

# A DISTRIBUTED ALGORITHM FOR CONGESTION-MINIMIZED MULTI-PATH ROUTING OVER AD HOC NETWORKS

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## ABSTRACT

When an ad hoc network with limited link capacities is used to transport high-rate, latency-constrained multimedia streams, it is important that routing algorithms not only yield high aggregated bandwidth, but also minimize network congestion to avoid excessive delay. We propose a distributed algorithm to achieve congestion-minimized multi-path routing. The optimal solution can be approximated by decomposing the total target rate into a sequence of rate increments, and using the classical Bellman-Ford algorithm to find a corresponding minimum-cost route for each increment. Simulation results for video streaming over an ad hoc network show that compared to a centralized scheme, the approximations introduced by the distributed algorithm lead to performance loss of less than 0.5 dB in PSNR of received video quality.

## 1. INTRODUCTION

In an ad hoc network, nodes communicate with each other in a peer-to-peer fashion. Each node can be a source, a destination, or a relay for traffic. The flexible configuration of such a network makes it appealing for many applications. For instance, a group of such nodes can be deployed in a residential neighborhood to collectively deliver multimedia content from one common Internet access point to a chosen household.

Multimedia delivery over ad hoc networks imposes many challenges, however. Limited resources in bandwidth and power at each node, combined with adverse wireless channel conditions and the impact of interference, typically lead to relatively low capacity on each individual link. On the other hand, multimedia content usually has a high data rate and, for streaming services, stringent latency requirements. Consequently, a high-rate media stream may need to be dispersed over multiple paths for delivery, so as to avoid unacceptably high delay over any individual link.

In the case of a wireless ad hoc network operating under a CDMA-like media access protocol, where all nodes can transmit simultaneously and the capacities are determined by the amount of interference observed on each link, it is shown in [1] that multi-path video streaming yields an efficient allocation of network resources. Overall network congestion is minimized and higher aggregate bandwidth are achieved, allowing video streams of higher rate and quality.

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To share the computational burden among many nodes and to avoid the overhead of collecting global network information, it is desirable to have a distributed algorithm that achieves the same objective. While research on ad hoc routing has been very active [2, 3], and while many distributed routing protocols such as DSR [4] and AODV [5] are tailored for ad hoc networks, the issue of congestion minimization is seldom addressed. As the protocols are designed with generic traffic in mind, the typical criterion for route selection is minimum number of hops, which may not lead to the best performance [6].

Our work, instead, seeks to minimize the overall network congestion, defined as the average delay per link for all packets in the network, in a distributed fashion. After explaining the objective function for flow assignment and route selection in Section 2, we show in Section 3 how to decompose the original problem as a series of minimum-cost routing problems, each of which can be readily solved by the distributed Bellman-Ford algorithm. Simulation results are presented in Section 4.

## 2. CONGESTION-OPTIMIZED MULTI-PATH ROUTING

In a bandwidth-limited network, where the capacity of each individual link is relatively low with respect to the supported traffic rate, it is important to disperse the total traffic rate over many different links, so that no individual link is overwhelmed and creates unacceptable queuing delay. In fact, it is a classic problem to minimize the network congestion via optimal flow assignment, as studied in [7] and [8]. Using the M/M/1 queuing model, the average delay on each link is proportional to  $1/(C - F)$ , where  $C$  is the link capacity and  $F$  is the rate of existing traffic flow. Therefore, the optimal flow assignment  $f_{ij}$ 's over links from node  $i$  to node  $j$  should achieve the following:

$$\min_{f'_{ij}s} \sum_{i,j} \frac{F_{ij} + f_{ij}}{C_{ij} - F_{ij} - f_{ij}}. \quad (1)$$

In (1), the  $C_{ij}$ 's denote the link capacity, the  $F_{ij}$ 's denote the rate of existing traffic and the  $f_{ij}$ 's denote the additional flow from a new stream. The  $f_{ij}$ 's also need to satisfy the flow positivity and continuity constraints, and add up to the total rate  $R$  of the new stream at the sender and receiver. Since the objective is a convex function of the flows,

and the constraints are all linear, the problem can be solved efficiently via the interior point method [9] in a centralized fashion.

Note, however, that the resulting flow assignments usually do not correspond directly to a routing solution. To bridge the difference, in [1], paths with highest bottleneck rates are extracted recursively from this initial flow assignment and then the total rate is re-partitioned among the extracted routes, while minimizing the average delay over the selected links. The drawback of such a centralized scheme is that all the computation is carried out by one node, which also needs to collect global information such as the capacity and the rate of existing traffic on all links in the network. The complexity of the algorithm and the overhead of information collection scale approximately with the square of the total number of nodes in the network. The centralized solution therefore becomes less attractive as the network size grows.

