BCN
Stability and Fairness

Yi Lu, Rong Pan, Balaji Prabhakar, Davide Bergamasco, Andrea Baldini, Valentina Alaria

Stanford University and Cisco Systems
Outline

1. Stability analysis
   - Explicit parameterization of stability region
   - Sufficient condition for overall stability

2. Self-increase
   - Stability
   - Fairness (?)
   - Flow completion time
Fluid-Model Equations

- The CP equations (not linearized)
  \[ \frac{dq(t)}{dt} = N \times R(t) - C. \]

- The RP equations

  \[ F_b(t) = - \left[ (q(t) - q_{eq}) + \frac{wS}{CP} \times \frac{dq(t)}{dt} \right] / S. \]

  If \( F_b(t - \tau) > 0, \)
  \[ \frac{dR(t)}{dt} = [G_i R_i \times F_b(t - \tau) \times R(t - \tau) \times P] / S. \]

  If \( F_b(t - \tau) < 0, \)
  \[ \frac{dR(t)}{dt} = [G_d \times R(t) \times F_b(t - \tau) \times R(t - \tau) \times P] / S. \]
Fluid-Model Equations

- Continuous time
- No stochastic processes
- No discrete packet sizes
- Assume infinite buffer size

- Control analysis stability
  - Help us set parameters
  - Prerequisite for stochastic stability
Stability Analysis

The linearized system is stable if

(i) \( G_i R_u w \leq \frac{S}{a \tau} \)

(ii) \( G_i R_u w^2 > \frac{PC}{b \sqrt{b^2 + 1}} \)

(iii) \( G_d w \leq \frac{SN}{a C \tau} \)

(iv) \( G_d w^2 > \frac{PN}{b \sqrt{b^2 + 1}} \)

where \( a \geq 1 \) and \( b/a + \arctan(b) = \pi/2 \)

\[
\begin{align*}
\text{atan}(x) & \quad \text{blue} \\
x/a & \quad \text{blue} \\
n & \quad \text{blue}
\end{align*}
\]

a bigger \( \rightarrow \) slower response

\( \rightarrow \) b bigger \( \rightarrow \) N can be bigger
Sufficient condition

(i) and (ii) corresponds to the source equation $F_b > 0$

(iii) and (iv) corresponds to the source equation $F_b < 0$

We show that these conditions are sufficient for the stability of the switching system.
Scenario

- Every 0.2 s, 50 new long-lived flows inserted
- Starting rate: 100 Mbps
- $q_{eq} = 16$
- Buffer size = 100 x 1500 Bytes
- $P = 0.01$
- $G_i = 4$, $Ru = 1e^6$, $w = 2$, $G_d = 1/128$
  obtained with $a = 5$ and $b = 2.2$
Stability
Self-increase: RP may gently increase its sending rate in various ways (see below), even when there are no BCN signals from its CP.

This is a good idea for several reasons:

- It is fail-safe (messages may be lost)
- Gently probe for extra bandwidth
- Very useful for fairness, as we shall see

Let’s consider 3 types of self-increase

1. At a fixed rate of \( A \) bps
2. At a rate \( AxR \) bps, where \( R \) is the current sending rate
3. At a rate \( A/(\# \text{ of negative feedback signals}) \)

Type 2 brings a bounded amount of extra work, regardless of the number of sources

Type 3 similar to type 1, but fairer
Self-increase: stability

Type 1: Gentle increase of 10 Mbps/s
Self-increase: stability

Type 1: Aggressive increase of 500 Mbps/s
Self-increase: stability

Type 2: Aggressive increase factor = 10/s
Self-increase: stability

Type 3: Aggressive increase of 500 Mbps/s
• Self-increase helps improve fairness properties
Fairness (unfairness index)

\[ \sum_i \frac{|x_i - \bar{x}|}{\bar{x}} \]
Fairness → Flow Completion Time

- Plots of fairness properties for infinitely long-lived flows are not very informative.

- We realize that fairness has its implications in scenarios with flows arriving and departing.

- Fairness can be translated into: For flows within a size range, the completion times are similar.

- Lack of fairness is hence reflected by the large variance in completion times.

- Simulations show that self-increase helps reduce the variance in completion time, and does not hurt the average.
Scenario

- Flow size distribution ~ Pareto 1.8
- Mean flow size 1 MB
- Arrival rate Poisson 1125 flow/sec
- 9 Gbps average traffic
- Starting rate 1Gbps
Average completion time
Normalized standard deviation