The Efficiency of Real-World Bargaining: Evidence from Wholesale Used-Auto Auctions

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January 21, 2020

Abstract

This study empirically quantifies the efficiency of a real-world bargaining game with two-sided incomplete information. Myerson and Satterthwaite (1983) and Williams (1987) derived the theoretical ex-ante efficient frontier for bilateral trade under two-sided uncertainty and demonstrated that it falls short of ex-post efficiency, but little is known about how well bargaining performs in practice. Using about 265,000 sequences of a game of alternating-offer bargaining following an ascending auction in the wholesale used-car industry, this study estimates (or bounds) distributions of buyer and seller values and evaluates where realized bargaining outcomes lie relative to efficient outcomes. Results demonstrate that the ex-ante and ex-post efficient outcomes are close to one another, but that the real bargaining falls short of both, suggesting that the bargaining is indeed inefficient but that this inefficiency is not solely due to the information constraints highlighted in Myerson and Satterthwaite (1983). Quantitatively, findings indicate that over one half of failed negotiations are cases where gains from trade exist, leading an efficiency loss of 12–23% of the available gains from trade.

Keywords: Bargaining, incomplete information, bounds identification, Myerson-Satterthwaite Theorem, efficiency, empirical market design, alternating-offers

JEL Classification: C57, C78, D44, D47, D82

*I thank Panle Jia Barwick, Glenn Ellison, and Stephen Ryan for invaluable help and advice. I would also like to thank Isaiah Andrews, Kyle Bagwell, Gabe Carroll, Mingli Chen, Victor Chernozhukov, Denis Chetverikov, Dominic Coey, Joachim Freyberger, Ken Hendricks, Kyoo-il Kim, Steven Lang, Ariel Pakes, Christopher Palmer, Parag Pathak, Brennan Platt, Zhaonan Qu, Dan Quint, Mark Satterthwaite, Paulo Somaini, Alan Sorensen, Evan Storms, Jean Tirole, Juuso Toikka, and Anthony Zhang for helpful suggestions. This paper also benefited from conversations with many other faculty and students at various institutions. I thank several anonymous auction houses and their employess for providing data and institutional details. I acknowledge support from the National Science Foundation under grants GRFP-0645960 and SES-1530632.

Whether haggling in an open-street market, deciding upon prices between an upstream supplier and downstream producer, or negotiating a corporate takeover deal, bargaining between a buyer and seller is one of the oldest and most common ways of transacting. When both parties have incomplete information, it is known that equilibrium outcomes are difficult to characterize.[1] Myerson and Satterthwaite (1983) demonstrated that *ex-post* efficiency—trading whenever the buyer values the good more than the seller—is not possible in negotiations with two-sided incomplete information. Myerson and Satterthwaite (1983) and Williams (1987) derived the theoretical *ex-ante* efficient frontier, but, as Williams emphasized, "little is known about whether or not these limits can be achieved with 'realistic' bargaining procedures."[2] This paper is the first attempt to bring data to this question. I develop a framework to estimate distributions of private values of both buyers and sellers who participate in bargaining following wholesale used-auto auctions. I then map these primitives into results from the theoretical mechanism design literature to compare real-world outcomes to efficient outcomes.

The question of whether real-world bargaining is efficient is one that cannot be addressed in a standard non-strategic framework (e.g. some form of Nash bargaining) or even a strategic alternating-offer game (e.g. Rubinstein 1982). These frameworks entail *complete information* and thus presume *a priori* that bargaining is perfectly efficient: in such a world, bargaining is never even attempted unless agreement is the efficient outcome. Treating bargaining as efficient, if it is in fact not, can result in incorrect market design recommendations or misleading calculations for welfare or pricing, or an incorrect understanding of bargaining power. The data and methodology I use in this paper allow me to study whether or not bargaining is actually efficient, rather than assuming it to be so.

Moreover, this question is indeed an empirical question—one that theory alone cannot address. Theoretical work by Chatterjee and Samuelson (1983), Satterthwaite and Williams (1989), Ausubel and Deneckere (1993), and Ausubel, Cramton, and Deneckere (2002) demonstrated that certain knife-edge or limiting cases of bargaining games may reach the theoretical ex-ante efficient frontier, but the limits of practical bargaining are unknown. Also, while the large theoretical literature on strategic bargaining with incomplete information has yielded valuable insights, it has done so

[1] Fudenberg and Tirole (1991) stated, "The theory of bargaining under incomplete information is currently more a series of examples than a coherent set of results. This is unfortunate because bargaining derives much of its interest from incomplete information." Fudenberg, Levine, and Tirole (1985) similarly commented, "We fear that in this case [of two-sided incomplete information], few generalizations will be possible, and that even for convenient specifications of the functional form of the distribution of valuations, the problem of characterizing the equilibria will be quite difficult." Very little work—theoretical or empirical—on bargaining with two-sided uncertainty and continuous values has been published before or after this time.

[2] In the language of Holmström and Myerson (1983), the term *ex-ante* refers to before the players learn their values and before the outcome of the bargaining is realized, and the term *ex-post* refers to after the values and bargaining outcomes are realized. As explained below, the ex-ante efficient frontier describes the limits on possible combinations of buyer and seller surplus that can be achieved under *any* bilateral bargaining mechanism in the presence of incomplete information.

primarily through a focus on special cases.[3]

The general case, with alternating offers, two-sided incomplete information, and continuous values has received little attention because it involves complex signaling and updating by both parties. It is known to have multiple equilibria, some of which are very inefficient, but no canonical model or equilibrium characterization exists for the general setting I study.

To overcome these challenges, I take advantage of a unique, new dataset and novel empirical approach. The data consists of several hundred thousand sequences of bargaining offers between buyers and sellers at wholesale used-car auctions. It is the first bargaining dataset of this volume and detail to be analyzed in the literature, containing not only final negotiated prices on consummated deals, as most empirical bargaining datasets likely would, but also all of the back-and-forth offers between negotiating parties, including cases where bargaining failed to yield an agreement. The data also contains detailed information on cars and sale characteristics.

The empirical setting, described in Section 2, is a large market of business-to-business transactions where new- and used-car dealers buy vehicles from other dealers as well as from rental companies, banks, and manufacturers. The industry is the backbone of the supply side of the US used-car market, with 15 million cars annually passing through its lanes, totaling over $80 billion in sales. For each car, the auction house runs a secret-reserve-price ascending auction, followed by bargaining if the auction price falls short of the reserve price (which occurs more than two-thirds of the time). It is in this bargaining stage of the game that trade can fail, and thus understanding the efficiency of the bargaining is key to understanding the efficiency of the overall market. Industry wide, about 40% of sales attempts result in no trade. Why do these trades fail? These could be cases where the seller values the good more than the buyer, and hence no trade should occur even in a fully efficient world; these could be cases where gains from trade do exist, but trade fails due to the information constraints highlighted by Myerson and Satterthwaite (1983); or these may be cases where trade fails because of the particular bargaining protocol employed or the particular equilibrium played. These questions are the focus of this paper.

The bargaining I study takes place after an auction.[4] This is not an unfortunate characteristic of the data, but rather a useful feature in studying what might otherwise be an intractable problem. Indeed, it is quite difficult to make any progress studying incomplete-information bargaining empirically without some kind of special lever. Specifically, given that no canonical model or even characterization of equilibria exists for such games, theory provides no obvious mapping from ob-

[3]See Appendix Table A9 for a summary of this literature. This, and all appendix material, is in the online appendix. The online appendix, along with the data and code for this paper, is available on my website and the journal's website.

[4]Many other settings similarly constitute an auction followed by bargaining, where one party collects initial bids from a number of different bidders in an auction-like stage and then selects a single bidder with whom to negotiate a final deal. Examples include business-to-business settings (e.g. procuring subcontractors), government settings (e.g. procuring services or selling government property), or private settings (e.g. selling a home). See examples in Elyakime, Laffont, Loisel, and Vuong (1997), Wang (2000), Huh and Park (2010), and An and Tang (2018).

servables to primitives. In my setting, however, the distribution of buyer values can be estimated using auction data, and bounds on the distribution of seller values can be obtained using sellers' responses to the first bargaining offer (the auction price). Each of these steps imposes only minimal assumptions on the structure of the bargaining game.

I lay out a model in Section 3 and demonstrate several theoretical properties that aid in estimating model primitives. I show that the precise effect of the auction on the bilateral bargaining game is twofold: first, the auction leads to a truncation of the lower bound of the support of types who bargain. Thus, the game I study is analogous to a setting of bargaining alone where the lower bound of the support of the types in the bargaining game differs across realizations of the game in a tractable manner determined by the realized auction price, and my results average over these realizations. Second, the auction price is the first offer in the bargaining game and provides a lower bound on achievable bargained prices, similar to how a list price provides an upper bound in many other real-world bargaining games, such as haggling over a car. I also demonstrate in the model that game-level heterogeneity affects the game's outcomes tractably.

My approach to bounding the distribution of seller values is similar in spirit to Haile and Tamer (2003), using inequalities implied by very basic assumptions about players' rationality to learn about model primitives without imposing a complete model of the game or solving for an equilibrium. The bargaining setting is more complicated than the auction setting in Haile and Tamer (2003), however, in that it is not necessarily the case that an upper and lower bound on the valuation is observed for each individual realization in the data; instead, I obtain conditional probability statements that bound the whole distribution of values. This methodology is new to the empirical bargaining literature, and can be applied to alternating-offer bargaining settings when the econometrician observes the first offer and first response, regardless of whether the bargaining follows an auction.

In order to compute expected gains from trade to measure efficiency in the real-world mechanism, it is necessary to know not only the distributions of values but also which player types trade and which do not. This is an equilibrium object and, as highlighted above, existing theory provides no guidance on identifying equilibria in games of two-sided incomplete-information bargaining. I demonstrate, however, that even without solving explicitly for equilibrium strategies, the direct-revelation mechanism corresponding to the equilibrium of the real-world game is identified in the data. This argument relies on the Revelation Principle, which has been exploited widely in the theoretical mechanism design literature. Applying this concept to my empirical setting allows me to avoid solving for or characterizing the actual equilibrium of the game and instead work with the direct mechanism corresponding to this game as implied by the data.

Section 4 describes each step of my estimation approach, which exploits the model's properties. After controlling for observable heterogeneity, I use a likelihood approach to deconvolve unobserved game-level heterogeneity and estimate buyer values using an order statistic inversion.

I then estimate bounds on seller values, exploiting revealed preferences inequalities. I estimate the mapping between auction prices and the lower bounds of the support of buyer and seller types in the bargaining game as well as the mapping corresponding to the direct revelation mechanism of the game. These mappings and the seller valuation bounds can each be estimated using flexible spline approximations within a constrained least-squares framework.

After estimating these structural objects, I describe in Section 5 how I compute welfare under counterfactual efficient bargaining mechanisms. These counterfactual mechanisms are related to results derived in Myerson and Satterthwaite (1983) and Williams (1987), but are more complex to compute than the mechanisms they study because the distributions I estimate do not satisfy the regularity assumptions of those studies. I must therefore impose incentive compatibility numerically. Also, having only bounds on seller values, and not point estimates, I must perform a large numerical search to find bounds on efficiency measures. I ease this computational burden by deriving useful monotonicity properties that yield bounds for some welfare measures.

In Section 6 I then measure efficiency by comparing outcomes under efficient bargaining to those under the real bargaining. The first type of efficiency loss I measure is the loss due solely to incomplete information. Ideally, a buyer and seller trade whenever the buyer values the good more than the seller (ex-post efficient trade). However, the celebrated Myerson and Satterthwaite (1983) Theorem demonstrated that, when the supports of buyer and seller types overlap, there does not exist any incentive-compatible, individually rational bargaining mechanism that is ex-post efficient and that also satisfies an ex-ante balanced budget. Williams (1987) derived the entire ex-ante efficient frontier for any range of relative weights placed on the buyer's and seller's expected gains from trade. This frontier describes the limits on buyer and seller surplus that can be achieved by any incentive-compatible, individually rational, budget-balancing mechanism. I study several mechanisms along the ex-ante efficient frontier, primarily the mechanism that places equal welfare weight on the buyer and seller surplus, which I refer to as *the second-best mechanism*. The gap between the ex-ante and ex-post efficient frontiers represents an efficiency loss due to the presence of incomplete information. Using the estimated distributions, I find that incomplete information per se need not be a huge problem in this market: the second-best mechanism achieves about the same range of expected surplus as the infeasible ex-post efficient mechanism. The efficiency loss due solely to incomplete information is about \$9–59. The second-best mechanism falls short of ex-post efficiency in terms of the probability of trade by 3 to 16 percentage points, but these trades that the second-best mechanism fails to capture appear to be low-surplus trades.

