

Suppose $A \subset [-\infty, \infty]$. By definition, $x \in [-\infty, \infty]$ is a lower bound for A if and only if $x \leq y$ for every $y \in A$. Equivalently, x is a lower bound for A unless there is a $y \in A$ such that $y < x$. If A is the empty set, then there is no such y , so every x is a lower bound. Thus the set of lower bounds is $[-\infty, \infty]$, so the infimum (which by definition is the greatest lower bound) is ∞ :

$$\inf \emptyset = \infty.$$

Likewise, $\sup \emptyset = -\infty$. Thus, perhaps surprisingly, $\sup \emptyset < \inf \emptyset$. (Of course for any nonempty $A \subset [-\infty, \infty]$, we have $\inf A \leq \sup A$.)