

Switching under Energy Constraints

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Abstract—The performance metrics that network designers typically optimize their design for are: throughput, delay and queue back-logs. These metrics have guided the design of wireline networks, like the Internet, and network components (e.g. switches, routers, etc). In wireless networks, the energy consumed by transmission nodes is a precious resource whose conservation is very important. This has motivated the research community to design wireless networks so as to deliver high throughput, achieve low delays, and minimize energy consumption. This paper makes a start at understanding how minimizing communication energy impacts the design of high-performance network algorithms.

Specifically, we consider a “switch” topology as this captures most of the constraints of a general acyclic wireless network. We study the problem of designing minimum energy transmission schedules which deliver maximum throughput. We show that the optimal policy is the solution of a convex optimization problem. We provide an iterative scheme that exploits the structure of the problem to obtain the solution. We also obtain approximations which are implementationally simpler and interesting in their own right.

I. INTRODUCTION

The design of network algorithms is usually guided by two main performance metrics: the throughput, and the delay experienced by a packet in the network. Both metrics determine the quality-of-service (QoS) perceived by user applications and observed by network operators.

The design of wireless networks is significantly influenced by another metric: the energy used to switch packet flows from their source to their destination. Minimizing energy prolongs the battery life duration, and permits greater node mobility. This paper is concerned with minimizing the energy *per* bit required for transmission. Since energy is directly related to power, our derivation of the energy consumed allows us to view energy-per-bit and power-costs interchangeably.

The ultimate aim of our work is to study the trade-off between energy, delay and throughput. However, in the present paper, we shall only consider the trade-off between energy and throughput while designing algorithms. As we shall see, the study of this trade-off is already interesting, and our models allow the incorporation of delay incrementally. Specifically, we shall assume that data arrives *deter-*

ministically at the transmission nodes. This allows us to design zero-delay switching algorithms. In future we plan to incorporate delays by considering random packet processes for data arrivals.

II. SWITCH CONSTRAINED NETWORKS

Consider a radio network with N transmitters (TXs) and N receivers (RXs). Assume the traffic is stationary, and follows the Strong Law of Large Numbers; λ_{ij} is the average arrival rate of the traffic coming at TX i and directed to RX j , $1 \leq i, j \leq N$; the arrival matrix is $\Lambda = [\lambda_{ij}]$ and it is known. In general, the connectivity between TX i and RX j is determined by the available channel capacity, the topology and other physical layer effects.

Let γ_{ij} be the fraction of time such that TX i is transmitting to RX j ; $\Gamma = [\gamma_{ij}]$ is the corresponding matrix and $\gamma = [\gamma_{ij}]$ is the vector obtained by stacking the columns of Γ . Consider a time interval of size T . We assume that TX i transmits to RX j in an interval of duration $\gamma_{ij}T$ using a constant power P_{ij} , achieving a rate R_{ij} (bps). The rate is related to the transmission scheme as will be described later.

To achieve 100% throughput and to expend minimum energy per bit, the following relation holds: $R_{ij}\gamma_{ij} = \lambda_{ij}$. Now P_{ij} is a function of the rate R_{ij} i.e.,

$$P_{ij}(R_{ij}) = g_{ij}(R_{ij}) \quad \Rightarrow \quad P_{ij}(\gamma_{ij}) = g_{ij}(\lambda_{ij}/\gamma_{ij})$$

where $g_{ij}(\cdot)$ is a convex function which depends on the transmission scheme (see [1]). Then, the total average power is $P(\gamma) = \sum_{i,j} P_{ij}(\gamma_{ij})$. Similarly, we can define the energy per bit E_{ij} :

$$E_{ij}(\gamma_{ij}) = \begin{cases} P_{ij}(\gamma_{ij})/\lambda_{ij} & \text{if } \lambda_{ij} > 0 \\ 0 & \text{if } \lambda_{ij} = 0 \end{cases}$$

and the total energy per bit $E(\gamma) = \sum_{i,j} E_{ij}(\gamma_{ij})$. Note that both $E(\gamma)$ and $P(\gamma)$ are convex functions.

