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Stanford University and NBER

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We Know Little About Bargaining in Field

• Bargaining is one of oldest and most common forms of transactions

• Lots of theory work

• Lots of experiments in econ and OB/psych

• Little known about how it plays out in the field

Figure: In the Souk, 1891 by Moritz Stifter
Research Questions

1. In a bargaining setting with incomplete information, can we learn anything about buyers’/sellers’ private values from data even if when we don’t have a complete model of equilibrium behavior?

2. When a negotiating buyer and seller disagree, how often is it the case that the buyer really values the good more that the seller and yet they still fail to trade (inefficient impasse/breakdown)?
Existing Empirical/Theory Frameworks Insufficient

Structural empirical bargaining work typically assumes *complete* information (e.g. Nash Bargaining).

Typically ignores

- incomplete information (of both parties)
- actual strategic process of bargaining
- possibility of inefficient impasse/breakdown
  (in spite of Myerson-Satterthwaite 1983, Williams 1987)

Arguably justifiable shortcomings given

- **Data**: often only *final negotiated price* for *successful trades*
- **Theory**: Incomplete info is challenging
  - two-sided signaling; multiple, qualitatively diff. equilibria
  - refinements often fail to narrow down (Gul-Sonnenschein 1988) or predict immediate ending (Perry 1986)
  - lit. focuses on special cases (TIOLI offer, one-sided inc. info)
New Empirical Bargaining Research
Detailed data on offers from field; opens up new possibilities

- Recent studies with detailed data on process (all offers)
  1. Keniston 2011 (auto-rickshaw rides in India)
  2. Prescott Spier 2014 (pre-trial settlement)
  3. Hernandez-Arenaz Iriberri 2018 (game show in Spain)
  4. Bagwell et al. 2019 (tariff negotiations)
  5. Backus et al. 2020 (eBay bargaining)
  6. Mateen et al. 2021 (housing)
  7. Larsen 2021 (used car B2B bargaining)
  8. Keniston et al. 2021 (studies fairness in above 7 settings)
  9. Dunn et al. 2021 (medical claim reimbursement)
  10. Byrne et al. 2019 (retail electricity service)

- This paper: Consumers bargaining on eBay
  - Assumptions on equilibrium behavior or common knowledge arguably even less likely to hold
New Empirical Bargaining Research

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This Paper: Robust Structural Bounds Analysis

Our approach:

• Make assumptions on eBay consumers’ behavior (without fully characterizing equilibria)

• Bound buyer and seller value distributions (objects needed for variety of counterfactuals)

• Menu of assumptions of varying strength ⇒ wider/tighter bounds

• Bound counterfactual first-best prob of trade, compare to data ⇒ measure of inefficient impasse

Could be called an auction approach to studying bargaining ⇒ allowing for private info, strategic behavior
Related Literature

• Theory on incomplete-info bargaining

• Experimental work on incomplete-info bargaining
  • Valley et al. 2002, Bochet et al. 2020, Huang et al. 2021

• eBay bargaining:
  • Backus et al. (*), Green-Plunkett 2021, Keniston et al. 2021

• Structural empirical work on incomplete-info bargaining

• Only study quantifying inefficient impasse in real-world bargaining: Larsen (2021), used cars
  • Has post-auction bargaining data (we don’t here)
  • Professionals (here we have consumers)
eBay Bargaining: “Best Offer” Platform

- Starts with seller list/Buy-It-Now price
- Buyer can make offer
- Seller accepts, counters, or quits
  ... (etc.)
Hi Brad,
Don't forget you have received a Best Offer for your item. cleansweepfive is waiting for your response - don't leave them hanging!

Brandon Sanderson
STORMLIGHT ARCHIVE Fantasy Series Col...

Pending offer: $20.00 expires Jun 08, 2018 20:14 PDT
Quantity: 1
Buy it now price: $25.00
Buyer: cleansweepfive (10)

Respond
Great news, your item sold. Now it’s time to get it ready.

Hi Brad,

Your buyer hasn't paid yet, so hold onto the item until you receive payment. We’ll send your buyer a reminder to pay within 48 hours.

Check payment status

Brandon Sanderson STORMLIGHT ARCHIVE
Fantasy Series Collection 1-3

Sale Price: $22.50

Item #: 263723734962
Date Sold: 06/08/2018
Quantity Sold: 1
Buyer: cleansweepfive (10 ★) 100% Contact Buyer
Views: 101
Watchers: 1
Descriptives for Top-Selling Product Per Category

- Subset of data from Backus et al. 2020. We limit to first seller a given buyer negotiates with, and vice versa
- Limit to 363 popular cataloged products (those with at least 100 sequences) ⇒ 70k obs. (sequences)

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Descriptives for Top-Selling Product Per Category

- Stats for product negotiated over most within category
- Reference price = avg. fixed price, same product
- $P(sale)$ is quite low (mostly 0.2–0.4)

**Question:** is this impasse efficient or inefficient?

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Incomplete Model of Bilateral Bargaining

- Buyer with value $B \sim F_B$ and seller with value $S \sim F_S$ bargain over a good. Supports of $B$ and $S$ are $[0, \infty)$

- If agree at price $P$, payoffs are $B - P$ for buyer, $P$ for seller

- If disagree, payoffs are 0 for buyer and $S$ for seller

- Assume $S, B$ stay constant during a given negotiation (i.e. players learn about opponent’s value, not own)