### 3. THE DISTRIBUTED ALGORITHM

As an alternative, we show in this section that the optimization in (1) can be approximately decomposed into a sequence of minimum-cost routing problems, each of which is readily solved by the well-known Bellman-Ford distributed algorithm [10].

Consider dividing the total target rate  $R$  into  $K$  small increments  $R = \sum_{k=1}^K \Delta R_k$ . Assuming that the optimal routes for  $k-1$  of the  $K$  increments are already determined, resulting in the additional flows  $f_{ij}^1, \dots, f_{ij}^{k-1}$  over the selected links, we now need to choose an appropriate route for the next increment  $\Delta R_k$ , such that:

$$\min_{f_{ij}^k} \sum_{i,j} \frac{F'_{ij} + f_{ij}^k}{C_{ij} - F'_{ij} - f_{ij}^k}, \quad (2)$$

where  $F'_{ij} = F_{ij} + \sum_{k'=1}^{k-1} f_{ij}^{k'}$  denotes the existing traffic plus the contributions from previous  $k-1$  rate increments. This is equivalent to optimizing the *increase* in the total network congestion:

$$\min_{f_{ij}^k} \sum_{i,j} \left( \frac{F'_{ij} + f_{ij}^k}{C_{ij} - F'_{ij} - f_{ij}^k} - \frac{F'_{ij}}{C_{ij} - F'_{ij}} \right) \quad (3)$$

$$= \sum_{i,j} \frac{C_{ij} f_{ij}^k}{(C_{ij} - F'_{ij})(C_{ij} - F'_{ij} - f_{ij}^k)} \quad (4)$$

$$\approx \sum_{i,j} \frac{C_{ij} f_{ij}^k}{(C_{ij} - F'_{ij})^2}. \quad (5)$$

The approximation holds when  $\Delta R_k$ 's are small, and consequently  $f_{ij}^k$ 's are small. One can further restrict the flow assignment for  $\Delta R_k$  to be over a single path from source to destination. This results in a sub-optimal solution to (2), but since the increment is small, the degradation in performance is expected to be insignificant.

Now, the problem of optimal flow assignment for  $\Delta R_k$

is converted into that of finding the single path  $\mathcal{P}_k$  from source to destination minimizing the increase in total network congestion. Since only links belonging to  $\mathcal{P}_k$  experience a change, the overall increment in network congestion is only affected on the links of the selected route. The optimization now becomes:

$$\min_{\mathcal{P}_k} \sum_{(i,j) \in \mathcal{P}_k} \frac{C_{ij}}{(C_{ij} - F'_{ij})^2} \Delta R_k. \quad (6)$$

This can be viewed as a minimum-cost routing problem, where the total cost from source to destination  $\sum_{(i,j) \in \mathcal{P}_k} C_{ij} / (C_{ij} - F'_{ij})^2$  is independent of the amount of traffic to be routed  $\Delta R_k$ . In fact, the link cost  $C_{ij} / (C_{ij} - F'_{ij})^2$ , as the derivative of the function  $F / (C - F)$  with respect to  $F$ , can be interpreted as the *sensitivity* of total network congestion to additional traffic rate on that link.

The minimum-cost path for  $\Delta R_k$  can be readily solved by the distributed Bellman-Ford algorithm [10], where each node maintains a minimum-cost path of itself to the source, exchanges this information with its neighbors, and updates to a better path if it discovers one through one of its neighbors. The minimization at node  $i$  can be formally expressed as:

$$\min_{j \in \mathcal{N}_i} PC_{sj} + LC_{ji}, \quad (7)$$

where  $\mathcal{N}_i$  denotes the collection of neighbors of node  $i$ ,  $PC_{sj}$  is the minimum cost for source node  $s$  to node  $j$ , and  $LC_{ji}$  is the link cost from node  $j$  to node  $i$  calculated as in (6). Note that this requires knowledge of the link capacity  $C_{ji}$ , the existing traffic  $F_{ji}$  and the total rate of previous increments that travel over this link  $\sum_{k':(j,i) \in \mathcal{P}_{k'}} \Delta R_{k'}$ . In a network of  $N$  nodes and diameter  $D$ , the algorithm is guaranteed to converge within  $D$  rounds of information exchange, by the end of which all nodes in the network find a minimum-cost path to the source. At the destination, a reply message is then sent backwards along the reverse path, to update the estimate of  $F'_{ij}$ 's on the affected links and to inform the sender of the chosen route.

By repeating this process for all increments from  $\Delta R_1$  to  $\Delta R_K$ , the proposed scheme can find up to  $K$  paths for the total rate  $R$ , as an approximation to the centralized congestion-minimized flow assignment solution. While there is much freedom in the design of the rate increment sequence  $\{\Delta R_k\}$ , which may potentially impact the routing performance, only the simple case of  $\Delta R_k = R/K$  is considered for the rest of the paper.