The second type of efficiency loss I study compares the real-world bargaining to the ex-ante efficient frontier. The real bargaining may fall short of this frontier for several reasons. First, it is well known that, unlike the mechanisms discussed in Myerson and Satterthwaite (1983) and Williams (1987), real-world bargaining with two-sided uncertainty has no clear equilibrium predictions due to signaling by both parties, and many qualitatively different equilibria exist (see Ausubel and Deneckere 1993). The equilibrium play observed in the data may correspond to a particularly inefficient equilibrium. Second, it may be the case that the alternating-offer protocol used in this market is inefficient regardless of the equilibrium played. Third, it may be that the real bargaining falls short of the theoretically efficient benchmark because that benchmark fails to satisfy other constraints that real-world bargaining is subject to, such as having rules that are simple for players to understand or being implementable without requiring the strong assumption that players and the market designer all have common knowledge of players' value distributions and beliefs (an assumption of traditional mechanism design critiqued in the influential Wilson doctrine, Wilson 1986). Because I place very little structure on the bargaining game, my analysis allows for any of these possibilities, each of which can lead to a gap between actual and efficient outcomes.

My findings indicate that the real bargaining falls short of the second-best by \$377–1,123 for cars sold by dealers and by \$223–834 for cars sold by large fleet or lease institutions. The losses of the real-world mechanism compared to the ex-post efficient frontier are similar in magnitude. These losses represent 17–23% of the ex-post gains from trade for cars sold by dealers and 12–20% for cars sold by large institutions. In terms of the probability of trade, the real-world bargaining falls short of the ex-post efficient outcome by 0.172–0.225 for cars sold by dealers and by 0.199–0.235 for cars sold by fleet and lease sellers. This implies that about 17–24% of negotiations constitute cases where the buyer indeed values the good more than the seller and yet the negotiation fails. Given that the overall rate of trade failure in the bargaining stage is about 35% in each sample, this suggests that over half of *failed* trades are cases where gains from trade exist but the parties do not trade, and the remainder of failed trades are cases where no gains from trade exist. The key takeaway of my analysis is that the real-world bargaining in this market is indeed inefficient and that this inefficiency is not solely due to the information constraints highlighted in Myerson and Satterthwaite (1983).

1 Related Literature

To my knowledge, this paper is the first to bring data to the bargaining efficiency framework of Myerson and Satterthwaite (1983). Unlike the vast structural auction literature—where researchers identify primitives to study various counterfactuals by modeling the game as one of incomplete information and strategic behavior—structural studies analyzing bargaining through a strategic, incomplete-information lens are rare.[5] Several exceptions that estimate models of one-sided in-

[5] In a separate strand of the structural bargaining literature, a number of papers have made valuable contributions by abstracting away from incomplete information and modeling negotiated prices as arising from a Nash-bargaining surplus-splitting rule, such as Crawford and Yurukoglu (2012) and other subsequent studies of bargaining settings with externalities, and other work studying post-auction bargaining settings (Elyakime, Laffont, Loisel, and Vuong 1997; An and Tang 2018). Merlo and Tang (2012) provided identification arguments for stochastic bargaining games of complete information, and Merlo and Tang (2018) and Watanabe (2009) studied complete-information games with asymmetric priors.

complete information include Sieg (2000) and Silveira (2017), who focused on take-it-or-leave-it bargaining in trial settings, and Ambrus et al. (2018), who studied pirate ransom negotiations and modeled bargaining following the theoretical work of Fudenberg, Levine, and Tirole (1985). Structural empirical work that highlights a role for *two-sided* uncertainty in bargaining (i.e., where both parties have private information) includes Genesove (1991), who discussed briefly the bargaining that takes place at wholesale auto auctions. Lacking detailed data on bargaining, he tested several parametric specifications for buyer and seller distributions and found that these assumptions performed poorly in explaining when bargaining occurred or when it was successful. Li and Liu (2015) studied identification of values in a static, two-sided incomplete-information bargaining game (a k double auction).

Another strand of the literature offers reduced-form analysis of implications of incomplete-information bargaining in home sales data (Merlo and Ortalo-Magne 2004), retail car survey data (Scott Morton et al. 2011), international trade negotiations (Bagwell et al. 2017), e-commerce bargaining (Backus et al. 2018, 2019), and hospital-supplier negotiations (Grennan and Swanson 2019).[6] The data I analyze is new to the literature, and is particularly novel in the opportunity it presents for analyzing bargaining in detail, as it contains hundreds of thousands of observations and rich information about the characteristics of the goods sold and the actions players take during each observation of the game.

The only previous structural analysis of actual back-and-forth offers is Keniston (2011), but the setting, methodology, and focus of the two papers are quite distinct. Keniston (2011) collected several thousand observations of back-and-forth bargaining offers between riders and autorickshaw drivers in India, whereas my setting studies professionals engaging in business-to-business negotiations. The model of Keniston (2011) allowed for two-sided incomplete information, like mine, but the author embedded this model in a search-and-matching framework to model agents' outside options, whereas my paper does not explicitly model players' continuation payoffs when bargaining fails. The method of Keniston (2011) requires estimating beliefs in the bargaining subgame, relying on the assumption of a stationary equilibrium, whereas my approach does not require stationarity assumptions or belief estimation. Keniston (2011) does not focus on the efficiency of bargaining or the Myerson-Satterthwaite Theorem, but instead compares welfare under bargaining to welfare under a fixed-price mechanism.

The approach I develop can be applied to other settings with alternating-offer data to identify and estimate bounds on the distribution of values for the player who responds to the first offer. Larsen and Zhang (2018) presented an approach that can be used to instead obtain the distribution of values in bargaining games for the player who *makes* (rather than responds to) the first offer. Larsen and Zhang (2018) applied their approach to a subset of the data used in this paper to

[6] Merlo et al. (2015) provided a structural model of the home-sales data from Merlo and Ortalo-Magne (2004) but abstracted away from bargaining actions in order to focus on the seller's dynamic choice of list price.

analyze the full auction-plus-bargaining mechanism rather than the bilateral bargaining studied in this paper, finding similar qualitative results regarding mechanism efficiency.

2 The Wholesale Used-Car Industry and the Data

The wholesale used-auto auction industry provides liquidity to the supply side of the US used-car market. Each year approximately 15 million used cars pass through wholesale auction houses in the United States. Cars from these auction houses make up the largest source of inventory for used-car dealers. Industry wide, about 60% of these cars sell, with an average price between $8,000 and $9,000, totaling to over $80 billion in revenue.[7] The industry consists of approximately 320 auction houses scattered across the country. Throughout the industry, the majority of auction house revenue comes from fees paid by the buyer and seller when trade occurs. Buyers attending wholesale auto auctions are used-car dealers. Sellers may be car dealers selling off extra inventory, or they may be large fleet/lease institutions, such as banks, manufacturers, or rental companies selling repossessed, off-lease, lease-buy-back, or old fleet vehicles.

Sellers bring their cars to the auction house, usually several days before the sale, and establish a secret reserve price. In the days preceding the sale, potential buyers may view car details and pictures online or may visit the auction house to inspect and test drive cars (although very few visit prior to the day of sale). The auction sale takes place in a large, warehouse-like room with 8–16 lanes running through it. In each lane there is a separate auctioneer, and lanes run simultaneously. A car is driven to the front of the lane and the auctioneer calls out bids, raising the price until only one bidder remains.

If the auction price exceeds the secret reserve price, the car is awarded to the high bidder. If the auction price is below the secret reserve price, the high bidder is given the option to enter into bargaining with the seller. If the high bidder opts to bargain, the auction house will contact the seller by phone (or in person, if the seller is present at the sale), at which point the seller can accept the auction price, end the negotiations, or propose some counteroffer higher than the auction price.[8] If the seller counters, the auction house calls the buyer. Bargaining continues in this fashion until one party accepts or terminates negotiations (with the typical time between calls being 2-3 hours). It is this bilateral bargaining that is the focus on this paper.

The dataset I use is new to the literature. It comes from six auction houses owned by one

[7] http://www.naaa.com/pdfs/2010_NAAAAnnualReview.pdf

[8] If the seller is absent and the auction price and reserve price are far enough apart that phone bargaining is unlikely to succeed, the auctioneer may reject the auction price on the seller's behalf. If both the buyer and seller are present, a quick round of bargaining may sometimes take place in person immediately following the auction, but such behavior is discouraged as it delays the next sale; each auction typically takes 30–90 seconds, and inserting in-person negotiations into that procedure could drastically increase that time. Furthermore, the auction house discourages in-person interactions lest they lead parties to transact off site in attempts to avoid auction house fees. Off-site transacting is generally prevented by social norms, but in extreme cases violators could be punished through revoked access to future sales.

Table 1: Descriptive Statistics and Game Outcome by Ending Period

A. Descriptive Statistics	Mean	S.D.		Mean	S.D.
Trade	0.705	0.456	Number of periods	2.096	0.681
Reserve price	\$7,405	\$5,196	Blue book	\$6,820	\$4,828
Auction price	\$6,253	\$4,881	Age (years)	6.769	3.369
Auction price if $\geq$ reserve	\$6,197	\$4,700	Odometer (miles)	97,938	46,445
Auction price if $<$ reserve	\$6,258	\$4,899	Number of bidders	2.924	0.34
Sample size	133,523				

B. Game Outcome by Ending Period

					Full Sample		Conditional on Sale		
Ending period	Player's turn	# Obs	% of Sample	% Trade	Reserve price	Auction price	Reserve price	Auction price	Final price
1	(Auction)	14,232	10.659%	88.58%	\$5,818	\$6,050	\$5,704	\$6,197	\$6,197
					(\$4,688)	(\$4,696)	(\$4,626)	(\$4,700)	(\$4,700)
2	S	99,708	74.675%	77.28%	\$7,632	\$6,379	\$7,132	\$6,003	\$6,003
					(\$5,245)	(\$4,936)	(\$4,922)	(\$4,651)	(\$4,651)
3	B	14,644	10.967%	11.81%	\$7,192	\$5,502	\$7,725	\$6,508	\$6,912
					(\$5,041)	(\$4,598)	(\$5,222)	(\$4,968)	(\$5,054)
4	S	2,916	2.184%	65.47%	\$7,828	\$6,369	\$7,678	\$6,359	\$6,608
					(\$5,168)	(\$4,833)	(\$5,022)	(\$4,733)	(\$4,798)
5	B	1,667	1.248%	38.15%	\$8,267	\$6,680	\$8,640	\$7,174	\$7,792
					(\$5,283)	(\$4,933)	(\$5,469)	(\$5,173)	(\$5,317)
6	S	190	0.142%	75.79%	\$8,655	\$7,068	\$8,995	\$7,456	\$7,965
					(\$5,286)	(\$4,988)	(\$5,460)	(\$5,171)	(\$5,298)
7	B	139	0.104%	54.68%	\$8,459	\$6,829	\$8,865	\$7,339	\$8,029
					(\$5,260)	(\$4,963)	(\$5,394)	(\$5,153)	(\$5,315)
8	S	20	0.015%	70.00%	\$8,935	\$7,575	\$9,446	\$8,157	\$8,725
					(\$4,933)	(\$4,888)	(\$5,669)	(\$5,614)	(\$5,692)
9	B	6	0.004%	66.67%	\$7,583	\$6,225	\$6,675	\$5,188	\$5,838
					(\$4,362)	(\$4,517)	(\$2,871)	(\$2,964)	(\$2,800)
10	S	1	0.001%	100.00%	\$14,500	\$11,000	\$14,500	\$11,000	\$13,600
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Notes: Panel A: Trade is an indicator for whether trade occurred between the buyer and seller. Number of periods is 1 if game ends through auction price exceeding reserve price or through buyer opting out of bargaining, 2 if seller accepts at her first bargaining turn, etc. Blue book is an estimate of the market value of the car, provided by the auction house. Number of bidders is the lower bound on the number of bidders, only observable in the bid log subsample (13,150 observations). Panel B: For each period, panel B reports the number of observations ending in that period, percent of total sample ending in that period, and percent of cases in which trade occurred. Table also reports reserve price and auction price for observations ending in a given period and, for those observations ending in trade, the reserve price, auction price, and final price conditional on trade.

company, each maintaining a large local market share. The sample period is from January 2007 to March 2010. An observation in the dataset represents a *run* of the vehicle, that is, a distinct attempt to sell the vehicle through the mechanism. For a given run, the data records the date, time, auction house location, and auction lane, as well as the seller's secret reserve price, the auction price, and, when bargaining occurs over the phone, the full sequence of buyer and seller actions (accept, quit, or counter) in each period of the game, and the amounts of any offers/counteroffers. The data also records detailed characteristics of each car and sale. I impose a number of sample restrictions, such as dropping observations with missing variables or extreme price realizations (lying outside the lowest or highest 0.01 percentiles). I also drop car types (make-model-year-trim-age combinations) that are not offered for sale at least ten times in my sample. Appendix D.1 lists all of my sample restrictions and the observable characteristics I use in estimation. In the end, I am left with 133,523 runs of cars sold by used-car dealers and another 131,443 sold by fleet/lease sellers. I focus on the dealers sample throughout the body of the paper.