- Bargain via alternating offers, starting with seller turn

- Data: observe offers and decisions at each turn for many instances of the game

- **Goal**: Bounds on $F_B$, $F_S$, and $P(S \leq B)$
Some Notation at Offer Level

i.e. at level of an individual player making a decision at a given period of a given bargaining sequence

- $t = 1, 2, \ldots$ denotes period in bargaining, starts with seller

- Seller’s decision at $t$ odd: $D^S_t \in \{A, Q, C\}$ to Accept ($A$), Quit ($Q$), or Counter ($C$)

- Buyer’s decision at $t$ even: $D^B_t \in \{A, Q, C\}$
  - Note: $D^B_2 = C$ in our subsample (cases with a buyer offer)

- Seller’s offer at odd $t$ (if $D^S_t = C$): $P^S_t$

- Buyer’s offer at even $t$ (if $D^B_t = C$): $P^B_t$
An Example: Cell Phone

$t = 1, 2, \ldots$ denotes period in bargaining, starts with seller

$t = 1$: Seller: $\$350$

$t = 2$: Buyer: $\$150$

$t = 3$: Seller: $\$325$

$t = 4$: Buyer: $\$225$

$t = 5$: Seller: $\$315$

$t = 6$: Buyer: Quit
An Example: Cell Phone (+ Notation)

$t = 1, 2, ...$ denotes period in bargaining, starts with seller

$t = 1$: Seller: \[ 350 = P^S_1 \]

$t = 2$: Buyer: \[ 150 = P^B_2, D^B_2 = C \]

$t = 3$: Seller: \[ 325 = P^S_3, D^S_3 = C \]

$t = 4$: Buyer: \[ 225 = P^B_4, D^B_4 = C \]

$t = 5$: Seller: \[ 315 = P^S_5, D^S_5 = C \]

$t = 6$: Buyer: Quit, \[ D^B_6 = Q \]
What Does Data + Rationality Imply for $B$ and $S$?

$t = 1, 2, \ldots$ denotes period in bargaining, starts with seller

$t = 1$: Seller: $350 \Rightarrow S \in [0, 350]$

$t = 2$: Buyer: $150$

$t = 3$: Seller: $325$

$t = 4$: Buyer: $225$

$t = 5$: Seller: $315$

$t = 6$: Buyer: Quit

Recall: Buyer gets $B - P$ if agree, 0 if disagree
Seller gets $P$ if agree, $S$ if disagree
What Does Data + Rationality Imply for B and S?

$t = 1, 2, \ldots$ denotes period in bargaining, starts with seller

$t = 1$: Seller: $\$350 \Rightarrow S \in [0, 350]$

$t = 2$: Buyer: $\$150 \Rightarrow B \in [150, \infty)$

$t = 3$: Seller: $\$325$

$t = 4$: Buyer: $\$225$

$t = 5$: Seller: $\$315$

$t = 6$: Buyer: Quit

Recall: Buyer gets $B - P$ if agree, 0 if disagree
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What Does Data + Rationality Imply for $B$ and $S$?

$t = 1, 2, \ldots$ denotes period in bargaining, starts with seller

$t = 1$: Seller: $350$

$t = 2$: Buyer: $150 \Rightarrow B \in [150, \infty)$

$t = 3$: Seller: $325 \Rightarrow S \in [0, 325]$  

$t = 4$: Buyer: $225$

$t = 5$: Seller: $315$

$t = 6$: Buyer: Quit

Recall: Buyer gets $B - P$ if agree, 0 if disagree  
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$t = 1, 2, ...$ denotes period in bargaining, starts with seller

$t = 1$: Seller: $350$

$t = 2$: Buyer: $150$

$t = 3$: Seller: $325 \Rightarrow S \in [0, 325]$

$t = 4$: Buyer: $225 \Rightarrow B \in [225, \infty)$

$t = 5$: Seller: $315$

$t = 6$: Buyer: Quit

Recall: Buyer gets $B - P$ if agree, 0 if disagree
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What Does Data + Rationality Imply for $B$ and $S$?

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$t = 1$: Seller: $350$  
$t = 2$: Buyer: $150$  
$t = 3$: Seller: $325$  
$t = 4$: Buyer: $225 \Rightarrow B \in [225, \infty)$  
$t = 5$: Seller: $315 \Rightarrow S \in [0, 315]$  
$t = 6$: Buyer: Quit

Recall: Buyer gets $B - P$ if agree, $0$ if disagree  
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What Does Data + Rationality Imply for $B$ and $S$?

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$t = 3$: Seller: $\$325$

$t = 4$: Buyer: $\$225$

$t = 5$: Seller: $\$315 \Rightarrow S \in [0, 315]$

$t = 6$: Buyer: Quit $\Rightarrow B \in [225, 315]$

Recall: Buyer gets $B - P$ if agree, 0 if disagree
Seller gets $P$ if agree, $S$ if disagree
What Does Rationality Imply for $B$ and $S$?

$t = 1, 2, ...$ denotes period in bargaining, starts with seller

$t = 1$: Seller: $350 = P^S_1$

$t = 2$: Buyer: $150 = P^B_2$

$t = 3$: Seller: $325$

$t = 4$: Buyer: $225 = X^B_{AC}$

$t = 5$: Seller: $315 = X^S_{AC}$

$t = 6$: Buyer: Quit

Recall: Buyer gets $B - P$ if agree, 0 if disagree
Seller gets $P$ if agree, $S$ if disagree

$X^B_{AC}$ is buyer’s max accept/counter; $X^S_{AC}$ is min for seller
What Does Rationality Imply for $B$ and $S$?