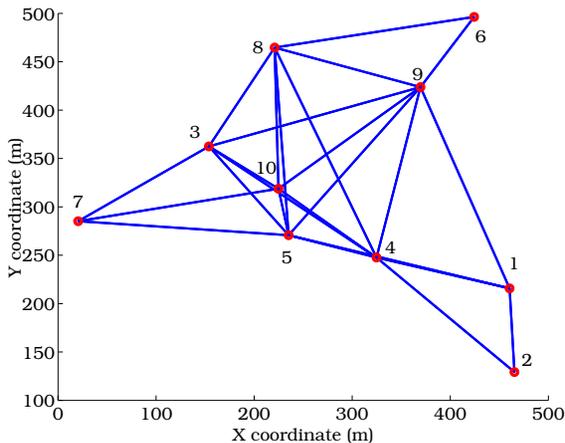
With a small  $K$ , fewer rounds of the minimum-cost routing procedure is needed, hence lower delay for route acquisition; when  $K$  is large, however, the distributed algorithm may select too many paths for practical considerations. The total number of paths can be further restricted by sorting all the selected routes according to the allocated traffic rate on each path, choosing the first  $L$  paths and re-allocating among them the total rate proportional to their initial rate allocations.

#### 4. SIMULATION RESULTS

Network simulations are performed in ns-2 [11] to compare the performance of the proposed distributed routing algorithm with the centralized scheme in [1]. As illustrated in Fig. 1, the network contains 10 nodes randomly placed in a 500m-by-500m square. Links are established between nodes within a distance of 250m. The capacity on each link is chosen randomly between 100kbps and 1Mbps. The rate of existing traffic on each link is also assigned a random value up to 50% of the link capacity. The existing traffic is simulated by packets with fixed size (500 bytes) and exponentially distributed arrival time.

A video streaming application is simulated by looping 300 frames of the *Foreman* CIF sequence, encoded with the H.264 codec at different quantization parameters. The frame rate is 30Hz, and the GOP length is 15, with a conventional IBBP... structure. The playout deadline is set at 500ms; packets arriving after the deadline are dropped. Routing for video streaming from Node 1 to Node 6 is carried out using either the centralized or the distributed scheme. In the case of distributed routing, the rate increment is set to be 20, 50 or 100kbps. Each simulation lasts for 500 seconds, i.e., 50 realizations of the decoded video sequence. Note that by using encoded packet size for video streams and fixed size packets for the existing traffic, the simulation captures the effect on performance of more realistic traffic patterns instead of the M/M/1 model used in the optimization.

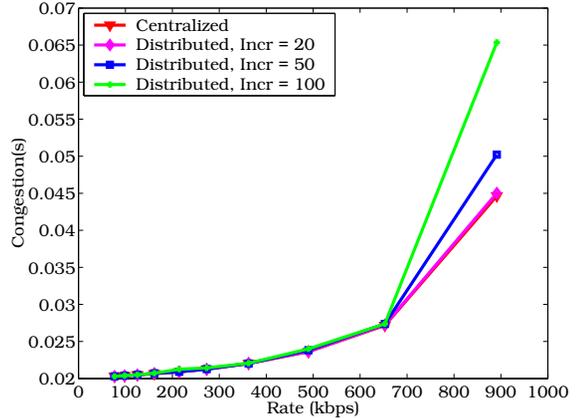
Figure 2 plots the network congestion, calculated ac-



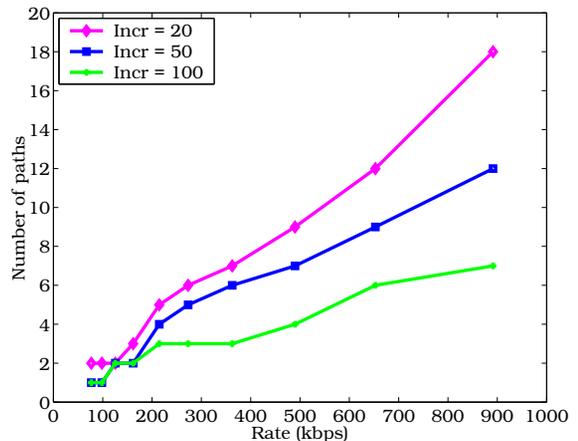
**Fig. 1.** A network with 10 nodes, randomly placed within a 500m-by-500m square.

ording to the M/M/1 model, resulting from video streaming at different rates. It can be seen that the distributed algorithm yields slightly higher network congestion than that of the centralized solution. The degradation is more pronounced with a coarser rate increment, as the approximation in (2) is less accurate.

