A Theoretical Model for QCN

Mohammadreza Alizadeh, Balaji Prabhakar
Stanford University
Overview

• The stability ("unit step response") of congestion control algorithms are analyzed theoretically in the following way
  – Write down equations describing evolution of algorithm
    • Usually, these are nonlinear delay-differential equations
  – Analyze these equations for stability
    • Usually, linearize equations around operating point and analyze linear system

• The reason QCN equations were hard to get were that the Fast Recovery cycle is different from the usual source behavior (there is usually no Target Rate--Current Rate)
  – We show how the equations can be obtained
  – And check their accuracy using simulations
Fluid Model for QCN

• We will model only the key features of the QCN protocol. Namely, we do not consider:
  - Timer, HAI, extra fast recovery, window jittering, drift increase.

• Switch behavior is not too different from what we have seen for BCN => Easy to describe

• But source behavior appears to have a new ‘memory’ element in the Fast Recovery phase. It’s not possible to model this with a single variable, namely the current rate at the source

• This motivates using two variables at the source: Current Rate, and Target Rate
Fluid Model for QCN

• Target Rate (TR) is the rate that the source tries to reach by successive phases of fast recovery
  – Anytime the source sends 100 packets, and it receives no congestion signals, CR increases to halve the distance between CR and TR, i.e.: \( \text{CR} \leftarrow (\text{CR} + \text{TR})/2 \)
  – Anytime the source receives a congestion signal, it multiplicatively decreases CR, i.e.: \( \text{CR} \leftarrow (1- G_d F_b) \text{CR} \)

• Upon receiving congestion signal, TR drops to CR, i.e.: \( \text{TR} \leftarrow \text{CR} \)

• In Active Increase, after sending 100 packets and not receiving congestion signals: \( \text{TR} \leftarrow \text{TR} + \alpha \)
  – \( \alpha = 5 \text{ Mbps} \)
Fluid Model for QCN

- Assume N flows pass through a single queue at a switch. State variables are $TR_i(t)$, $CR_i(t)$, $q(t)$, $p(t)$.

\[
\frac{dTR_i}{dt} = -(TR_i(t) - CR_i(t)) \times CR_i(t - \tau) p(t - \tau) + (1 - p(t - \tau))^{500} \times \alpha \times \frac{CR_i(t - \tau)}{100}
\]

\[
\frac{dCR_i}{dt} = -(G_d F_b(t - \tau) CR_i(t)) \times CR_i(t - \tau) p(t - \tau) + \frac{TR_i(t) - C_i(t)}{2} \times \frac{CR_i(t - \tau) p(t - \tau)}{1 - p(t - \tau)^{100}} - 1
\]

\[
\frac{dq}{dt} = \sum_{i=1}^{N} CR_i(t) - C
\]

\[
F_b(t) = q(t) - Q_{eq} + \frac{W}{Cp(t)} \times \left( \sum_{i=1}^{N} CR_i(t) - C \right)
\]

\[
\frac{dp}{dt} = (\Phi(F_b(t)) - p(t)) \times 500
\]
10 sources, 100 us
10 sources, 300 us
10 sources, 500 μs
10 sources, 1 ms

Graph 1: 10 sources, 1 ms

Graph 2: 10 sources with 1 ms