Descriptive statistics for this sample are displayed in panel A of Table 1.[9] The probability of trade is 0.705; this trade probability is higher than the industry-wide average highlighted above (due primarily to my sample restrictions, such as focusing on certain make-model-year-trim-age combinations). The average auction price is over \$1,000 below the average reserve price and about \$600 below the average blue book price. Cars are on average seven years old and have nearly 100,000 miles on the odometer.

Panel A also shows information on the number of bidders participating in the auction. A precise measure of the number of bidders is difficult to obtain at these auctions, as many sales take place simultaneously in different auction lanes and bidders are not required to register for the sale of a specific car. However, for some auction sales, the company offers live video streaming and a web-based portal for remote bidding, and for these sales I can obtain an auction-by-auction lower bound on the number of bidders from bid logs.[10] This lower bound rarely falls below 2 (only in 0.37% of observations). The mean of this lower bound conditional on it being at least 2 is 2.924. I use the distribution of this lower bound in estimation (Section 4).

As data on actual back-and-forth offers is rare in the literature, I provide a period-by-period summary of this data in Table 1 panel B. Outcomes in this panel are separated by the period of the game in which the observed sequence ends. Period 1 is the auction. Observations ending in period 1 represent cases that end with auction price exceeding the reserve price or with the auction price falling short of the reserve price and the buyer opting out of bargaining. The remaining periods are

[9] Additional descriptive statistics (including for the fleet/lease sample) are in Appendix Tables A2 and A3.

[10] Bid logs record each bid and the identity of the bidder if the bidder participated online. If the bidder was instead physically present on the auction house floor, the bid log only records the amount of the bid and an indicator, "floor", rather than an identity. A lower bound on the number of distinct bidders is given by the number of distinct online identities who placed bids plus 1 if the log records any floor bids or plus 2 if the log records two consecutive floor bids (assuming no bidder bids against himself). I observe bid logs for 13,150 sales in the dealers sample.

labeled with even numbers for seller turns and odd numbers for buyer turns. Table 1 demonstrates that in 10.66% of the sample the game ends at the auction (thus, what happens *after* the auction plays a major role in the market), and in these cases the final price when parties agree (which occurs 88.58% of the time) is naturally the auction price. The remainder of the time, the buyer opts out of bargaining. Observations ending in trade in the second period also have the final price equal to the auction price (as the auction price is the first bargaining offer). Consider now the fifth period of the game. Only 1.25% of the full sample reaches this period, but this still consists of nearly 1,700 observations. In the fifth period, when trade does occur, it occurs at an average final price of $7,792, which is over $600 above the average auction price ($7,174), but still does not reach as high as the average reserve price ($8,640). Overall, Table 1 suggests that observations ending in later periods have somewhat higher reserve prices than those ending in earlier periods. Only one buyer-seller pair in the data endures ten periods of the game, coming to agreement in the end, at a price $2,600 above the auction price.

3 Model

This section presents a model of the game played in wholesale used-car markets. Prior to stating the assumptions of the model, I first restate the timing of the game, which is as follows:

1. Seller sets a secret reserve price, R.
2. N bidders bid in an ascending auction.
3. If the auction price, P^A, exceeds the secret reserve price, the high bidder wins the item.
4. If the auction price does not exceed the secret reserve price, the high bidder is given the opportunity to walk away, or to enter into bargaining with the seller.
5. If the high bidder chooses to enter bargaining, the auction price becomes the first bargaining offer, and the high bidder and seller enter an alternating-offer bargaining game.

Throughout I maintain the following assumptions:

Assumptions.

(A1) $N \geq 2$ *risk-neutral bidders participate in an ascending button auction with zero participation costs. For* $i = 1, ..., N$, *each buyer* i *has a private value* $\tilde{B}_i = W + B_i$, *with* $B_i \sim F_B$ *and* $W \sim F_W$, *and with* $(W, N, \{B_i\}_{i=1}^N)$ *mutually independent..*

(A2) A risk-neutral seller has a private value $\tilde{S} = W + S$, *with* $S \sim F_S$ *and with* S *independent of* $(W, N, \{B_i\}_{i=1}^N)$.

(A3) The bargaining lasts for up to $T < \infty$ periods; buyers incur a common bargaining cost, $c_B > 0$, for each offer made; and sellers incur a common bargaining cost, $c_S > 0$, for each offer made.

(A4) The seller's expected payoff in the bargaining subgame is continuous in the auction price.

(A5) S has density f_S and B_i has density f_B, where f_B is positive on $[\underline{b}, \overline{b}]$.

The motivation for the independent private values framework is that, according to market participants, buyers—as well as dealer-type sellers—have values arising primarily from their local demand and inventory needs.[11] Also, seller values can depend on the value at which the car was assessed as a trade-in; for a bank or leasing company, values can arise from the size of the defaulted loan.[12] The button auction assumption simplifies the analysis of the auction, but is also not an unreasonable approximation, as it is the auctioneer in this market who raises the price and not the bidders (unlike in oral English auction) and bid increments are small. The assumption of symmetric buyers is not restrictive in this setting given that the high bidder's identity is generally not known to the seller during bargaining and given that, in a private values ascending auction, bidders' auction strategies will not depend on the identities of other participants. The assumption that N is independent of buyer values rules out endogenous entry. In Appendix D.3.2, I document some evidence supporting this assumption, following the intuition derived in Aradillas-Lopez, Gandhi, and Quint (2016).

The form of bargaining costs in Assumption $A3$ is found elsewhere in the theoretical bargaining literature (e.g. Perry 1986 and Cramton 1991), and prevents players from continuing to bargain when no surplus exists. The cap on the number of periods T simplifies the proofs of many of the model properties. T is assumed to be known to the players but not necessarily to the econometrician (and similarly for c_B and c_S). Assumption $A4$ is a technical condition required for the differentiability of the seller's payoff, exploited in the proof of Proposition 3 to prove strict monotonicity of the seller's secret reserve price strategy.

The assumption of positive density for B_i in Assumption $A5$ only plays a role in preventing division by zero when I prove strict monotonicity of the seller's secret reserve price (Proposition 3) and when I prove identification arguments in Appendix D.4–D.6. For the support of the seller

[11] These buyers come from a wide geographic area, with some participants driving long distances or even flying to attend the auction sale, and thus strong correlations between local demands and inventory needs among these buyers are not likely a major concern. While there is likely some common values component to wholesale auto auctions, accounting for this in estimation would be beyond the state of the methodological literature (positive identification results do not exist for interdependent-values ascending auctions; see Athey and Haile 2007). In conversations with market participants, buyers often claim to decide upon their willingness to pay before bidding begins, sometimes having a specific retail customer lined up for a particular car, also suggesting a strong private component to valuations (see also popular industry blogs: http://www.thetruthaboutcars.com/2011/09/hammer-time-the-chosen). Studying similar auto auctions in Korea, Roberts (2013) and Kim and Lee (2014) provided evidence that private values models fit bidder behavior well in these settings.

[12] These explanations for seller values are due to conversations with industry professionals. Note also that adverse selection from the seller possessing more knowledge about car quality than the buyer is likely small because of auction house information-revelation requirements and because sellers are not previous owners/drivers of the vehicles.

density, f_S, I will use the notation $[\underline{s}, \overline{s}]$. My results do not rely on specifying whether the supports of B_i and S are finite or infinite. In estimation and in computing welfare measures, I choose large, finite values for the support bounds of B and S. In pinning down one tail condition empirically (discussed in estimation step 4 in Section 4), I will also assume that $\underline{s} \geq \underline{b}$, given that the seller will be guaranteed a price of at least $\underline{b}$ from the auction.

The random variable W in Assumptions $A1$ and $A2$ is observed by all buyers and the seller and represents game-level heterogeneity. Conditional on W, buyers and sellers have independent private values, but unconditional on W values are correlated. In estimation, in Section 4, I consider W to be unobserved to the econometrician, and I incorporate an additional, additively separable game-level heterogeneity term $X'\gamma$ that is observable to both the econometrician and to the players. Incorporating this latter term, the seller's value is $S + W + X'\gamma$ and buyer i's value is $B_i + W + X'\gamma$. I do not assume that B_i and S take on only positive values; this is because these random variables represent how the players value the car relative to the game-level heterogeneity component they all observe (so a negative B_i or S means that a buyer or seller values the car less than the observable value of the car). Because these objects can be negative, I assume nothing that prevents the model from suggesting that players' overall values may be negative. However, in practice, when I estimate the pieces of my model, I find that the majority of the variation in these values arises from the observable heterogeneity term $X'\gamma$, and that this term has most of its mass above zero, and thus the additively separable model does not appear to be a bad approximation. I discuss this in Appendix D.2.2.

For the next several subsections, I will discuss properties of the game *conditional* on a realization of W, and thus I will omit W for notational simplicity and return to it when I incorporate game-level heterogeneity in Proposition 5. I ignore auction house fees in this analysis but discuss them in detail in Appendix E.

3.1 Payoffs

I model the game as follows. In period $t = 0$, the seller chooses her secret reserve price, $R = \rho(S)$, knowing only her type S. This choice of reserve price is not revealed to buyers, before or after the auction. In period $t = 1$, the ascending auction takes place. Let β_i denote bidder i's auction strategy (a price at which bidder i drops out of the auction), and let the final auction price be denoted P^A. If $P^A \geq R$, the high bidder wins the car and the game ends. If $P^A < R$, the high bidder is given the opportunity to walk away (denoted $D_1^B = 1$), which ends the game, or not walk away ($D_1^B = 0$), entering into bargaining with the seller . If the buyer chooses not to walk away, the buyer enters an alternating-offer bargaining game with the seller. In doing so the buyer immediately incurs a bargaining cost, $c_B > 0$, and this c_B will be incurred by the buyer at every offer he makes. The seller will incur a bargaining cost, $c_S > 0$, at each offer she makes. The first offer of the bargaining game is P^A. The game moves to period 2 of the game, in which the seller chooses $D_2^S \in \{A, Q, C\}$—a choice to accept (A), quit (Q), or counter (C). If the seller chooses Q or A the game ends. If the seller chooses C, the seller specifies a counteroffer P_2^S, and play continues to period 3, with the buyer choosing $D_3^B \in \{A, Q, C\}$, and so on up to period T. If period T is reached, the player whose turn it is can only choose to accept or quit.

Throughout the game, bargaining offers must be weakly greater than the auction price. Although this relationship is not an explicit rule of the auction house, the data show that bargaining prices lie above the auction price nearly 100% of the time. This feature means that the auction price plays a similar role for the seller that a list price would play for a buyer in many other real-world haggling scenarios.[13]

The payoffs in the game are as follows. If a buyer of type B and a seller of type S agree to trade at a price P, the buyer's payoff is $B - P$ less the per-offer bargaining costs the buyer has incurred up to that point. If trade occurs in round 1 of the game (i.e. at the auction), the buyer's payoff will be $B - P$, with $P = P^A$, the auction price. Similarly, if the buyer and seller agree to trade at a price of P, the seller's payoff is P, less any incurred bargaining costs.

When disagreement occurs, the buyer receives a payoff of zero and the seller a payoff of S (less any incurred costs). This modeling choice is one of the abstractions (and limitations) of the model. In practice, a buyer who fails to acquire a car may choose to later re-enter the market to bid on a similar car. The approach I adopt—treating buyers' outside option as a 0 payoff—means that the object I model as the buyer's value is actually the buyer's full willingness to pay minus a discounted continuation value of re-entering the market. Similarly, what I model as the seller's value S is in practice the seller's discounted continuation value of re-entering the market to attempt to sell the car again at the auction house, at a competing wholesale outlet, or at her own lot. These abstractions are appropriate under the following interpretation of my counterfactual exercises: For a given buyer and seller pair who meet in bargaining today, holding fixed their continuation values of re-entering the market, how would their expected gains from trade improve if today's bargaining game were efficient? Where these abstractions become a limitation is that they do not allow me to model how players' continuation values might change if the bargaining mechanism were to change *permanently*. In Appendix B.3, I demonstrate that my qualitative and quantitative findings are similar in several analyses that cut the data based on variables related to players' continuation values. These analyses do not alleviate all concerns associated with ignoring these continuation-game dynamics. I ignore these dynamics across games in order to focus on dynamics within the game; studying instead the dynamics across games would be an interesting avenue for future research.[14]

[13]In haggling in the presence of posted list price, a seller and buyer negotiate over prices in a range *below* the list price but are unlikely to negotiate above the list price.