$t = 1, 2, ...$ denotes period in bargaining, starts with seller

$t = 1$: Seller: $350 = P_1^S$
$t = 2$: Buyer: $150 = P_2^B$
$t = 3$: Seller: $325$
$t = 4$: Buyer: $225 = X_{AC}^B$
$t = 5$: Seller: $315 = X_{AC}^S$
$t = 6$: Buyer: Quit

Recall: Buyer gets $B - P$ if agree, 0 if disagree
Seller gets $P$ if agree, $S$ if disagree

$X_{AC}^B$ is buyer’s max accept/counter; $X_{AC}^S$ is min for seller

$X_Q^B = 315$ (buyer’s quit price) and $X_Q^S = 0$ (seller never quit)
Summarizing These Initial Assumptions

Assumption 1 (*Unconditional Assumptions*)

(i) Buyer never accepts or counters at a price higher than $B$, and seller never accepts or counters at a price lower than $S$

(ii) If buyer quits at price $P$, it must be that $B - P \leq 0$; if seller quits at price $P$, it must be that $S \geq P$

Under these assumptions, across multiple obs. (sequences),

- Empirical CDFs of $X_{AC}^S$ and $X_{Q}^S$ bound $F_S$
- Empirical CDFs of $X_{AC}^B$ and $X_{Q}^B$ bound $F_B$
Discussion of Initial Bounds

• Bounds are pointwise sharp (can’t reject they hold with equality)

• Bounds do not rely on
  • Any notion of equilibrium (allow multiple equilibria)
  • Any common knowledge assumption
  • Any structure on joint distribution of buyer/seller values

• Robust to incomplete info (or even complete info)

• Robust to unobserved heterogeneity. Examples:
  • Additively separable: $B + W, S + W$
  • Multiplicatively separable: $BW, SW$

• But... weak assumptions can lead to wide bounds
Unconditional Bounds on $F_B$, Cell Phone Product

Number of sequences in data for this product: 2,501

- y-axis: upper and lower bounds on buyer CDF, $F_B$
- x-axis: buyer values, normalized by reference price for this product
  (reference price is avg. fixed price; for this product = $225)
Unconditional Bounds on $F_B$, Cell Phone Product

Number of sequences in data for this product: 2,501

- Upper bound is empirical CDF of $X^B_{AC}$
  (maps to every point in [0,1] because $P^B_2$ always observed)
- Lower bound is CDF of $X^B_Q$
  (Note $X^B_Q = \infty$ in a sequence where buyer doesn’t quit)
Unconditional Bounds on $F_S$, Cell Phone Product

Number of sequences in data for this product: 2,501

- CDF bounds tighter for seller values than for buyer
- Seller lower bound maps to all point in [0,1] because $P_1^S$ always observed (and that enters into $X^S_{AC}$)
- Upper bound is empirical CDF of $X^S_Q$ (0 if seller doesn’t quit)
What Stronger Assumptions Could We Impose?

• Few models study (i) sequential offers, (ii) cont. values, (iii) 2-sided incomplete info, (iv) 2 parties can make offers

• Two examples:
  • Perry (1986): game ends immediately (only 1 offer in equil.)
  • Cramton (1992): At most 2 meaningful offers in equil.

• A property satisfied in both models: Monotonicity
  • $P^S_1$ weakly increases in $S$
  • $P^B_2$ weakly increases in $B$ conditional on $P^S_1$

• A property of info environ. in both models: Independence
  • $P^S_1$ independent of $B$
  • $P^B_2$ independent of $S$ conditional on $P^S_1$
Bounds Implied by Monotonicity

- Recall $X_{AC}^S$ is lowest price in this negotiation sequence at which the seller indicated a willingness to sell

- Let
  - Seller’s first offer be $P_1^S = y$
  - $X_{AC}^{S^*}(y) = $ lowest offer accepted/countered by any sellers whose first offer is weakly higher than $y$

- **Monotonicity** of seller’s first offer $P_1^S$ in $S \Rightarrow X_{AC}^{S^*}(y) \geq S$

\[ \Rightarrow P(S \leq x) \geq \int 1(X_{AC}^{S^*}(y) \leq x) dF_{P_1^S}(y) \]

- Similar upper bound on $F_S$ and bounds on $F_B$
Bounds Implied by Monotonicity

• Recall $X_{AC}^S$ is lowest price in this negotiation sequence at which the seller indicated a willingness to sell

• Let
  • Seller’s first offer be $P_1^S = y$
  • $X_{AC}^{S*}(y) = \text{lowest offer accepted/countered by any sellers whose first offer is weakly higher than } y$

• **Monotonicity** of seller’s first offer $P_1^S$ in $S \Rightarrow X_{AC}^{S*}(y) \geq S$

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• Similar upper bound on $F_S$ and bounds on $F_B$

• Bounds are pointwise sharp
Bounds Implied by Independence

• **Independence** of $B$ and seller’s first offer $P_S^1$: For any $y$,

$$P(B \leq x) = P(B \leq x | P_S^1 = y)$$

• Therefore,

$$\max_{y'} P(B \leq x | P_S^1 = y') = F_B(x) = \min_{y'} P(B \leq x | P_S^1 = y')$$