We assume that the maximum power available at the TX is *limited* and equal to P_{MAX} , corresponding to a transmission rate $R_{MAX_{ij}}$ for the particular link from TX i to RX j . Hence,

$$R_{ij} \leq R_{MAX_{ij}} \quad \Rightarrow \quad \gamma_{ij} \geq \frac{\lambda_{ij}}{R_{MAX_{ij}}} \triangleq \hat{\gamma}_{ij} \quad (1)$$

where $\{\hat{\gamma}_{ij}\}$ are normalized arrival rates.

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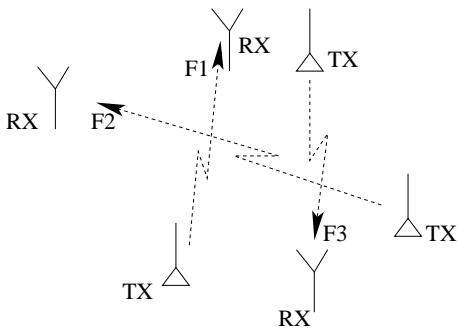


Fig. 1. Example of 3 TX-RX couples communicating each other, at the same time, using 3 different frequencies F_1 , F_2 and F_3 .

So far, the described model is general and does not depend on the transmitting and receiving constraints which are involved in practical systems. We now model these communication constraints.

All the TXs and RXs use some kind of diversity scheme to communicate between themselves without conflicts. The diversity scheme can be frequency-based, time-based, spatial-based, code-based or some combination of them. For simplicity, we focus our attention on a frequency-based scheme, but the same holds for all the other schemes, where “frequency” in the following description is substituted by “time-slot”, “antenna-direction” or “CDMA-code”.

Consider each RX tuned on a fixed frequency. Each TX transmits using the frequency corresponding to the destination RX. To avoid interference, we assume that no more than one TX can transmit at the same time using the same frequency; the scheduler selects a set of packets to be transferred satisfying these transmission constraints. Figure 1 depicts such a scenario. Packets which are at the TX and cannot be transmitted are stored in FIFO queues. To achieve the maximum throughput and avoid the well-known HoL blocking problem [2], we assume that at every TX i we have separate queues for all RX j . Hence, the problem of scheduling can be modeled as a problem of finding matchings in a bipartite graph with appropriate costs associated with each edge.

Radio networks with such communication constraints are referred to as *switch constrained networks*.

A. Previous work

The throughput of scheduling policies in switch constrained networks was first studied in [3]. Later papers [4], [5] have addressed the same problem when the network connectivity is time-varying. Similar results were also found in the context of scheduling for input-queued switches [6], [7]. But past work doesn’t consider the prob-

lem of minimizing energy. As we shall see, this extra constraint influences the design of scheduling algorithms significantly.

III. THE MINIMUM-ENERGY SCHEDULING PROBLEM

The minimum-energy scheduling problem can be modeled as a matching problem with an additional energy constraint. Formally, the **min-energy optimization problem** is:

$$\min E(\gamma) \quad (2)$$

$$\sum_i \gamma_{ij} = 1 \quad \sum_j \gamma_{ij} = 1 \quad (3)$$

$$\gamma_{ij} \geq \hat{\lambda}_{ij} \quad (4)$$

where (3) corresponds to the non-idling constraints, and (4) follows from (1). Note that the same problem can be formulated in terms of power-minimization and there the cost function will be $P(\gamma)$.

Now suppose we can efficiently solve the above optimization problem and obtain Γ . Then given that Γ is a doubly-stochastic matrix, the Birkhoff-von-Neumann (BvN) decomposition allows us to write it as (for example, see [8]):

$$\Gamma = \sum_k \alpha_k M_k \quad \sum_k \alpha_k = 1$$

where M_k are permutation matrices. The scheduling algorithm then chooses matching M_k for α_k fraction of time.