[14]Note that players' continuation values *within* a given game are addressed in the model; see Appendix A; it is only players' continuation values *across* instances of the game that I abstract away from.

3.2 Equilibrium Concept

In what follows, I focus on pure strategy Bayesian Nash equilibria (BNE). A BNE of the game is as follows. Let H_t represent the history of offers, including the auction price, up through period $t-1$ of the bargaining game. The strategy of a buyer of type b_i is a history-contingent set of actions $\sigma_i^B(b_i) = \{\beta_i, \{D_t^B|H_t\}_i, \{P_t^B|H_t\}_i\}$, where the decisions D_t^B and offers P_t^B included are those for periods in which it is the buyer's turn. The strategy of a seller of type s is a history-contingent set of actions $\sigma^S(s) = \{\rho, \{D_t^S|H_t\}, \{P_t^S|H_t\}\}$, where the decisions and offers are those for periods in which it is the seller's turn. I do not allow for strategies that directly depend on N, W, or the drop-out prices of non-price-setting bidders. A set of strategies $\sigma^{B*}(b_i)$ for all buyers and $\sigma^{S*}(s)$ for the seller constitutes a BNE of this game if, for each player, his or her strategy is a best response to opponents' strategies and players update their beliefs about opponent values using Bayes rule at each history of the game that is reached with positive probability.[15]

It is simple to derive a multiplicity of equilibria of the game, such as the following three examples (none of which need violate Assumption $A4$):

Three Examples of Equilibria of the Bargaining Subgame:

1. *Sellers only accept or quit at* $t = 2$*, and buyers reject all (off-equilibrium) offers at* $t = 3$.

2. *Sellers make uninformative offers (equal to* $\bar{s}$*, say) at* $t = 2$*, buyers counter at* $t = 3$*, and sellers only accept or quit at* $t = 4$*. Buyers reject all off-equilibrium offers at* $t = 3$ *or* $t = 5$.

3. *All offers and counteroffers must lie within a particular set of possible values, and in the (off-equilibrium) case in which any player deviates from these offers, the opponent responds by quitting.*

Ausubel and Deneckere (1993) provided a discussion of other partial-pooling equilibria for a similar bargaining game but with one-sided offers, and Ausubel, Cramton, and Deneckere (2002) suggested that such arguments can be extended to two-sided offer games as well.

3.3 Mechanism Design Framework for Evaluating Bargaining Efficiency

Prior to deriving the properties of BNE of this game, I describe the mechanism design framework I use to assess efficiency of bargaining, as it is the motivation for deriving some of the game's properties. By the Revelation Principle, any BNE of an incomplete-information trading game has

[15] Note that Perfect Bayes Equilibrium (PBE) is a refinement of BNE (and thus, every PBE is also a BNE) requiring that the researcher also specify how beliefs are updated at histories of the game that are never reached in equilibrium. I focus on the broader equilibrium concept, BNE, because the PBE concept does not meaningfully narrow down the set of equilibria in sequential bargaining games of incomplete information (see discussion in Gul, and Sonnenschein 1988) and because none of my identification or estimation arguments rely on specifying how beliefs are updated after zero-probability events. See Appendix B.1 of Larsen (2019) for further discussion.

a payoff-equivalent, direct-revelation mechanism. In a direct mechanism, a buyer of type b and seller of type s report their true types to the mechanism designer and then trade occurs with probability $x(s, b)$ (the allocation function), where this allocation function is determined so that players receive the same expected outcomes as in the original game.

The allocation function corresponding to ex-post efficient trade is simply $x^*(s, b) \equiv 1\{s \leq b\}$. The allocation function corresponding to a given point along the ex-ante efficient frontier, on the other hand, will maximize a convex combination of the buyer's and seller's ex-ante expected gains from trade, with weight η given to the seller's gains and weight $1 - \eta$ given to the buyer's. I will use the notation $x^{\eta}(\cdot)$, for a given $\eta \in [0, 1]$, to denote the allocation function corresponding to a point on the ex-ante efficient frontier. Computing $x^{\eta}(\cdot)$ boils down to solving a linear programming problem, described in Section 5 and Appendix D.7. The direct mechanism corresponding to the real-world bargaining, which I denote $x^{RW}(\cdot)$, can be estimated directly from the data, as described in Section 4. Computing each of these allocation functions requires estimates of F_B and F_S, the distributions of buyer and seller values. Thus, a key focus of this paper is the estimation of these distributions without imposing *a priori* any restrictions on how efficient the real-world bargaining is relative to these counterfactual benchmarks.

3.4 Model Properties

I now describe a number of properties of this game that will hold in any equilibrium.[16] These properties will then be exploited in Section 4 to estimate the distributions of buyer and seller values, the support of types who enter the bargaining game, and the allocation function corresponding to the real-world mechanism.

Bidding Behavior. The first property concerns bidding in the auction. In the button auction, the auctioneer continuously raises the price and bidders decide to remain in the bidding or drop out. Any bidder who drops out receives a payoff of zero. When only one bidder is left in the bidding, the auction ends immediately at the current price. I define a bidder's strategy as *bidding truthfully* if the bidder drops out only once the current price exceeds the bidder's value.

Proposition 1. *Suppose Assumption A1 holds and consider an arbitrary bidder i. In any BNE, holding fixed the strategies of all players in the continuation game, the strategies of other bidders in the auction, and the reserve price strategy of the seller, it is a weak best response for i to play the following strategy: (i) bid truthfully in the auction and (ii) enter bargaining only if doing so yields a non-negative expected payoff.*

[16] Appendix B.4 discusses an extension of this model in which sellers have some uncertainty about the distribution of buyer values when choosing the reserve price, which addresses explicitly why sellers may make or accept offers below their secret reserve price. Appendix B.4 also provides several other explanations of this phenomenon.

The intuition behind the proposition is as in a standard ascending button auction: a bidder will not find it optimal to drop out before the current price reaches his value because doing so would make the bidder miss out on a chance to win the auction. A bidder will also not find it optimal to remain in the auction once the current price passes his value because doing so will yield a negative payoff if the bidder does end up winning.[17]

The best response correspondence of bidder i can contain strategies other than the one described in Proposition 1. For example, consider the following BNE: the seller sets a very high reserve price, all bidders drop out immediately, and, off the equilibrium path, bidders always enter bargaining and sellers always reject the auction price. In this example, bidders receive a payoff of zero, but would receive no less by following the strategy in Proposition 1. To rule out such cases, and motivated by Proposition 1, I make the following assumption:

Assumption. *(A6) Bidders bid truthfully in the auction and enter bargaining only if doing so yields a non-negative expected payoff.*

Seller's Choice to Accept the Auction Price or Quit. I now demonstrate that bounds on the distribution of seller values can be achieved by an argument similar to the Haile and Tamer (2003) bounds in English auction settings. The argument differs from Haile and Tamer (2003), however, in that it is not possible here to construct both an upper and lower bound on the seller's value for each individual realization of the game. This is because, as shown below, a lower bound on a seller's value is only observed when the seller chooses to quit. Therefore, rather than observation-level bounds, I will obtain bounds on the *distribution* of seller values relying on probability statements formed from observations of many sellers' decisions to accept or walk away from an offer on the table.

Let $D^S = A$, without a t subscript (to distinguish this from the period-specific action described in Section 3.1), represent the event in which the seller takes an action in period 1 or 2 that results in the game ending in agreement at the auction price. This event occurs either when 1) the auction price exceeds the reserve price or 2) the auction price fails to meet the reserve price, the high bidder does not opt out of bargaining, and the seller accepts the auction price on her first bargaining turn. Similarly, let $D^S = Q$ represent the event in which the seller takes an action in period 2 that results in the game ending in disagreement at the auction price. This event happens when the auction price fails to meet the reserve price, the high bidder does not opt out of bargaining, and the seller quits on her first bargaining turn rather than accepting the auction price or making a counteroffer.[18]

[17] Appendix B.1 expounds on this result and proves that bidders bidding above their values and then attempting to bargain to a lower final price later could not occur in equilibrium even if bargained prices below the auction price were allowed by the auction house.

[18] In mathematical notation, $1\{D^S = A\} \equiv 1\{P^A \geq R \vee (P^A < R \wedge D_1^B = 0 \wedge D_2^S = A)\}$ and $1\{D^S = Q\} \equiv 1\{P^A < R \wedge D_1^B = 0 \wedge D_2^S = Q\}$. Note that the events $D^S = A$ and $D^S = Q$ are observable to the econometrician for every instance of the game recorded in the data, not just those in which bargaining occurs.

I exploit the following assumption:

Assumption. *(A7) The seller never (i) accepts an auction price below her value or (ii) walks away from (quits at) an auction price above her value.*

The conditions in Assumption $A7$ will be satisfied in any BNE of the game in which the bargaining stage is reached with positive probability (see Lemma 3 in Appendix A). These conditions imply that, if the realized auction price is p^A and the seller accepts, it must be the case that the seller values the good *less* than p^A. Similarly, if the seller quits when the auction price is p^A, it must be the case that the seller values keeping the car herself *more* than p^A. These conditions imply bounds on distribution of S:

$$\Pr(D^S = A|P^A = p^A) \leq \Pr(S \leq p^A) = F_S(p^A)$$
$$\Pr(D^S = Q|P^A = p^A) \leq \Pr(S \geq p^A) = 1 - F_S(p^A) \Rightarrow \Pr(D^S \neq Q|P^A = p^A) \geq F_S(p^A)$$

I state these bounds as the following proposition, where $\mathcal{F}$ represents the space of all possible CDFs (i.e. right-continuous, weakly increasing functions approaching 0 to the left and 1 to the right):

Proposition 2. *Under Assumptions $A2$ and $A7$, for any $v \in [\underline{s}, \overline{s}]$, any CDF of seller values $F_S \in \mathcal{F}$ must satisfy $F_S(v) \in [\Pr(D^S = A|P^A = v), \Pr(D^S \neq Q|P^A = v)]$.*

I now highlight several interesting features of these bounds. First, the bounds do not cross, because $D^S = A \Rightarrow D^S \neq Q$, and therefore $\Pr(D^S = A|P^A = v) \leq \Pr(D^S \neq Q|P^A = v)$. Second, these bounds rely only on Assumption $A2$ (that is, that buyer and seller values are independent) and the conditions in Assumption $A7$. Under these assumptions alone, the bounds are sharp. However, under the additional assumptions imposed elsewhere in the paper (namely, that actions correspond to some BNE), the bounds are not necessarily sharp, but are conservative. Appendix B.2 discusses sharpness formally.

Third, the width of these bounds will be determined by the frequency with which (i) the seller chooses to make a counteroffer in response to the first offer or (ii) the buyer opts out of bargaining. Specifically, the bounds can be re-written

$$\Pr(D^S = A|P^A = p^A) \leq F_S(p^A) \leq \Pr(D^S = A|P^A = p^A) + \underbrace{\Pr(D^S \neq A \ \wedge \ D^S \neq Q|P^A = p^A)}_{\text{Prob. seller counters or buyer opts out}}$$

The object $\Pr(D^S \neq A \ \wedge \ D^S \neq Q|P^A = p^A)$ is the probability that either the seller makes a counteroffer in response to the first offer or the buyer opts out of bargaining. If, at a given $P^A = p^A$, the buyer does not opt out and the seller only accepts or quits (does not counter), this probability will be zero, and the bounds will collapse to a point equal to the probability of acceptance at that p^A, $\Pr(D^S = A|P^A = p^A)$.

These bounds can be applied to other alternating-offer bargaining settings, independent of whether the bargaining follows an auction. In such cases, the decisions $D^S = A$ or $D^S = Q$ would represent the first action taken by the player in the bargaining game who responds to the first offer.