• To get bounds from data, replace $B$ with $X_{AC}^B$ and $X_Q^B$:

$$\max_{y'} P(X_{AC}^B \leq x | P_S^1 = y') \leq F_B(x) \leq \min_{y'} P(X_{AC}^B \leq x | P_S^1 = y')$$
Bounds Implied by Independence

• **Independence** of $B$ and seller’s first offer $P^S_1$: For any $y$,

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• To get bounds from **data**, replace $B$ with $X^B_{AC}$ and $X^B_Q$:

$$\max_{y'} P(X^B_Q \leq x | P^S_1 = y') \leq F_B(x) \leq \min_{y'} P(X^B_{AC} \leq x | P^S_1 = y')$$

• Bounds are pointwise sharp
These Assumptions Can Be Violated

- Cramton (1992), Perry (1986) both satisfy monotonicity and independence

- Consider modified model, with game-level heterogeneity, $W$ shifting players’ values ($B + W$, $S + W$)
  - $W$ unobserved to econometrician but observed to players
  - Captures cracked screens, missing/extra pieces, etc.
  - Introduces correlation between seller and buyer values

- Unobserved heterogeneity can (but won’t necessarily) violate both monotonicity and independence in both models
Some Alternative, Weaker Assumptions

- Recall that *Monotonicity* is
  - $P^S_1$ weakly increases in $S$
  - $P^B_2$ weakly increases in $B$ conditional on $P^S_1$

- A *weaker* alternative: *Stochastic Monotonicity*
  - $P^S_1$ stochastically increases in $S$
  - $P^B_2$ stochastically increases in $B$ conditional on $P^S_1$
Some Alternative, Weaker Assumptions

- Recall that *Monotonicity* is
  - $P_1^S$ weakly increases in $S$
  - $P_2^B$ weakly increases in $B$ conditional on $P_1^S$

- A *weaker* alternative: *Stochastic Monotonicity*
  - $P_1^S$ stochastically increases in $S$
  - $P_2^B$ stochastically increases in $B$ conditional on $P_1^S$

- Recall that *Independence* is
  - $P_1^S$ independent of $B$
  - $P_2^B$ independent of $S$ conditional on $P_1^S$

- A *weaker* alternative: *Positive Correlation*
  - $P_1^S$ has non-negative correlation with $B$
  - $P_2^B$ has non-negative correlation with $S$ conditional on $P_1^S$
Some Alternative, Weaker Assumptions

- Recall that *Monotonicity* is
  - $P_1^S$ weakly increases in $S$
  - $P_2^B$ weakly increases in $B$ conditional on $P_1^S$

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- Recall that *Independence* is
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- A *weaker* alternative: *Positive Correlation*
  - $P_1^S$ has non-negative correlation with $B$
  - $P_2^B$ has non-negative correlation with $S$ conditional on $P_1^S$

In Cramton (1992) and Perry (1986), both weaker assumptions are satisfied under unobserved heterogeneity
Stochastic Monotonicity and Positive Correlation Bounds

- **Stochastic monotonicity** of seller’s first offer in $S$:

  $P(S \leq x | P^S_1 = y)$ is decreasing in $y$

  $\Rightarrow P(S \leq x) = \int P(S \leq x | P^S_1 = y) dF_{P^S_1}(y)$

  $= \int \max_{y' \geq y} P(S \leq x | P^S_1 = y') dF_{P^S_1}(y)$

  $\geq \int \max_{y' \geq y} P(X^S_{AC} \leq x | P^S_1 = y') dF_{P^S_1}(y)$
Stochastic Mon. and Pos. Correlation Bounds

- **Stochastic monotonicity** of seller’s first offer in $S$:

  $P(S \leq x|P_1^S = y)$ is decreasing in $y$

  $$\Rightarrow P(S \leq x) = \int P(S \leq x|P_1^S = y) dF_{P_1^S}(y)$$

  $$= \int \max_{y' \geq y} P(S \leq x|P_1^S = y') dF_{P_1^S}(y)$$

  $$\geq \int \max_{y' \geq y} P(X_{AC}^S \leq x|P_1^S = y') dF_{P_1^S}(y)$$

- **Positive correlation** of $B$ and seller’s first offer $P_1^S$:

  $P(B \leq x|P_1^S = y)$ is decreasing in $y$

  $$\Rightarrow P(B \leq x) = \int \max_{y' \leq y} P(B \leq x|P_1^S = y') dF_{P_1^S}(y)$$

  $$\leq \int \max_{y' \leq y} P(X_{AC}^B \leq x|P_1^S = y') dF_{P_1^S}(y)$$
## A Menu of Assumptions and Bounds

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
<th>Stochastic Monotonicity</th>
<th>Monotonicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconditional</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Positive Correlation</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Independence</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
When Do Bounds Help Over Unconditional Bounds?