We demonstrate the above concept with a simple example with $N = 2$. For simplicity, $R_{MAX_{ij}} = 1$, and it is given that:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

Any feasible solution can be written as:

$$\Gamma(x) = \begin{bmatrix} x & 1-x \\ 1-x & x \end{bmatrix}$$

with (4) becoming $x \geq x_L$ and $x \leq x_H$, where $x_L = \max\{\lambda_{11}, \lambda_{22}\}$ and $x_H = \min\{1 - \lambda_{12}, 1 - \lambda_{21}\}$. The optimization problem is $\min_{0 \leq x \leq 1} E(x)$ and its optimal solution x_{opt} can be easily computed:

$$x_{opt} = \begin{cases} x^* & \text{if } x_L \leq x^* \leq x_H \\ x_L & \text{if } x^* < x_L \\ x_H & \text{if } x^* > x_H \end{cases}$$

where x^* solves $(dE(x)/dx)|_{x=x^*} = 0$.

A. Example of cost function

In this section we discuss specific energy and power cost functions to show that the assumptions in our model are realistic. More complex radio-communication models also satisfy these assumptions [1]. Let the transmit power at TX be P ; then the received power at the RX is $P \times G$ where G is a gain factor (due to propagation loss, $G < 1$). The channel capacity C , measured in bps is (Shannon capacity):

$$C = \frac{1}{2} \log_2 \left(1 + \frac{PG}{N} \right)$$

where N is the noise power. Hence to transmit at rate C , the power required is: $P = N/G \times (2^{2C} - 1)$.

The total amount of energy spent $E_{ij}(T)$ is given by:

$$E_{ij}(T) = T\gamma_{ij}P_{ij} = T\gamma_{ij}\frac{N}{G_{ij}}(2^{2R_{ij}} - 1) \quad (5)$$

Hence, the average power spent on long term is given by:

$$\begin{aligned} P_{ij}(\gamma_{ij}) &= \lim_{T \rightarrow \infty} \frac{1}{T} E_{ij}(T) \\ &= \gamma_{ij} \frac{N}{G_{ij}} (2^{2R_{ij}} - 1) = \gamma_{ij} \frac{N}{G_{ij}} (2^{2\lambda_{ij}/\gamma_{ij}} - 1) \end{aligned}$$

It can be shown that both $P_{ij}(\gamma_{ij})$ and $E_{ij}(\gamma_{ij})$ are convex functions, and hence the assumptions of our model are met.

IV. SOLUTION OF THE MIN-ENERGY SCHEDULING PROBLEM

The minimum-energy problem for N TXs-RXs can be solved as a convex optimization problem, involving N^2 variables satisfying $N^2 + 2N$ constraints. Standard methods to solve this problem involve inverting a $(N^2 + 2N) \times (N^2 + 2N)$ matrix, whose complexity is approximately $O(N^6)$. We now wish to exploit the structure of the problem to obtain faster methods to solve the problem.

We have developed two methods: the first is an iterative procedure and the second one is an ϵ -approximation scheme for any $\epsilon > 0$. In the following, we will refer to $f(\gamma)$ as the generic cost function ($E(\gamma)$ or $P(\gamma)$) to be minimized.

A. Optimal Iterative solution for the Scheduling problem

The proposed iterative procedure requires the computation of an initial γ^0 which is a feasible solution, satisfying (3) and (4). We refer to [8]:

Proposition 1: If a matrix $\hat{\Lambda} = [\hat{\lambda}_{ij}]$ is doubly sub-stochastic, then there exists a doubly stochastic matrix $S = [s_{ij}]$ such that: $s_{ij} \geq \hat{\lambda}_{ij}, \forall i, j$.

During the initialization of our algorithm, γ_{ij}^0 is set equal to any s_{ij} satisfying the previous proposition. An algorithm to compute S with a computational complexity of $O(N^3)$ is proposed in [8].