The Lower Support of Buyer and Seller Types Who Bargain. One advantage of studying bargaining following an ascending auction is that the auction outcome affects the bargaining game in a tractable manner, allowing me to isolate the bargaining game from the auction. Let $\pi^B(p^A, b)$ represent the buyer's expected payoff from entering into bargaining conditional on his value b and the realization of the auction price. Let $\chi(b)$ be defined by $\pi^B(\chi(b), b) = 0$. The high bidder will end up in bargaining when $P^A < R$ and when $\pi^B(P^A, b) \geq 0$. The object $\chi^{-1}(p^A)$ then represents the buyer type that would be indifferent between bargaining and not bargaining when the realized auction price is p^A. As above, $\rho(\cdot)$ is the seller's secret reserve price strategy; that is, $R = \rho(S)$.

Proposition 3. *If Assumptions $A1$–$A6$ hold, then in any BNE, conditional on an auction price $P^A = p^A$ and conditional on bargaining occurring, the support of seller types in the bargaining game is $[\underline{s}(p^A), \overline{s}]$ and the support of buyer types is $[\underline{b}(p^A), \overline{b}]$, where $\underline{s}(\cdot) \equiv \rho^{-1}(\cdot)$ and $\underline{b}(\cdot) \equiv \chi^{-1}(\cdot)$. Moreover, $\rho(\cdot)$ and $\chi(\cdot)$ are strictly increasing, with $\rho(s) \geq s$ and $\chi^{-1}(p^A) > p^A$.*

The intuition behind this result is as follows. When the auction price is p^A and bargaining occurs, it will be common knowledge among the two bargaining parties that the seller's type s satisfies $\rho(s) \geq p^A$ (i.e. the reserve price is above the auction price), implying $s \in [\rho^{-1}(p^A), \overline{s}]$. Similarly, bargaining occurring means the buyer did not opt out, so $\chi(b) \geq p^A$, implying $b \in [\chi^{-1}(p^A), \overline{b}]$. Thus, the game I study is analogous to a setting of bargaining alone where the lower bound of the support of the types in the bargaining game differs across realizations of the game as determined by the realization of the auction price, and when I present welfare results later they will average over these realizations. The clean relationship between the auction and the bargaining game obtained in Propositions 1 and 3 would not exist if the pre-bargaining stage were a first-price auction rather than an ascending auction; the first-price auction would affect the bargaining (and vice versa) in an intractable manner.[19] The proof of Proposition 3 also addresses the seller's choice of reserve price, using Assumption $A4$ and a monotone comparative statics result to show that $\rho(\cdot)$ is strictly monotone.

Proposition 3 implies that, given an auction price p^A, the distributions of buyer and seller types in bargaining are given by $\frac{F_B(b)}{1-F_B(\chi^{-1}(p^A))}$ and $\frac{F_S(s)}{1-F_S(\rho^{-1}(p^A))}$, respectively. These distributions correspond precisely to on-equilibrium-path Bayes updating of the buyer's and seller's beliefs about their opponents' types given the actions occurring prior to the bargaining, as highlighted in Section 3.2. Also, the seller's beliefs in the bargaining game do not condition on N, the number of bidders.

[19] Elyakime, Laffont, Loisel, and Vuong (1997) discussed this issue and adopted a model in which a first-price auction takes place under incomplete information and post-auction bargaining takes place under complete information (Nash bargaining).

This is due to a convenient property of the symmetric independent private values button auction: the distribution of the maximum order statistic (here, the value of the buyer entering bargaining) conditional on a lower order statistic (here, the auction price), does not depend on N, a result first shown in Song (2004) and extended to the unobserved heterogeneity case in Freyberger and Larsen (2017). Thus, the number of bidders does not enter into the seller's beliefs about the density of buyer values she faces in bargaining once she knows the realization of P^A.

The Real-World Mechanism. The allocation function corresponding to the real-world mechanism, x^{RW}, satisfies the following property:

Proposition 4. *Under Assumptions A1–A6, in any BNE the allocation function x^{RW} can be written as $x^{RW}(r, b; p^A) \equiv 1\left\{b \geq g\left(r, p^A\right)\right\}$, where $g(r, p^A)$ is weakly increasing in r.*

Proposition 4 demonstrates that x^{RW} depends on a cutoff function defining the boundary between those types who trade and those who do not. Ausubel and Deneckere (1993) referred to this property as the "Northwestern Criterion" as it implies that trade occurs if and only if players' types lie northwest of a boundary defined by g. The proof of Proposition 4 exploits the strict monotonicity of $\rho(\cdot)$ proved in Proposition 3, which makes it possible to model the allocation conditional on a realization of the reserve price, $R = r$, rather than conditional on the seller's type. This is particularly useful in that it allows me to evaluate the allocation function for the real-world bargaining without knowing where the true distribution of seller values lies within the bounds from Proposition 2.

Game-level Heterogeneity. The above results are derived conditional on a given realization of game-level heterogeneity. I now consider the additively separable structure of buyer and seller values in the common component W.

Proposition 5. *Fix any BNE and suppose Assumptions A1–A6 hold. Suppose, when $W = 0$, the equilibrium is such that the reserve price is r; the auction price is p_A; the lowest buyer type who would choose to bargain is $\chi^{-1}(p^A)$; and, for each period t at which the game arrives, the offer is given by $P_t = p_t$ and the decision to accept, quit, or counter is given by $D_t = d_t$. Then, when $W = w$, the equilibrium will be such that the reserve price is $\tilde{r} = r + w$; the auction price is $\tilde{p}_A = p_A + w$; the lowest buyer type who would choose to bargain is $\chi^{-1}(\tilde{p}^A - w) + w$; the period t decision is d_t; and, for any period t offer that is accepted with positive probability, the period t offer is $p_t + w$.*

Proposition 5 is similar to results used elsewhere in the empirical auctions literature (Haile et al. 2003) but is a generalization specific to this setting. It implies that continuous actions of the game (reserve prices, auction prices, and bargaining offers) will be additively separable in W; choice probabilities for discrete actions (opting into bargaining; accepting, declining, or countering in response to an offer) will be unaffected by the value of W. An immediate implication of Proposition 5 is that the allocation function is invariant to game-level heterogeneity; that is, $x^{RW}(r+w, b+w; p^A+w) = x^{RW}(r, b; p^A)$.

4 Estimating Value Distributions and the Bargaining Mechanism

In this section, I exploit the model properties derived above in order to estimate the distribution of buyer and seller values and the bargaining mechanism. Identification and estimation require the following additional assumptions on the data. Below, let F_R, F_{P^A}, and F_W represent the cumulative distribution functions of R, P^A, and W.

Assumptions.

(A8) *F_R, F_{P^A}, and F_W have densities f_R, f_{P^A}, and f_W satisfying the following: (i) the characteristic functions of f_R and f_W have only isolated real zeros; (ii) the real zeros of the characteristic function of f_{P^A} and the real zeros of its derivative are disjoint; and (iii) E[W]=0.*

(A9) *The supports of S and B satisfy $\underline{s} \geq \underline{b}$.*

(A10) *Observations of random variables (S,B_i,W,N) across instances of the game are identically and independently distributed.*

(A11) *All observations in the data are generated by the same equilibrium.*

Assumption $A8$ lists the sufficient conditions from Evdokimov and White (2012) for proving identification of f_R, f_{P^A}, and f_W.[20] I use Assumption $A9$ in pinning down the left tail of the upper bound on the seller value CDF. The motivation for this assumption ($\underline{s} \geq \underline{b}$) is that any seller is guaranteed a price of at least $\underline{b}$ from participating.

Assumption $A10$ is common in the empirical games literature, and it abstracts away from dynamics across instances of the game.[21] Assumption $A11$ is not required for steps 1–4 below but is required for steps 5–6. For example, even if different equilibria of the bargaining subgame are played in different observations of the data, the distribution of buyer values can still be estimated (step 3) using the distribution of auction prices, as described below. Similarly, the revealed preference arguments used to bound the distribution of seller values (step 4) will still hold even if $A11$ fails. Steps 5–6, however, require inverting policy functions that will depend on the equilibrium of the game. Fortunately, none of the steps, including 5–6, require fully specifying or solving the equilibrium. Like Assumption $A10$, Assumption $A11$ is common in the structural literature. The

[20] Evdokimov and White (2012) demonstrated that these are weaker conditions than those used previously in the empirical auctions literature in settings relying on convolution arguments. The assumption that $E[W] = 0$ is a location normalization, and this normalization could alternatively be placed on R or P^A without loss of generality.

[21] Appendix B.3 and D.1 discuss some simple ways in which I do analyze inter-game dynamics.

typical approach in the literature to handling cases where particular subsamples of the data are believed to have been generated by different equilibria is to estimate the model separately in these subsamples. In line with this, throughout the estimation, I treat the dealers and fleet/lease samples separately because, according to conversations with industry professionals, this is likely the most important division of the data in which behavior may differ; it turns out that my findings for the fleet/lease sample—found in Appendix C—are very similar to the dealers sample results discussed in the body of the paper.

I now provide an overview of each estimation step. I do not describe all of the technical details for each step here, but include them in Appendix D. Appendix D also contains nonparametric identification proofs, arguments for consistency of the estimates, and evidence of goodness of fit for each estimation step.

Step 1) Accounting for Observed Heterogeneity Empirically. To account for game-level characteristics that are observed to the econometrician as well as the players, I apply Proposition 5. Let R^{raw} and $P^{A,raw}$ be random variables representing the reserve price and auction price in the raw data, prior to any adjustments for heterogeneity. As above, let W be a random variable representing game-level heterogeneity that observable to players but not the econometrician. Let X be a random variable representing game-level heterogeneity that is instead observed by the econometrician and players, with X independent of W, S, B, N. Let realizations of R^{raw}, $P^{A,raw}$, X, and W for game j be denoted by lower case letters with subscript j.

I specify the total game-level heterogeneity (observed plus unobserved) for observation j to be $x_j'\gamma + w_j$, where γ is a vector of parameters to be estimated. Proposition 5 implies that auction prices and reserve prices can be "homogenized" (Haile et al. 2003) by estimating the following joint regression of reserve prices and auction prices on observables:

$$\begin{bmatrix} r_j^{raw} \\ p_j^{A,h} \end{bmatrix} = \begin{bmatrix} x_j'\gamma \\ x_j'\gamma \end{bmatrix} + \begin{bmatrix} \tilde{r}_j \\ \tilde{p}_j^A \end{bmatrix},$$

where $\tilde{r}_j = r_j + w_j$, $\tilde{p}_j^A = p_j^A + w_j$. In the vector x_j I include a rich vector of controls, including flexible mileage terms, dummies for each make-model-year-trim-age combination, and a number of other factors described in detail in Appendix D.1. An estimate of $\tilde{r}_j$ is then given by subtracting $x_j'\hat{\gamma}$ from r_j^{raw}, and similarly for $\tilde{p}_j^A$. Variation in these two quantities is then attributed to unobserved game-level heterogeneity and to players' private values, as detailed below.

Step 2) Accounting for Unobserved Heterogeneity Empirically. To account for heterogeneity W in the game that is observed by the players but not by the econometrician, I apply a result due to Kotlarski (1967), which implies that observations of $\tilde{R} = R + W$ and $\tilde{P}^A = P^A + W$ (which are additively separable in W by Proposition 5) are sufficient to recover the densities f_W,

f_R, and f_{P^A}. This result has been applied elsewhere in first-price auction work (e.g., Li et al. 2000; Krasnokutskaya 2011); my application of this deconvolution argument using instead an *ascending auction bid* and a *reserve price* to identify unobserved heterogeneity parallels Freyberger and Larsen (2017) and Decarolis (2018). I estimate these densities using a flexible maximum likelihood approach, where the likelihood of the joint density of $(\tilde{R}, \tilde{P}^A)$ is given by

$$\mathcal{L}(f_{P^A}, f_R, f_W) = \prod_j \left[\int f_{P^A}(\tilde{p}^A_j - w) f_R(\tilde{r}_j - w) f_W(w) dw \right] \tag{1}$$

I approximate each of the densities f_{P^A}, f_R, and f_W as Hermite polynomials, as suggested by Gallant and Nychka (1987) (I use fifth-order polynomials). This also yields estimates of the CDFs F_W, F_R, and F_{P^A}. Appendix D.2 describes technical details and nonparametric identification.