Consider how seller monotonicity ($P_1^S$ increasing in $S$) affects lower bound on $F_S$

- Will tighten $F_S$ lower bound if some sellers who start with relatively low $P_1^S$ end up accepting or countering later in the game at a relatively high final offers (i.e. high $X_{AC}^S$)

- Can arise from
  - randomness in buyer who the seller gets matched with
  - anything later in game (which we assume nothing about)

- If the assumption doesn’t tighten the bounds, doesn’t mean it isn’t satisfied, it means that $X_{AC}^S$ is also monotonic in $S$

- If bounds cross $\Rightarrow$ assumption is violated
Estimation

- Each bound nonparametrically identified (combinations of conditional probabilities)

- Each can be estimated using combination of nonparametric regressions (e.g., series or kernel)

- Data requirements: For each bargaining interaction, observe any offers \{P^S_1, P^B_2, P^S_3, \ldots\} and decisions \{D^S_1, D^B_2, D^S_3, \ldots\}

Next: Look at CDF bound estimates to see
- which properties satisfied
- which bounds are wide/narrow
Monotonicity Bounds for Seller CDF Cross

Number of sequences in data for this product: 2,501

- Assumptions are falsifiable: Crossing bounds means assumption can’t be satisfied
- Seller **monotonicity** \((P_1^S \text{ increasing in } S)\) is too strong for this product
No evidence **stochastic monotonicity** violated (insights for theory)

But assump. does little to improve upon rationality alone
Stochastic Monotonicity Bounds on $F_B$, Cell Phone

Number of sequences in data for this product: 2,501

- Assuming **stochastic monotonicity** does little to improve upon assuming rationality
- Doesn’t mean stoch. mon. violated
- Bounds pointwise sharp $\Rightarrow$ need even stronger assumptions if want tighter bounds
Monotonicity Bounds on $F_B$, Cell Phone

Number of sequences in data for this product: 2,501

- Assuming **monotonicity** tightens lower bound a lot
- Recall monotonicity for buyer means $P^B_2$ weakly increasing in $B$ **conditional** on $P^S_1$
  (this is a weaker condition than seller monotonicity)
Independence Bounds for Buyer CDF Cross Slightly

Bounds assuming $B$ independent of $P_1^S$ (seller’s first offer) are violated

- Assumption of buyer independence is not satisfied for this product
  (unobserved heterogeneity could be one potential culprit)
Independence Bounds for Seller Don’t Cross

Bounds assuming $S$ independent of $P_2^B$ conditional on $P_1^S$ don’t cross

- Independence assumption for seller is weaker than for buyer
- Only assumes $S$ independent of $P_2^B$ conditional on $P_1^S$
All Buyer CDF Bounds, Cell Phone

Note: Representative of other products: independence crosses 48%; monotonicity 0%
All Seller CDF Bounds, Cell Phone

Note: Representative of other products: independence crosses 2%; monotonicity 96%
Verifying Seller CDF Bounds

Special feature in the data: auto accept/decline prices

- Seller can report a hidden “auto-accept” and “auto-decline” price to eBay

- Any buyer offers below auto-decline price are automatically declined by platform on behalf of seller

- Any buyer offers above auto-accept price are automatically accepted

- Empirical CDFs of auto-accept/decline prices provide bounds on true seller CDF
Seller CDF Bounds + Auto Accept/Decline

Bounds/assumptions are consistent with auto accept/decline prices

<table>
<thead>
<tr>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Uncond. upper bound</td>
<td>Uncond. lower bound</td>
<td>Stoch mono upper bound</td>
<td>Stoch mono lower bound</td>
<td>Auto accept/decline</td>
</tr>
</tbody>
</table>
Counterfactual: Efficient Bargaining
How well does bargaining work in practice in this online market?

• In first-best world, agents trade whenever $S \leq B$

• Myerson-Satterthwaite Theorem says no mechanism will reach first-best surplus level if supports of $B$ and $S$ overlap

• Open question: How close does bargaining get in practice?

• $P(S \leq B)$ lower bound can be compared to $P(sale)$ in data
  • Informative about degree of inefficient impasse in data
    (Cases where $B \geq S$ yet no trade occurs)

• $P(S \leq B)$ upper bound can be compared to 1
  • If upper bound < 1, agents must have uncertainty about whether gains from trade exist
Assumptions to Obtain Bounds on $P(S \leq B)$

1. Rationality alone is uninformative
   - Yields $P(S \leq B) \in [P(sale), 1]$
     (because $P(X^S_{AC} \leq X^B_{AC}) = P(X^S_{AC} = X^B_{AC}) = P(sale)$)

Alternative assumptions:

2. Weak:
   - Assume $B - S$ stoch. increasing in $P^B_2$ conditional on $P^S_1$
   - Yields bounds akin to buyer-offer stochastic monotonicity bounds but on difference $B - S$

3. Moderate:
   - Assume $B - S$ weakly increasing in $P^B_2$ conditional on $P^S_1$
   - Yields bounds akin to buyer-offer monotonicity bounds but on difference $B - S$

4. Strong (violated in data):
   - Assume buyer-offer and seller-offer monotonicity
   - Yields $P(S \leq B) \in [P(X^S_{AC} \leq X^B_{AC}), P(X^S_{Q} \leq X^B_{Q})]$
How Assumptions For $P(S \leq B)$ Work

- Assumptions from previous slide help us say when disagreement is inefficient

- Consider two sequences, both with $P^S_1 = 300$
  - In first seq., $P^B_2 = 200$, and bargaining ends in trade
  - In second seq., $P^B_2 = 290$, and bargaining ends in no trade

- Assumptions imply:
  - Assump 2 $\Rightarrow$ it is likely true $S \leq B$ in second seq.
  - Assump 3 $\Rightarrow$ it is definitely true $S \leq B$ in second seq.
Bounds on $P(S \leq B)$ For All 363 Products Separately

Red line = 45 degree; yellow $\times$ = upper; blue $\circ$ = lower; purple = violations