Let γ^k be the solution at the end of iteration k . We obtain the next iterate as follows:

Step I: Building the residual graph

Build the following bipartite direct graph, with a set of left-most vertices V_I and a set of right-most vertices V_O . Each vertex $v \in V_I$ corresponds to a TX, and each vertex in $v \in V_O$ corresponds to a RX. An edge (i, j) connects $v_i \in V_I$ to $v_j \in V_O$ if $\gamma_{ij}^k < 1$; its capacity K_{ij} is set equal to $K_{ij} = 1 - \gamma_{ij}^k$. An edge (j, i) connects $v_j \in V_O$ to $v_i \in V_I$ if $\gamma_{ij}^k - \hat{\lambda}_{ij} > 0$; its capacity K_{ji} is set equal to $K_{ji} = \gamma_{ij}^k - \hat{\lambda}_{ij}$. Edge (i, j) is associated cost c_{ij} :

$$c_{ij} = \left. \frac{\partial}{\partial \gamma_{ij}} f(\gamma^k) \right|_{\gamma_{ij}=\gamma_{ij}^k}$$

or, equivalently, the cost vector is $\nabla f(\gamma^k)$. A cycle in such a directed bipartite graph is a collection of edges forming two matchings π^+ and π^- with π^+ having all edges from V_I to V_O and π^- having edges from V_O to V_I . The cost of such a cycle is given by $\langle \nabla f(\gamma^k), \pi^+ - \pi^- \rangle$, where the operator $\langle \cdot, \cdot \rangle$ is the scalar product.

Step II: Finding a negative-cycle

In the above constructed residual-bipartite graph, search for any cycle with negative cost. There are two possibilities:(A) a negative cycle exists, or (B) no such cycle exists. If its case (B) then the algorithm stops. Next we consider the case (A):

Such a cycle can be decomposed into two matchings π^+ and π^- such that $\langle \nabla f(\gamma^k), \pi^+ - \pi^- \rangle < 0$. Note that for all the nodes outside the cycle, it is possible to connect any unmatched left-most node with any unmatched right-most node. Add these edges to both π^+ and π^- to obtain complete matchings π^+ and π^- .

Now compute:

$$\begin{aligned} b^+ &= \min\{K_{ij} : (i, j) \in \pi^+, \gamma_{ij}^k < 1\} \\ b^- &= \min\{K_{ji} : (j, i) \in \pi^-, \gamma_{ij}^k > \hat{\lambda}_{ij}\} \\ b^*, &\text{ if exists, is such that } f(\gamma^k - b^*\pi^+ + b^*\pi^-) < f(\gamma^k) \end{aligned}$$

Note that $b^+, b^- > 0$. b^* can be computed as follows:

$$b_0^* = \arg \min_{\epsilon \geq 0} \{f(\gamma^k - \epsilon\pi^+ + \epsilon\pi^-) - f(\gamma^k)\} \quad (6)$$

Let,

$$\epsilon(k) = \min\{b^+, b^-, b^*\}$$

Then the new iterate γ^{k+1} is obtained as follows:

$$\gamma^{k+1} = \gamma^k - \epsilon(k)\pi^+ + \epsilon(k)\pi^-$$

Before proceeding further, observe that (6) requires a local minimum search procedure. This can be done using, for example, any dichotomic procedure.¹

Theorem 1: Let $\bar{\gamma}$ be the optimal solution. If the algorithm stops at iteration k then $\gamma^k = \bar{\gamma}$.

Proof: If the algorithm stops at iteration k , then there does not exist any cycle in the graph with cost < 0 . We now show that this implies: $\gamma^k = \bar{\gamma}$.

Denote γ^k by γ^A . Suppose γ^k is not the optimal solution, that is, $\gamma^A \neq \bar{\gamma}$. Then $f(\gamma^A) > f(\bar{\gamma})$. Consider the following vector:

$$v(\theta) = \gamma^A + \theta(\bar{\gamma} - \gamma^A) \quad \theta \in [0, 1]$$

Since $f(\gamma)$ is a convex function, $\bar{\gamma}$ is the only optimal solution,

$$f(v(\theta)) = f((1-\theta)\gamma^A + \theta\bar{\gamma}) \leq (1-\theta)f(\gamma^A) + \theta f(\bar{\gamma}) < f(\gamma^A) \quad \text{for } \theta \in (0, 1] \quad (7)$$

By recalling the definition of the objective function, we can write:

$$f(\gamma) = \sum_{i,j} g_{ij}(\gamma_{ij}) \quad \Rightarrow \quad f(v(\theta)) = \sum_{i,j} g_{ij}(v_{ij}(\theta))$$