Step 3) Estimating the Distribution of Buyer values. I recover the distribution of buyer values, F_B, from the distribution of auction prices, F_{P^A}, which, by Proposition 1, will coincide with the distribution of the second order statistic of buyer values. The relationship of F_{P^A} and $\Pr(N = n)$ (the distribution of the number of bidders) to F_B is as follows:

$$F_{P^A}(v) = \sum_n \Pr(N = n) \left[n F_B(v)^{n-1} - (n-1) F_B(v)^n \right] \tag{2}$$

The right-hand side of (2) is strictly monotonic in $F_B(\cdot)$, and thus F_B is nonparametrically identified by $\Pr(N = n)$ and F_{P^A} (see, for example, Athey and Haile 2007). I estimate the object F_B by solving (2) numerically on a grid of values for v, plugging in an estimate of $\widehat{\Pr}(N = n)$ and the maximum likelihood estimate $\hat{F}_{P^A}(v)$ from step 2.

To estimate $\widehat{\Pr}(N = n)$, I use the subsample of the data for which bid logs are available, in which I observe a lower bound on N that varies from auction to auction (see discussion in Section 2). I set $\widehat{\Pr}(N = n)$ equal to the empirical frequency with which this lower bound equals n. This treats the distribution of the lower bound as though it were the true distribution of the number of bidders. I gathered some additional independent data supporting this choice by physically attending over 200 auction sales and recording the number of bidders (see Appendix D.3.1). It turns out, however, that the choice of $\Pr(N = n)$ is, perhaps surprisingly, not critical to the welfare estimates of this paper. Specifically, the choice of $\Pr(N = n)$ affects the estimate of the full underlying buyer distribution, F_B, but has a negligible effect on the transformation of F_B used in evaluating welfare, which is the distribution of the high bidder's value conditional on the auction price integrated against the auction price density (i.e. the maximum order statistic distribution). In Appendix D.3.1, I demonstrate numerically that this latter object is not sensitive to the choice of $\Pr(N = n)$. I also provide a new mathematical result, stated as Proposition 10, proving that, for a class of possible choices of $\Pr(N = n)$ (Poisson distributions with mean λ), conditional on $\hat{F}_{P^A}$, the inferred maximum order statistic distribution has a derivative that is identically zero with respect to the choice of λ.[22]

Step 4) Estimating Bounds on the Distribution of Seller values. Proposition 2 demonstrates that, absent unobserved game-level heterogeneity, the objects $\Pr(D^S = A|P^A = v)$ and $\Pr(D^S \neq Q|P^A = v)$ will provide bounds on $F_S(v)$. The same revealed preference arguments used to derive that result—Assumption A7—extend to the case of unobserved heterogeneity, providing bounds on the distribution of $\tilde{S} \equiv S + W$ conditional on $\tilde{P}^A \equiv P^A + W$. Here I also incorporate the bound provided by secret reserve prices themselves: $R \geq S \Rightarrow F_S(v) \geq F_R(v)$.

To describe these bounds, let $q(v; F_S) = \int F_S(v-w)\frac{M_S(v,w)}{\int M_S(v,z)dz}dw$, where $M_S(v,w) \equiv f_{P^A}(v-w)f_W(w)$ is the joint density P^A and W.[23] Bounds on F_S, which I denote $[F_S^L(v), F_S^U(v)]$, are given by the solution to the following minimization problem:

$$\min_{(F_S^L, F_S^U)\in\Phi} \left\{ \left| \Pr(D^S = A|\tilde{P}^A = v) - q\left(v; F_S^L\right) \right|^2 + \left| \Pr(D^S \neq Q|\tilde{P}^A = v) - q\left(v; F_S^U\right) \right|^2 \right\} \tag{3}$$

where $|\cdot|^2$ represents the L^2-norm, and the set Φ is the set of feasible pairs of CDFs that can be bounds on F_S; that is, $\Phi = \left\{F_S^L \in \mathcal{F}, F_S^U \in \mathcal{F} : F_S^U(v) \geq F_S^L(v) \geq F_R(v) \ \ \forall \ \ v \in [\underline{s}, \overline{s}]\right\}$, where $\mathcal{F}$ is the set of possible CDFs, as defined in Section 3.4. Appendix D.4 demonstrates that these bounds are nonparametrically identified.

The objects F_S^L and F_S^U, as well as the functions χ^{-1} and g in steps 5 and 6 below, can be estimated using a minimum-distance, constrained least-squares procedure. I will describe this approach in slightly more detail in this step and be more brief in my description in steps 5 and 6. Additional technical details for each of these steps are found in Appendices D.4–D.6. To estimate the functions F_S^L and F_S^U, I first parameterize each as a very flexible piecewise linear spline; I denote these approximations $F_S^L(\cdot, \theta^{S,L})$ and $F_S^U(\cdot, \theta^{S,U})$. Denote the fixed vector of spline knots $\{v_k^S\}_{k=1}^{K_S}$. I choose $K_S = 200$; as discussed in Appendix D.4, the estimates are not sensitive to this choice. I estimate the parameter vectors $\theta^{S,L}$ and $\theta^{S,U}$ using the following objective function:

$$\begin{aligned}\min_{\theta^{S,L},\theta^{S,U}} \sum_{k=1}^{K_S} \Bigg\{ & \left[\widehat{\Pr}(D^S = A|\tilde{P}^A = v_k^S)\left(\int \hat{M}_S(v_k^S, z)dz\right) - \int F_S(v_k^S - w; \theta^{S,L})\widehat{M}_S(v_k^S, w)dw\right]^2 \\ & + \left[\widehat{\Pr}(D^S \neq Q|\tilde{P}^A = v_k^S)\left(\int \hat{M}_S(v_k^S, z)dz\right) - \int F_S(v_k^S - w; \theta^{S,U})\widehat{M}_S(v_k^S, w)dw\right]^2 \Bigg\}\end{aligned} \tag{4}$$

This approach searches for the lowest and highest possible values of F_S that can rationalize the observed behavior in the data described by the conditional probabilities $\Pr(D^S = A|\tilde{P}^A = v)$ and

[22] This result is new to the literature and suggests that order-statistic-inversion approaches to auctions may in some cases result in estimated welfare effects that are not sensitive to the choice of $\widehat{\Pr}(N = n)$ used in the inversion.

[23] This object $q(v; F_S)$ is the conditional CDF of $\tilde{S}$, evaluated at v and conditional on the auction price with unobserved heterogeneity included ($\tilde{P}^A$) being equal to v. Thus, $q(v; F_S)$ could alternatively be written $F_{\tilde{S}|\tilde{P}^A}(v|v)$. I adopt the notation $q(v; F_S)$ to be clear that this functional depends on F_S, the object I wish to bound.

$\Pr(D^S \neq Q|\tilde{P}^A = v)$. I impose several constraints on the minimum distance problem in (4): (i) F_S^L lies graphically above F_R and graphically below F_S^U; (ii) F_S^L and F_S^U lie in $[0,1]$; (iii) F_S^L and F_S^U are weakly increasing; and (iv) $F_S^L(v)$ and $F_S^U(v)$ are equal to 0 for any $v < v_1^S$ and equal to 1 for any $v > v_{K_S}^S$. These last three constraints ensure that F_S^L and F_S^U will correspond to proper distribution functions. The only constraint of (iv) that binds in practice is that of the left tail of F_S^U. My approach essentially bounds that left tail below by $\underline{b}$, as stated in Assumption $A9$. This is discussed in more detail in Appendix D.4.

Computing (4) requires first-step estimates of several other objects, including $\hat{F}_R$, $\hat{f}_{P^A}$, and $\hat{f}_W$, which come from the maximum likelihood procedure in (1). The procedure also requires the objects $\widehat{\Pr}(D^S = A|\tilde{P}^A = \tilde{p}^A)$, and $\widehat{\Pr}(D^S \neq Q|\tilde{P}^A = \tilde{p}^A)$, which I estimate using local linear regressions.

Step 5) Estimating the Lower Support of Bargaining Types. By Proposition 3, both $\underline{b}(\cdot) \equiv \chi^{-1}(\cdot)$ and $\underline{s}(\cdot) \equiv \rho^{-1}(\cdot)$ are increasing functions, and correspond to the lower support of buyer and seller types who enter the bargaining game. For any function $F_S(\cdot)$ lying in the estimated bounds $[\hat{F}_S^L(\cdot), \hat{F}_S^U(\cdot)]$, the function $\rho(s)$ can be constructed as $\rho(s) = F_R^{-1}(F_S(s))$, with F_R replaced with the estimated $\hat{F}_R$ from (1). Similarly, $\rho^{-1}(r)$ can be constructed as $\rho^{-1}(r) = F_S^{-1}(F_R(r))$.

To describe the identification and estimation of $\chi^{-1}(\cdot)$, let $D_1^B = 0$ represent the buyer's decision to *not* walk away (and let $D_1^B = 1$ represent walking away) when informed that the high bid does not meet the reserve price, which occurs with the following conditional probability:

$$\Pr(D_1^B = 0|\tilde{P}^A = \tilde{p}^A, \tilde{P}^A < \tilde{R}) = \int \frac{1 - F_B(\chi^{-1}(\tilde{p}^A - w))}{1 - F_B(\tilde{p}^A - w)} \left(\frac{M_\chi(\tilde{p}^A, w)}{\int M_\chi(\tilde{p}^A, z)dz} \right) dw \tag{5}$$

where $M_\chi(\tilde{p}^A, w) \equiv f_{P^A}(\tilde{p}^A - w)(1 - F_R(\tilde{p}^A - w))f_W(w)$ is the likelihood of the event $(P^A = \tilde{p}^A - w, \tilde{P}^A < \tilde{R}, W = w)$. Appendix D.5 demonstrates that $\chi^{-1}(\cdot)$ is nonparametrically identified. For estimation, I approximate $h_\chi(\cdot) \equiv 1 - F_B(\chi^{-1}(\cdot))$ as a flexible piecewise linear spline parameterized by θ^χ. Like the bounds on seller values, θ^χ can be estimated using constrained least squares. I do so by evaluating the left-hand side and right-hand side of (5) on a fixed grid of points for the auction price $\tilde{p}^A$ and search for the value of the vector θ^χ that minimizes the distance between the left- and right-hand sides. This procedure requires estimates of densities and CDFs from above, as well as an estimate of the conditional probability of not walking away, $\Pr(D_1^B = 0|\tilde{P}^A = \tilde{p}^A, \tilde{P}^A < \tilde{R})$ which I estimate using a local linear regression. Technical details are found in Appendix D.5.

Step 6) Estimating the Direct Mechanism Corresponding to Real-World Bargaining. Proposition 4 demonstrates that the allocation function corresponding to the real-world mechanism can be written as $x^{RW}(r, b; p^A) \equiv 1\left\{b \geq g\left(r, p^A\right)\right\}$ for some function $g(\cdot)$. The empirical object that can be used to identify this function $g(\cdot)$ is the probability of trade conditional on a realization of $\tilde{R}$ and $\tilde{P}^A$. Let $\mathcal{A} \in \{0, 1\}$ be a random variable indicating whether or not trade occurs in a given instance of the game. The conditional probability of trade is given by

$$\Pr(\mathcal{A} = 1 | \tilde{R} = \tilde{r}, \tilde{P}^A = \tilde{p}_A) = \int \frac{1 - F_B\left(g(\tilde{r} - w, \tilde{p}^A - w)\right)}{1 - F_B\left(\tilde{p}^A - w\right)} \left(\frac{M_g(\tilde{r}, \tilde{p}^A, w)}{\int M_g(\tilde{r}, \tilde{p}^A, z)dz}\right) dw \tag{6}$$

where $M_g(\tilde{r}, \tilde{p}^A, w) \equiv f_R(\tilde{r} - w) f_{P^A}(\tilde{p}^A - w) f_W(w)$ is the joint density of (R, P^A, W). Appendix D.6 demonstrates that $g(\cdot)$ is nonparametrically identified. For estimation, I approximate $h_g(r, p_A) \equiv \frac{1 - F_B(g(r, p^A))}{1 - F_B(p^A)}$ using a flexible bilinear spline parameterized by θ^g. As with the estimation of the seller CDF bounds and the estimation of $\chi^{-1}(\cdot)$, I obtain an estimate of θ^g using constrained least squares. I do so by evaluating the left-hand side and right-hand side of (6) on a fixed grid of points and searching for the parameters θ^g to minimize the distance between the left- and right-hand sides. As with preceding steps, I estimate the conditional probability $\Pr(\mathcal{A} = 1 | \tilde{R} = \tilde{r}, \tilde{P}^A = \tilde{p}_A)$ in a first step; for this two-dimensional conditional probability, I use a tensor product of cubic b-spline functions. The other objects in (6) consist of densities and CDFs estimated above. Technical details are found in Appendix D.6.