Figure 3 shows the number of paths initially found by the distributed algorithm. With a higher rate or a finer rate increment, more paths are needed for dispersing the total rate among multiple links. Consequently, when the total



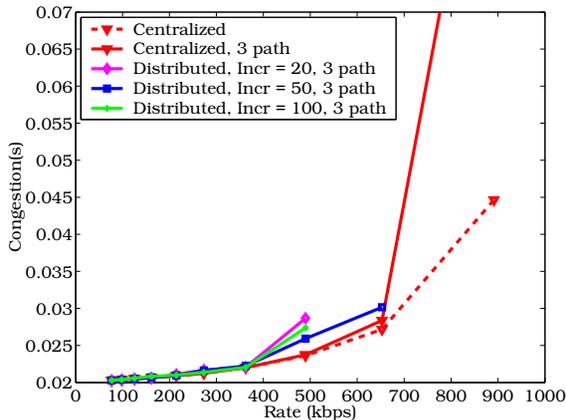
**Fig. 2.** Network congestion calculated by the M/M/1 formula for the centralized and distributed algorithms without path constraints.



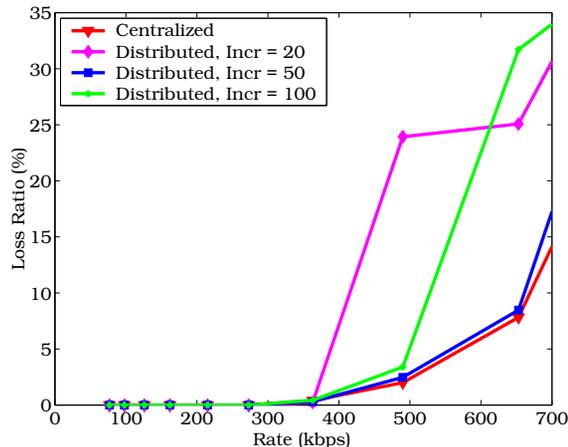
**Fig. 3.** Total number of paths selected by the distributed routing algorithm, with different incremental rates

number of paths are further restricted due to practical considerations, the routing performance with a finer rate increment will be affected more severely. When confining the total number of paths to 3 in our experiments, there is a change in the relative performance of the algorithms, as indicated in Fig. 4. The lowest congestions are now achieved by the intermediate rate increment of 50 kbps. The loss of performance due to the restriction on total number of paths is also illustrated in the figure for the centralized scheme.

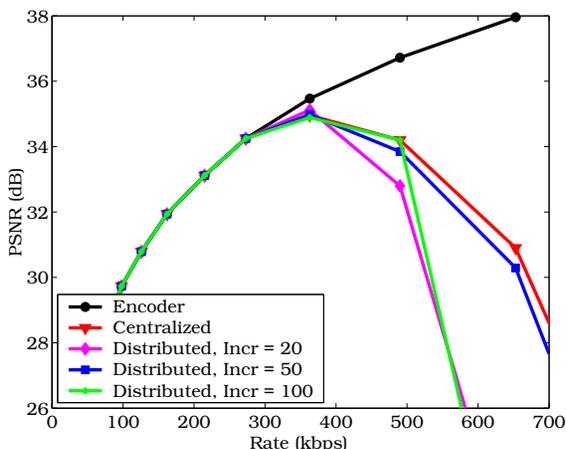
Figure 5 plots the rate-PSNR performance of video streaming over 3 paths in the network, together with the encoder rate-PSNR curve for reference. The corresponding packet loss ratios are shown in Fig. 6. Beyond the rate of 653 kbps, the high loss ratios indicate unacceptable received video quality. Below this rate, it can be seen that the distributed algorithm with the intermediate rate increment of 50 kbps performs most closely to the centralized scheme. The difference in PSNR of the received video quality is within 0.5 dB.



**Fig. 4.** Network congestion calculated by the M/M/1 formula for the centralized and distributed algorithms restricted to 3 paths



**Fig. 6.** Packet loss ratio for different routing schemes at different rate of the video



**Fig. 5.** Rate-PSNR performance of video streaming over 3 paths in a 10-node network, using centralized or distributed routing with different incremental rates.

## 5. CONCLUSIONS

We propose a distributed algorithm for congestion-minimized multi-path routing. By decomposing the congestion optimization process into a sequence of minimum-cost routing problems, and applying the Bellman-Ford distributed algorithm to find each minimum-cost route, we show that the distributed solution yields a close approximation to its centralized counterpart. Simulation results for video streaming over an ad hoc network confirm that the difference in the received video quality between the centralized and distributed schemes is less than 0.5 dB with properly chosen rate increments.

## 6. REFERENCES

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