(a) Weaker Assumptions (2)
- Plots rank products on x-axis by $P(sale)$
- y-axis shows lower and upper bounds on $P(S \leq B)$
- Weakest assump. have no power for any product
(b) Strongest Assumptions (4)
- Strongest assumptions rejected by data (42/363 products)
Bounds on $P(S \leq B)$ For All 363 Products Separately

Red line = 45 degree; yellow $\times$ = upper; blue $\circ$ = lower; purple = violations

- Moderate assumptions not rejected for any product
- Bounds width differs across products (unlike bounds based on weakest assumption) $\Rightarrow$ some power to detect different inefficiency across products
Bounds on $P(S \leq B)$ For All 363 Products Separately

Red line = 45 degree; yellow $\times$ = upper; blue $\circ$ = lower; purple = violations

(d) Moderate Assumptions (3)

- Lower bounds suggest inefficient impasse
- For median product, trade fails in $\geq 43\%$ of cases even when trade gains exist (i.e. $1 - P(sale) / P(S \leq B)$)
- Percentage ranges from 8.3–69% across products
Bounds on $P(S \leq B)$ For All 363 Products Separately

Red line = 45 degree; yellow $\times$ = upper; blue $\circ$ = lower; purple = violations

(e) Moderate Assumptions (3)

- Upper bounds suggest agents indeed negotiate under uncertainty about gains from trade
- Ranges from 0.673 to 1.00 (median product: 0.94) ⇒ even in first-best world, only trade 94% of time
Studies Quantifying Degree of Inefficient Impasse

Experiments with incomplete-info bargaining:
- Valley et al. 2002
  - Inefficient impasse 46% of time
  - Reduced to 15% if agents allowed to communicate
- Bochet et al. 2020
  - Inefficient impasse 30% of time
- Huang et al. 2021
  - Inefficient impasse 17% of time

Field data:
- Larsen (2021): Dealer-to-dealer used-car negotiations
  - Inefficient impasse 21% of time
  - Skilled mediators reduce this (Larsen et al. 2021)
- This paper: 8–69%. Median = 43%
Conclusion

• Study real-world alternating-offer bargaining (eBay)

• Derive bounds on value dist. robust to incomplete info
  • Bounds don’t rely on equil. concept or solving specific model (as theory provides little guidance)
  • Only rely on assumptions partially characterizing relationships between primitives and actions

• Bound counterfactual first-best trade prob.

• Finding:
  • Some properties of game theory models (seller monotonicity, buyer indep.) too strong; rejected by data
  • Under preferred assumptions, inefficient impasse/breakdown clearly present (8–69% of the time, depending on product)
Thank you
How to Think about Dynamics Across Negotiations

- We could instead write buyer gets $V - P$ if agree and $\mu$ if disagree (we’d then have $B = V - \mu$ as buyer’s WTP)

- Recall seller gets $P$ if agree and $S$ if disagree (so $S$ is WTS)

- We allow for fully flexible dynamics/continuation values within a sequence; don’t consider across sequences

- $\mu$ and $S$ are continuation values across sequences
  - could involve re-entering market, or going elsewhere, or giving up

- For our focus, not important to fully specify dynamic disagreement payoff
  - Our focus = bounds on WTP, WTS, and $P(S \leq B)$ for those negotiations that occur in our data
What if $B$ or $S$ Changes *Within* a Given Negotiation?

Suppose buyer or seller learns not only about opponent value but *own* value during the negotiation

- If $B$, $S$ can change during negotiation
  - can think of our unconditional bounds (for example) as tightest bounds implied by rationality that are not inconsistent with any *observed* bargaining actions
  - Can’t say anything about unobserved actions
  - Easier to think about assumptions (e.g. monotonicity) if $B$, $S$ constant within negotiation

Separate point: Could get tighter bounds by assuming $B$ stays constant for given buyer *across* sellers she engages with

- We aren’t currently using that info
- Recall: bounds pointwise sharp, so this would only tighten bounds because it would be a stronger assumption
Some Notation at *Sequence Level*

i.e. at level of a given bargaining sequence observed in the data

In a given bargaining sequence,

1. Smallest offer seller accepts or makes as a counteroffer:

\[ X_{AC}^S = \min \{ \{ P_t^B : D_{t+1}^S = A \} , \min_t \{ P_t^S : D_t^S = C \} \} \]

2. Offer seller quits at:

\[ X_Q^S = \max \{ 0 , P_t^B : D_{t+1}^S = Q \} \]

3. Largest offer buyer accepts or makes as a counteroffer:

\[ X_{AC}^B = \max \{ 0 , \{ P_t^S : D_{t+1}^B = A \} , \max_t \{ P_t^B : D_t^B = C \} \} \]

4. Offer buyer quits at:

\[ X_Q^B = \min \{ \infty , P_t^S : D_{t+1}^B = Q \} \]
Example: Me Vacation Rental Haggling in 2011

Seller: $1,275
Brad: $800
Seller: $1,125
Brad: $800
Seller: $1,025
Brad: Accept
Example: Me Vacation Rental Haggling in 2011

Seller: $1,275 — $P_S^1$

Brad: $800 — $P_B^2$

Seller: $1,125

Brad: $800

Seller: $1,025 — $X_S^{AC}$ (seller’s min accept/counter price)

Brad: Accept — $X_B^{AC} = $1,025 (buyer’s max A/C price)