Summary of Identification. Appendices D.2–D.6 provide proofs for the nonparametric identification of each of the objects I estimate. Here I provide a brief summary of the identification. The step 1 regression controlling for observable heterogeneity identifies the joint distribution of $(\tilde{R}, \tilde{P}^A)$ (the residuals). This joint distribution identifies the marginal distributions F_R, F_{P^A}, and F_W, estimated in step 2. The underlying buyer distribution, F_B, estimated in step 3, is identified by the probability mass function, $\Pr(N = n)$, and by the marginal distribution of auction prices, F_{P^A}. Bounds on the distribution of seller values, estimated in step 4, are identified by F_R, f_{P^A}, and f_W, and by the conditional probabilities of sellers accepting and not quitting, $\Pr(D^S = A | \tilde{P}^A = \tilde{p}^A)$ and $\Pr(D^S \neq Q | \tilde{P}^A = \tilde{p}^A)$. The object $\chi^{-1}(\cdot)$, estimated in step 5, is identified by F_R, F_B, f^{P^A}, and f_W, and by the conditional probability of buyers not walking away from bargaining, $\Pr(D_1^B = 0 | \tilde{P}^A = \tilde{p}^A, \tilde{P}^A < \tilde{R})$. The object $\rho^{-1}(\cdot)$ is identified by F_R for any F_S lying in the seller CDF bounds. Finally, the object $g(\cdot)$, estimated in step 6, is identified by F_B, f_R, f_{P^A}, and f_W, and by the conditional probability of trade, $\Pr(\mathcal{A} = 1 | \tilde{R} = \tilde{r}, \tilde{P}^A = \tilde{p}_A)$.

5 Computing Bargaining Efficiency

To evaluate efficiency, I consider several welfare measures and compute these measures for the real-world bargaining and for ex-ante and ex-post efficient mechanisms. Each welfare measure depends on the estimated densities, CDFs, and lower support functions ($\underline{b}(\cdot) \equiv \chi^{-1}(\cdot)$ and $\underline{s}(\cdot) \equiv \rho^{-1}(\cdot)$) obtained in the estimation steps 2–5 above. Each welfare measure will also depend on an allocation function, x. As discussed in Section 3.3, the allocation function corresponding to the real-world bargaining is x^{RW} (estimated in step 6); the ex-post efficient allocation function is $x^*(s, b; p^A) \equiv$

$1\{s \leq b\}$; and the ex-ante efficient frontier consists of allocation functions, which I denote x^{η}, that place welfare weight of η on the seller's expected utility and $1-\eta$ on the buyer's. Several mechanisms along the ex-ante efficient frontier are of particular interest: x^1, the allocation function corresponding to a take-it-or-leave-it offer by the seller; x^0, the allocation function corresponding to a take-it-or-leave-it offer by the buyer; and $x^{\frac{1}{2}}$, the equal-weighted ex-ante efficient mechanism. In discussing results below I will refer to $x^{\frac{1}{2}}$ as *the second-best mechanism*, x^1 as the *seller-optimal mechanism*, and x^0 as the *buyer-optimal mechanism*.

The first welfare measure I consider is the overall expected gains from trade. For a given allocation function x and densities f_S and f_B, this is given by

$$\int_{\underline{b}}^{\bar{b}} \left[\int_{\underline{b}(p^A)}^{\bar{b}} \int_{\underline{s}(p^A)}^{\bar{s}} (b-s)x(s,b;p^A) f_S(s|p^A) f_B(b|p^A) ds\, db \right] f_{p^A}(p^A) dp^A \tag{7}$$

where $f_S(s|p^A) = \frac{f_S(s)}{1-F_S(\underline{s}(p^A))}$ and $f_B(b|p^A) = \frac{f_B(b)}{1-F_B(\underline{b}(p^A))}$ are the Bayes-updated beliefs of agents about their opponents' types when bargaining starts. This welfare measure, along with the others I consider, is integrated over realizations of the lower bound of the support of buyer and seller types (i.e. integrated over the realized auction price p^A). I also evaluate several other welfare measures that are related to (7): the buyer's or seller's gains from trade (constructed by replacing $b-s$ in (7) with just b or s) and the probability of trade (constructed by replacing $b-s$ with 1).

The efficiency loss due to incomplete information—the loss highlighted in Myerson and Satterthwaite (1983)—can be estimated by evaluating the gains from trade in (7) using the ex-post efficient allocation function (i.e. replacing $x(s,b;p^A)$ in (7) with $x^*(s,b;p^A)$) and comparing this to (7) evaluated using the second-best allocation function ($x^{\frac{1}{2}}$). The efficiency loss due to other sources beyond those highlighted in Myerson and Satterthwaite (1983) can be estimated by comparing this second-best efficient outcome to (7) evaluated at the real-world mechanism (x^{RW}).[24]

Evaluating the efficiency of bargaining at a given allocation function x is easy once the densities, CDFs, and lower support of the bargaining types are known; it simply involves numerically evaluating integrals like (7). Computing the ex-ante efficient allocation functions themselves (x^{η}), however, is extremely computationally involved in my setting. This is because it must be done at each realization of the lower bound of the support (each p^A) and because the type distributions I estimate are *irregular*, in the sense of Myerson (1981). F_B and F_S are referred to as regular if $b - \frac{1-F_B(b)}{f_B(b)}$ and $s + \frac{F_S(s)}{f_S(s)}$, the *virtual values* of buyers and sellers, are increasing. Myerson and Satterthwaite (1983) and Williams (1987) derived convenient solutions for the mechanisms along the ex-ante efficient frontier under the assumption of regularity. Without regularity, I am forced to numerically enforce a large number of incentive compatibility constraints (see Appendix D.7). Furthermore, although I have point estimates of F_B, I only have bounds on F_S, and, without fur-

[24]For the expected gains from trade in the real bargaining, I also incorporate an upper bound on the amount of bargaining costs incurred in the real-world mechanism (see Appendix D.7.3).

Table 2: Monotonicity Results for Welfare Measures (Proposition 6)

A. Levels	Ex-post	Second-best	Buyer-optimal	Seller-optimal	Real bargaining
Total gains	↓	↓	–	–	↓
Buyer gains		–	↓	–	↓
Seller gains		–	–	↓	↓
Prob. of trade	↓	*	*	↓	↓

B. Differences	Ex-post minus second-best	Second-best minus real	Ex-post minus real
Total gains	*	↓	↓
Prob. of trade	*	*	↓

Notes: Table displays monotonicity results for welfare measures proved in Proposition 6. Each cell marked with a ↓ signifies that the specified measure will decrease given a first-order stochastically dominating shift in F_S. Each cell marked with an asterisk indicates that there is no mathematical proof of a monotonicity result and that the bounds must be determined numerically. Cases marked with "–" in the second-best column indicate that I will report bounds on these quantities corresponding to the F_S leading to the maximum and minimum bounds on the total expected gains from trade. Cases marked with "–" in the buyer-optimal column indicate that I will report bounds on these quantities corresponding to the F_S leading to the maximum and minimum bounds on the buyer gains from trade. Cases marked with "–" in the seller-optimal column indicate that I will report bounds on these quantities corresponding to the F_S leading to the maximum and minimum bounds on the seller gains from trade.

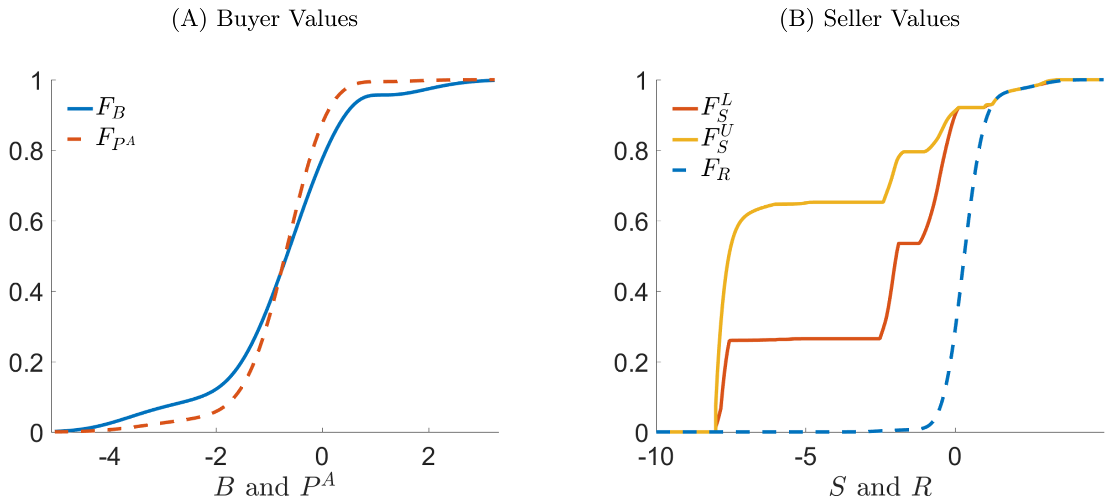
ther theory, obtaining bounds on welfare measures would require evaluating each mechanism at all possible distributions F_S within the estimated bounds. Fortunately, I am able to derive a number of useful monotonicity results that simplify this computation greatly. These results are summarized in Table 2 and in the following proposition:

Proposition 6. *A first-order stochastically dominating change in F_S will lead to the monotonic changes in welfare measures described in Table 2.*

Each cell in Table 2 marked with a ↓ signifies that the specified measure will decrease given a first-order stochastically dominating shift in the distribution of seller values (e.g. a shift from F_S^U to F_S^L). Each cell marked with an asterisk indicates that I have no proven monotonicity result for that welfare measure. For these latter welfare measures, I am forced to obtain bounds numerically through a massive grid search. This still yields valid bounds on welfare, but is very computationally expensive.[25] Appendix D.7 contains technical details on this numerical procedure and the procedure for computing the ex-ante efficient allocation functions, x^η, which builds on results from Myerson and Satterthwaite (1983) and Williams (1987).

[25]To give a rough idea of the computational burden, even with the techniques I introduce here to reduce this burden, it takes about one year of computation time for a single machine to compute all of the estimates and confidence intervals reported in the paper. I parallelize these computations on a high-performance computing cluster to reduce this time to less than one week.

Figure 1: Distribution Estimates



Notes: Panel A displays estimated distribution of auctions prices (dashed line) after removing observable and unobservable game-level heterogeneity, and estimated distribution of buyer values (solid line). Panel B displays estimated distribution of reserve prices (dashed line) after removing observable and unobservable game-level heterogeneity, and estimated lower and upper bounds on distribution of seller values (solid lines). Units = $\$1,000$.

6 Putting It All Together: How Efficient Is Bargaining?

6.1 Distribution Estimates

This section presents the distributions of buyer and seller values estimated using the procedures described in Section 4. In each figure and table that follows, monetary values are denoted in units of $1,000. Figure 1, panel A, displays the distribution of the auction price net of unobserved heterogeneity, F_{P^A} (the dashed line), and the estimated underlying distribution of buyer values, F_B. F_{P^A} does not entirely dominate F_B in a first-order stochastic dominance sense. This is due to the distribution of the number of bidders, $\Pr(N = n)$, having much of its mass at two or three bidders.

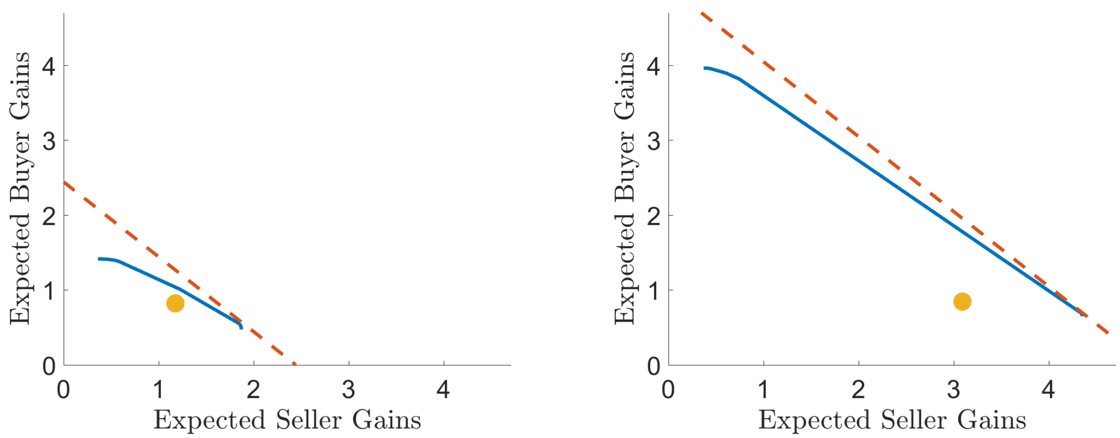
The dashed line in panel B of Figure 1 shows the distribution of secret reserve prices net of unobserved heterogeneity, F_R, and the solid lines show the estimated upper and lower bounds of the seller value CDF, F_S^L and F_S^U. These bounds suggest that, when the first bargaining offer (the auction price) is about -$1,000 (i.e. $1,000 lower than would be predicted based on car-level heterogeneity), sellers choose to accept this offer or walk away from it with frequencies that imply that the probability of S being less than -$1,000 is in the range $[0.56, 0.80]$.