Observe that:

$$\frac{\partial}{\partial \theta} f(v(\theta)) = \sum_{i,j} \frac{\partial g_{ij}(v_{ij})}{\partial v_{ij}} \frac{\partial v_{ij}(\theta)}{\partial \theta}$$

and:

$$v_{ij}(\theta) = \gamma_{ij}^A + \theta(\bar{\gamma}_{ij} - \gamma_{ij}^A) \quad \Rightarrow \quad \frac{\partial}{\partial \theta} v_{ij}(\theta) = \bar{\gamma}_{ij} - \gamma_{ij}^A$$

Note that vector $\nabla f(v)$ is given by terms in the form $\partial g_{ij}(v_{ij})/\partial v_{ij}$; hence,

$$\left. \frac{\partial}{\partial \theta} f(v(\theta)) \right|_{\theta=0} = \langle \nabla f(\gamma^A), \bar{\gamma} - \gamma^A \rangle$$

At the same time,

$$\left. \frac{\partial}{\partial \theta} f(v(\theta)) \right|_{\theta=0} = \lim_{\theta \rightarrow 0} \frac{f(v(\theta)) - f(v(0))}{\theta} < 0$$

where the last inequality holds because of (7). Hence,

$$\langle \nabla f(\gamma^A), \bar{\gamma} - \gamma^A \rangle < 0 \quad (8)$$

¹For example, say ϵ_0 be the minimum precision set to compute the minimum. Let $h(x) = f(\gamma^k - x\pi^+ + x\pi^-) - f(\gamma^k)$. Let $x_L(0) = 0$ and $x_H(0) = \min\{b^+, b^-\}$. At iteration s do the following. If $h(x_L(s)) < h(x_H(s))$ then $x_L(s+1) = x_L(s)$ and $x_H(s+1) = (x_L(s) + x_H(s))/2$. Otherwise, $x_L(s+1) = (x_L(s) + x_H(s))/2$ and $x_H(s+1) = x_H(s)$. If $(x_H(s+1) - x_L(s+1)) < \epsilon_0$ then stop, otherwise start iteration $s+1$.

Now we can write $\bar{\gamma} - \gamma^A = \sum_i \sigma_i^+ \pi_i + \sum_j \sigma_j^- \pi_j$, with two matchings π_i, π_j and $\sum_i \sigma_i^+ + \sum_j \sigma_j^- = 0$, with $\sigma_i^+ > 0$ and $\sigma_j^- < 0$. We can now compute:

$$\langle \nabla f(\gamma^A), \bar{\gamma} - \gamma^A \rangle = \langle \nabla f(\gamma^A), \sum_i \sigma_i^+ \pi_i + \sum_j \sigma_j^- \pi_j \rangle \quad (9)$$

Note that $\langle \nabla f(\gamma^A), \sigma_i^+ \pi_i \rangle \geq \langle \nabla f(\gamma^A), \sigma_i^+ \pi_\star \rangle$ and $\langle \nabla f(\gamma^A), \sigma_j^- \pi_j \rangle \geq \langle \nabla f(\gamma^A), \sigma_j^- \pi^\star \rangle$, being π^\star the maximum weight matching and π_\star the minimum weight matching. Hence, (9) becomes:

$$\begin{aligned} \langle \nabla f(\gamma^A), \bar{\gamma} - \gamma^A \rangle &\geq \langle \nabla f(\gamma^A), \sum_i \sigma_i^+ (\pi_\star - \pi^\star) \rangle \\ &= (N \sum_i \sigma_i^+) \langle \nabla f(\gamma^A), \pi_\star - \pi^\star \rangle \end{aligned}$$

By recalling (8), $\langle \nabla f(\gamma^A), \pi_\star - \pi^\star \rangle < 0$. This means that π^\star and π_\star form a cycle with a negative cost and this contradicts our assumption. ■

A.1 Complexity of the optimal algorithm

Consider a single iteration of the algorithm. Step I of the algorithm requires one to update the costs and capacities of $O(N^2)$ edges. Step II requires to find any negative-cost cycle, and this can be done using a traditional depth-first search, with complexity $O(N^3)$. The binary-search of a ‘‘good’’ $\epsilon > 0$ for augmenting flows on a bounded size interval (to some fixed-granularity) is of constant complexity. Hence, the overall complexity of each iteration is $O(N^3)$.