$X_B^Q = \infty$ (buyer never quit) and $X_S^Q = 0$ (seller never quit)
Table: Bound Crossings Under Diff. Assumptions, All 363 Products

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Seller Bounds</th>
<th>Buyer Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction</td>
<td>Integrated</td>
</tr>
<tr>
<td></td>
<td>Bounds</td>
<td>Violation</td>
</tr>
<tr>
<td></td>
<td>Crossing</td>
<td>Error</td>
</tr>
<tr>
<td>Unconditional</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Monotonicity</td>
<td>0.9642</td>
<td>0.1683</td>
</tr>
<tr>
<td>Stochastic Monotonicity</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Independence</td>
<td>0.0248</td>
<td>0.0001</td>
</tr>
<tr>
<td>Positive Correlation</td>
<td>0.0055</td>
<td>0</td>
</tr>
<tr>
<td>Pos. Corr. &amp; Stoch. Mon.</td>
<td>0.0055</td>
<td>0</td>
</tr>
<tr>
<td>Indep. &amp; Stoch. Mon.</td>
<td>0.0689</td>
<td>0.0004</td>
</tr>
<tr>
<td>Pos. Corr. &amp; Mon.</td>
<td>0.9642</td>
<td>0.1683</td>
</tr>
<tr>
<td>Indep. &amp; Mon.</td>
<td>0.9642</td>
<td>0.1683</td>
</tr>
</tbody>
</table>

- Table shows fraction of products with lower bound crossing upper
- Integrated violation error = *by how much it crosses, on average* (ranges between 0 and 1)
Bound Crossings Across All Products

**Table: Bound Crossings Under Diff. Assumptions, All 363 Products**

<table>
<thead>
<tr>
<th></th>
<th>Seller Bounds</th>
<th>Buyer Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction Bounds Crossing</td>
<td>Integrated Violation Error</td>
</tr>
<tr>
<td></td>
<td>Unconditional (A1)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Monotonicity</td>
<td>0.9642</td>
</tr>
<tr>
<td></td>
<td>Stochastic Monotonicity</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Independence</td>
<td>0.0248</td>
</tr>
<tr>
<td></td>
<td>Positive Correlation</td>
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<td>0.0055</td>
</tr>
<tr>
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<td>0.0689</td>
</tr>
<tr>
<td></td>
<td>Pos. Corr. &amp; Mon.</td>
<td>0.9642</td>
</tr>
<tr>
<td></td>
<td>Indep. &amp; Mon.</td>
<td>0.9642</td>
</tr>
</tbody>
</table>

- Monotonicity $\approx$ always violated for $S$, never for $B$
- Independence frequently violated for $B$, rarely for $S$
- Stochastic monotonicity always satisfied, pos. corr. almost always
Table: Statistics Across All 363 Products on Width of Bounds

<table>
<thead>
<tr>
<th>Seller Bounds</th>
<th>Buyer Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Min</strong></td>
<td><strong>Mean</strong></td>
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<tr>
<td>Unconditional</td>
<td>0.328</td>
</tr>
<tr>
<td>Monotonicity</td>
<td>0.000</td>
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<tr>
<td>Stochastic Monotonicity</td>
<td>0.320</td>
</tr>
<tr>
<td>Independence</td>
<td>0.137</td>
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<tr>
<td>Positive Correlation</td>
<td>0.144</td>
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<tr>
<td>Pos. Corr. &amp; Stoch. Mon.</td>
<td><strong>0.146</strong></td>
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<tr>
<td>Indep. &amp; Stoch. Mon.</td>
<td>0.121</td>
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<tr>
<td>Pos. Corr. &amp; Mon.</td>
<td>0.000</td>
</tr>
<tr>
<td>Indep. &amp; Mon.</td>
<td>0.000</td>
</tr>
</tbody>
</table>

- Table: Takes average width of bounds for a product, then reports min, mean, max across products
- Some assumptions help little beyond unconditional
- Some assumptions help more (stoch. mon. + pos. corr. for $S$, or monotonicity for $B$)
### Table: Fraction of cases (among 16 products with \( \geq 100 \) seq. with auto accept/decline) where auto CDFs fail to intersect estimated bounds

<table>
<thead>
<tr>
<th>Condition</th>
<th>Lower Bound v. Auto-Decline</th>
<th>Upper Bound v. Auto-Accept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction Bounds Crossing</td>
<td>Integrated Violation Error</td>
</tr>
<tr>
<td>Unconditional (A1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Monotonicity (A2)</td>
<td>0.9375</td>
<td>0.0493</td>
</tr>
<tr>
<td>Stochastic Monotonicity (A3)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Independence (A4)</td>
<td>0.4375</td>
<td>0.0047</td>
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<tr>
<td>Positive Correlation (A5)</td>
<td>0.125</td>
<td>0.0001</td>
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<tr>
<td>Pos. Corr. &amp; Stoch. Mon. (A5 + A3)</td>
<td>0.125</td>
<td>0.0002</td>
</tr>
<tr>
<td>Indep. &amp; Stoch. Mon. (A4 + A3)</td>
<td>0.5</td>
<td>0.0059</td>
</tr>
<tr>
<td>Pos. Corr. &amp; Mon. (A5 + A2)</td>
<td>0.9375</td>
<td>0.0493</td>
</tr>
<tr>
<td>Indep. &amp; Mon. (A4 + A2)</td>
<td>0.9375</td>
<td>0.0493</td>
</tr>
</tbody>
</table>

