1)}(n-1)!\, \frac{n^{(n-2)}}{(n-1)!} = \frac{1}{n}. \quad (44)$$

Substituting (44) into (42), we obtain $\Pr((S_n/n) > \max_{1 \leq i < n}(S_i/i)) = 1/n$.

For part 4), write $\mathrm{E}(I_n) = \mathrm{E}(I_n | I_n > 0) \Pr(I_n > 0)$. Or, more explicitly, $\mathrm{E}(I_n) = \mathrm{E}((S_n/n) - Z_{n-1} | (S_n/n) > Z_{n-1}) \Pr((S_n/n) > Z_{n-1})$. From part 3), $\Pr((S_n/n) > Z_{n-1}) = 1/n$, and from part 1), $\mathrm{E}(I_n) = 1/\lambda n^2$, so $\mathrm{E}((S_n/n) - Z_{n-1} | (S_n/n) > Z_{n-1}) = 1/\lambda n$. This result is interesting because it says that given the current time average exceeds the previous maximum, the average amount of the excess is exactly $1/\lambda n$. Finally, part 5) follows by setting $b = 0$ in Corollary 1. ■

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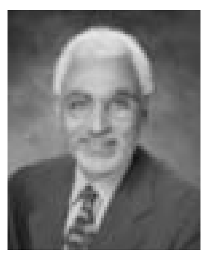

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