The maximum value of objective function $f(\cdot)$ of any feasible γ is bounded above by the value of objective function for $\Gamma = \hat{\Lambda}$. Since the optimal value is non-negative, the above iterative algorithm does not need to decrease the value of energy by more than a constant amount. Given that the energy function is continuous, and has continuous bounded derivatives, which are null only at the ‘unconstrained’ optimal value; the number of iterations taken to reach a value at most δ away from the optimal value is $O(1/\tilde{\delta})$ where $\tilde{\delta}$ is a function of δ and depends on the derivatives of the cost function.

B. Approximate solution for the scheduling problem

The algorithm presented above obtains the exact solution via an iterative procedure. In this section we present a scheme that obtains an ϵ -approximate solution in one-iteration. The techniques used in this algorithm are well known in slightly different forms in other convex optimization problems.

Let $f(\gamma)$ be the convex objective function to be minimized with constraints as before. By continuity of f , there exists a $\delta(\epsilon) > 0$ such that, given any feasible γ_1 and γ_2 :

$$|\gamma_1 - \gamma_2|_\infty \leq \delta(\epsilon) \Leftrightarrow |f(\gamma_1) - f(\gamma_2)| \leq \epsilon \quad (10)$$

Next create a bipartite graph G^ϵ as follows: as before, there are N vertices in one partition representing N TXs, and N vertices in the other partition representing N RXs. Add the edges between TX i and RX j as follows:

- (i) one edge of capacity $\hat{\lambda}_{ij}$ and cost “ $-\infty$ ” (equivalently large enough negative constant).
- (ii) $l_{ij} = \lceil (1 - \hat{\lambda}_{ij})/\delta(\epsilon) \rceil$ edges, $l_{ij} - 1$ of them are each of capacity $\delta(\epsilon)$, and the last one is such that the net capacity of all edges between TX i and RX j is 1. The cost of n^{th} edge is, for $n = 1, \dots, l_{ij} - 1$:

$$g_{ij}(\hat{\lambda}_{ij} + n\delta(\epsilon)) - g_{ij}(\hat{\lambda}_{ij} + (n - 1)\delta(\epsilon))$$

and the cost of the last edge is: $g_{ij}(1) - g_{ij}(\hat{\lambda}_{ij} + (l_{ij} - 1)\delta(\epsilon))$.

Add a source s and sink t to the G^ϵ such that, s has edges to all N TXs and t has edges to from all N RXs. The cost of each of these edges is 0 and the capacity is 1.

Solve for the *min-cost maximum flow* between s and t in graph G^ϵ . By construction the value of *maximum flow* is N . Let γ^ϵ be value of such *min-cost maximum flow* for the graph G^ϵ for the edges between N TXs and N RXs. We claim the following theorem:

Theorem 2: Let the actual minimization problem have value $OPT = f(\bar{\gamma})$, then $f(\gamma^\epsilon) \leq f(\bar{\gamma}) + \epsilon$.

Proof: Provided in the longer version of the paper. ■

B.1 Complexity of the approximated algorithm

The algorithm for this solution is effectively a *min-cost maximum flow* algorithm for a network with $|E| = N^2/\delta$ edges and $|V| = 2N$ nodes. The best known algorithm has a complexity of $O(|V||E|\log|V|/\log|E|)$, that is, $O(N^3/\delta)$.

V. CONCLUSIONS

This paper considered the minimum energy/bit transmission scheduling problem, arising in switch-constrained networks. The problem is shown to be a constrained convex optimization problem, whose optimal solution minimizes the energy spent while maximizing the throughput. Two algorithms were developed; one for finding the optimal solution, and another for finding an efficient approximation.

A number of related problems remain open and compelling. First, the delay performance needs to be characterized under generic stochastic traffic. Second, the assumption that the arrival rate matrix is known ought to be relaxed to better model realistic conditions, and for developing efficient on-line algorithms based on the state of the system.

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