- Seller monotonicity violated (not surprising)
- When violations occur, not by much (IVE is low)
Estimated Lower Bounds on $P(S \leq B)$

Results under weakest assumptions (1) to strongest (4)

<table>
<thead>
<tr>
<th>A. Lower Bounds</th>
<th>$n$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronics</td>
<td>577</td>
<td>0.319</td>
<td>0.348</td>
<td>0.622</td>
<td>0.842</td>
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<tr>
<td></td>
<td></td>
<td>[0.281,0.357]</td>
<td>[0.317,0.400]</td>
<td>[0.583,0.662]</td>
<td>[0.813,0.872]</td>
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<tr>
<td>Cameras</td>
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<td>0.384</td>
<td>0.509</td>
<td>0.679</td>
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<tr>
<td></td>
<td></td>
<td>[0.302,0.453]</td>
<td>[0.296,0.446]</td>
<td>[0.431,0.587]</td>
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<td>Sports</td>
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<td>0.311</td>
<td>0.321</td>
<td>0.563</td>
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<td></td>
<td></td>
<td>[0.245,0.376]</td>
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<td>[0.492,0.634]</td>
<td>[0.708,0.829]</td>
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<td>Video Games</td>
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<td>0.29</td>
<td>0.328</td>
<td>0.472</td>
<td>0.739</td>
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<tr>
<td></td>
<td></td>
<td>[0.249,0.330]</td>
<td>[0.274,0.355]</td>
<td>[0.428,0.517]</td>
<td>[0.700,0.778]</td>
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<tr>
<td>Musical</td>
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<td>0.724</td>
<td>0.794</td>
<td>0.902</td>
<td>0.894</td>
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<tr>
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</table>
### Estimated Lower Bounds on $P(S \leq B)$

**Results under weakest assumptions (1) to strongest (4)**

<table>
<thead>
<tr>
<th>A. Lower Bounds</th>
<th>$n$</th>
<th>(1)</th>
<th>(2)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Electronics</td>
<td>577</td>
<td>0.319</td>
<td>0.348</td>
<td>0.622</td>
<td>0.842</td>
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<td></td>
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- **Computers** $P(sale)$ is 0.22. Under
  - strongest assumptions (4), $P(S \leq B) \geq 0.76$
  - moderate assum. (3), $P(S \leq B) \geq 0.37$

$\Rightarrow 40\%$ of time $1-0.22/0.37$ trade fails even when $S \leq B$
Estimated Lower Bounds on $P(S \leq B)$

Results under weakest assumptions (1) to strongest (4)

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</table>

- **Cell Phone** $P(\text{sale})$ is 0.13
  - Under strongest assumptions (4), $P(S \leq B) \geq 0.93$
  - Turns out to violate upper bound (0.49)

Upper and lower bounds under (3) not violated
## Estimated Upper Bounds on $P(S \leq B)$

### Results under weakest assumptions (1) to strongest (4)

<table>
<thead>
<tr>
<th>B. Upper Bounds</th>
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<td>[0.731,0.806]</td>
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- Upper bounds largely uninformative
- Under strongest assumptions, upper < lower (cell phone)
- Under moderate case (3), cell phone negotiations would only result in trade 68% of time in first-best world
Simple Mechanical Setup For Simulating Bargaining

Draw many realizations $B, S$ from known $F_B, F_S$

1. $t = 1$: Seller offers some $P^S_1 > S$

2. $t = 2$: Buyer counters at some $P^B_2 < \min\{B, P^S_1\}$

3. $t = 3$
   - If $P^B_2 < S$, seller quits (with prob $p_{SQ}$) or counts $P^S_3 < P^S_1$
   - If $P^B_2 \geq S$, seller accepts (with prob $p_{SA}$) or counts $P^S_3 < P^S_1$

4. $t = 4$ (if reached)
   - If $P^S_3 > B$, buyer quits (with prob $p_{BQ}$) or counts $P^B_4 < B$
   - If $P^S_3 \leq B$, buyer accepts (with prob $p_{BA}$) or counts $P^B_4 < B$

... (etc.)

We vary probabilities, degree of shading when a party counters
Monte Carlo Results: Unconditional

(f) Unconditional Bounds, Seller (Wide)  
(g) Unconditional Bounds, Seller (Tight)

- **Wide:** Seller makes offers high above $S$ and rarely quits
- **Tight:**
  - Seller makes offers a little above $S$
  - Buyer counters at price little below $S$ (and below $B$)
  - Seller then quits
Monte Carlo Results: Monotonicity

(h) Monotonicity Bounds, Seller (Wide)
(i) Monotonicity Bounds, Seller (Tight)

- **Wide**: Final accept/counter prices \( (X_{AC}^S) \) deterministically monotonic in \( S \)
- **Tight**: Sellers who start with relatively low \( P_1^S \) end game at relatively high \( X_{AC}^S \)

(Can occur due to randomness in buyer to whom seller is matched or due to features later in bargaining game)
Monte Carlo Results: Independence

(j) Independence Bounds, Buyer (Wide)

• Wide: $X^B_{AC}$ and $X^B_Q$ indep of $P^S_1$ (just like $B$ is)

(k) Independence Bounds, Buyer (Tight)

• Tight: $X^B_{AC}$ and $X^B_Q$ depend directly on $P^S_1$