Comparing panels A and B, it is clear that there is overlap in the support of buyer and seller values. This feature illustrates what is referred to in the theoretical bargaining literature as the "no gap" case (i.e., the upper bound of the support of seller valuations is higher than the lower bound of the support of buyer valuations, and hence there is uncertainty as to whether gains from trade actually exist), and is the case motivating Myerson and Satterthwaite (1983) (see Fudenberg

Figure 2: Graphical Evaluation of Bargaining Efficiency

(A) Gains Using Seller Lower Bound

(B) Gains Using Seller Upper Bound



Notes: Panel A displays estimated expected seller and buyer gains on ex-post efficient frontier (dashed line), on ex-ante efficient frontier (solid line), and in real-world bargaining (solid dot) using the seller distribution lower bound. Panel B displays the same estimates at the seller distribution upper bound. Units = $1,000.

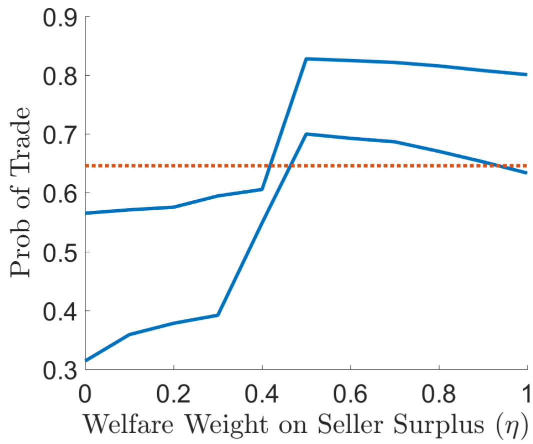
and Tirole 1991). However, the actual overlap in terms of mass appears to be small, as most seller values (at least 80%) lie below zero—in some cases, far below zero—while buyer values are centered around zero and are much less dispersed. This implies that the actual efficiency loss due to incomplete information may be small in this setting. Below I provide a more precise quantitative analysis of the overlap in buyer and seller mass taking into account the support of the types who actually end up in bargaining.

6.2 Graphical Analysis of Bargaining Efficiency

Using the approach described in Section 5, I compute buyer gains and seller gains in the real-world bargaining mechanism as well as the ex-post and ex-ante efficient frontiers. The performance of the real-world bargaining relative to these theoretical benchmarks is displayed in Figure 2. The dashed line displays the ex-post efficient frontier in the space of buyer gains (the vertical axis) and seller gains (the horizontal axis). The solid line displays the ex-ante efficient frontier. The solid dot indicates the expected gains in the real-world mechanism. Panel A uses the seller value CDF lower bound and panel B the upper bound.

Comparing the ex-ante efficient frontier to the ex-post efficient frontier provides an indication of the size of efficiency loss due strictly to incomplete information. In each case in Figure 2, the ex-ante efficient frontier lies close to the ex-post efficient frontier. This suggests that, in this market, incomplete information per se may not be leading to large inefficiencies, likely due to the limited overlap in buyer and seller distributions suggested by Figure 1. Comparing the real-world bargaining outcome to the ex-ante efficient frontier, on the other hand, provides an indication of

Figure 3: Graphical Evaluation of Trade Probability



Notes: Figure shows, in solid lines, the estimated probability of trade at each η (seller's welfare weight) along the ex-ante efficient frontier evaluated at the seller value distribution upper and lower bounds. The dashed line displays the probability of trade in the real-world mechanism for comparison.

the size of efficiency loss due to the fact that the mechanism used in practice (alternating-offer bargaining) or the equilibrium played may be inefficient. Panel A demonstrates that the real-world mechanism lies not far from the ex-ante efficient frontier when evaluated at the seller value CDF lower bound, but panel B shows a clear shortfall in efficiency when evaluated at the CDF upper bound.

Figure 3 displays the probability of trade along the ex-ante efficient frontier evaluated at the seller value CDF upper and lower bounds (the solid lines). The horizontal axis represents the weight $\eta \in [0, 1]$ given to the seller's surplus in evaluating ex-ante efficiency. The dashed line represents the probability of trade in the real bargaining (conditional on bargaining occurring). I find that the probability of trade is higher when η is closer to 0.5, and decreases slightly as η approaches 1. The probability of trade decreases dramatically as the seller's welfare weight goes to 0, and in this range of η the real-world bargaining outperforms the ex-ante efficient mechanisms in terms of the probability of trade.

6.3 Quantitative Analysis of Bargaining Efficiency

The graphical analysis in the preceding section does not capture bounds on the difference in welfare between different mechanisms; it only evaluates these mechanisms at the upper and lower bounds on seller value CDF. This section presents a quantitative analysis of the bounds on welfare and bounds on differences in welfare described in Section 5. Table 3 contains numerical values for

Table 3: Bounds on Welfare Measures

A. Levels	Ex-post	Second-best	Buyer-optimal	Seller-optimal	Real bargaining
Expected gains from trade	[2.442, 5.045] (2.184, 5.247)	[2.397, 5.029] (2.149, 5.227)	[1.776, 4.332] (1.473, 4.492)	[2.344, 4.999] (2.096, 5.207)	[1.993, 3.933] (1.812, 4.085)
Buyer gains		[0.553, 0.701] (0.521, 0.733)	[1.416, 3.962] (1.159, 4.102)	[0.474, 0.649] (0.457, 0.662)	[0.822, 0.845] (0.779, 0.913)
Seller gains		[1.844, 4.328] (1.578, 4.519)	[0.360, 0.370] (0.190, 0.441)	[1.870, 4.349] (1.612, 4.561)	[1.171, 3.088] (0.947, 3.202)
Probability of trade	[0.818, 0.871] (0.806, 0.894)	[0.699, 0.828] (0.672, 0.857)	[0.266, 0.609] (0.250, 0.637)	[0.634, 0.801] (0.627, 0.811)	[0.646, 0.646] (0.638, 0.672)

B. Differences	Ex-post minus second-best	Second-best minus real	Ex-post minus real
Expected gains from trade	[0.009, 0.059] (0.004, 0.066)	[0.377, 1.123] (0.318, 1.202)	[0.422, 1.139] (0.355, 1.219)
Probability of trade	[0.034, 0.128] (0.015, 0.142)	[0.052, 0.182] (0.026, 0.194)	[0.172, 0.225] (0.159, 0.236)

C. Random Bidder	Ex-post	Second-best	Real bargaining	Ex-post minus real
Expected gains from trade	[2.515, 4.888] (2.113, 5.049)	[2.423, 4.837] (2.026, 4.991)	[1.539, 2.684] (1.189, 2.867)	[0.976, 2.204] (0.787, 2.675)

Notes: Bounds on welfare measures under ex-post efficient, second-best, buyer-optimal, and seller-optimal mechanisms compared to real-world mechanism. Panel A displays levels and panel B displays differences. Panel C displays expected gains when a random buyer (rather than the high bidder) bargains with a seller. As explained in Section 6.4, this setting is computed by setting the auction price to a low quantile of the auction price distribution. Estimated bounds are in square braces and 95% confidence set is in parentheses. Gains are in $1,000 units.

each of these bounds in square brackets and confidence sets in parentheses.[26] Panel A displays the expected gains from trade, buyer and seller gains, and probability of trade for the ex-post efficient mechanism, the second-best mechanism, the buyer-optimal mechanism, the seller-optimal mechanism, and the real-world bargaining. Panel B displays the expected gains and probability of trade for the differences between ex-post efficiency and the second-best, the second-best and the real-world mechanism, and ex-post efficiency and the real-world mechanism. Gains are reported in units of $1,000.[27]

I begin by discussing the estimates in panel A of Table 3. The first column demonstrates that,

[26]Confidence sets are constructed using a nonparametric bootstrap of the full estimation procedure (steps 1–6) and the counterfactual computations. See Appendix D.8.

[27]I do not include the auction house revenue in the total expected gains from trade in these tables, but I do in my analysis of auction house fees in Appendix E.

in a full-information world, where ex-post efficiency would be achievable, the gain from trade for the bargaining game would lie in a range of $2,442–5,045. Taking the lower bound ($2,442) for illustrative purposes, this number suggests that if a car sells for $7,000, the seller might have been willing to sell the car for as low as $5,558 and the buyer willing to buy the car for as much as $8,000 (because $8,000 - 5,558 = 2,442$). The probability of trade under ex-post efficiency ranges from 0.818–0.871. This quantity (or rather, one minus this quantity) serves as a direct measure of the amount of overlap in mass between buyer and seller values.

The second column of panel A displays the second-best mechanism—the direct revelation mechanism maximizing the equally weighted expected gains subject to information constraints. I find that the range of surplus for this mechanism is only slightly below that of ex-post efficiency, suggesting that there is little loss due solely to incomplete information in this setting. Moving to panel B of Table 3, the results in the first column confirm this finding, where I display bounds on the difference between the ex-post and second-best gains from trade and probability of trade. These bounds indicate that the second-best gains from trade fall below ex-post efficiency by $9–59. Interestingly, however, the probability of trade in the second-best mechanism can be substantially lower than under ex-post efficiency (a lower bound of 0.699 as opposed to 0.818 in panel A, and an upper bound on the ex-post efficient probability of trade minus the second-best of 0.128 in panel B). Thus the second-best mechanism can miss out on trades that would be ex-post efficient (i.e. where the buyer values the car more than the seller), but in these missed trades the difference in values is small, and hence the surplus level is still close to ex-post efficient.

The final column of panel A indicates that the expected gains from trade in the real-world mechanism range from $1,993 to $3,933, with the buyer's expected gains lying in $822–845 and the seller's lying in $1,171–3,088. Relative to the real-world bargaining outcomes, column 3 of panel B indicates that ex-post efficiency would entail an increase in expected surplus of $422–1,139 per bargaining transaction. This surplus lost in the real bargaining represents 17–23% of the ex-post efficient surplus. This lost surplus is a deadweight loss, uncaptured by either party (or by the auction house).

Efficient bargaining would also yield a higher conversion rate. Table 3 shows that the probability of trade in the real-world bargaining is 0.646, meaning trade fails 35.4% of the time.[28] The final column of Panel B demonstrates that the probability of trade would increase by 0.172–0.225 under ex-post efficiency. This implies that 17.2–22.5% of negotiating pairs consist of cases where the buyer values the car more than the seller but trade fails. Comparing this to the overall failure rate (35.4%) suggests that approximately half (48.5–63.6%) of failed negotiations are cases where gains

[28]This estimated probability of trade comes from integrating the estimated allocation function $x^{RW}(\rho(S), B; P^A)$ over all three of its arguments, as described in Section 5. The corresponding raw probability of trade in the data for the bargaining stage can be calculated by combining periods 2 and later from Table 1, yielding 0.684. The proximity of these numbers (0.646 and 0.684) is one indication of good fit for the overall estimation exercise; Appendix D discusses other measures of fit, showing that the integrated absolute error for each estimation step is small.

from trade do exist but fail to be realized.

Panel B of Table 3 also demonstrates that the probability of trade would increase by up to 0.182 when moving from real-world bargaining to the second-best mechanism. Note that the lower bound on the improvement in the probability of trade is much lower in the second-best mechanism (0.052), implying that I cannot reject the possibility that the real bargaining achieves only a slightly lower trade volume than the second-best mechanism. This again highlights the feature that the second-best mechanism would guarantee an increase in expected surplus relative to the real bargaining by capturing more valuable trades, not necessarily a higher volume of trade. However, industry participants suggest that it is high conversion—a high probability of trade—that is the primary goal of wholesale auto auction houses (see Lacetera et al. 2016), and thus the real-world mechanism may be achieving this goal relatively well.[29]

The third and fourth columns of panel A in Table 3 display bounds on welfare outcomes under the buyer-optimal and seller-optimal mechanisms. These mechanisms also lie along the ex-ante efficient frontier, but place all of the welfare weight on one party or the other. One interesting feature of these mechanisms is that they are easy to implement; they simply require letting one party make a take-it-or-leave-it offer to the opposing party. I find that the buyer-optimal mechanism would yield a much higher payoff for the buyer ($1,416–3,962) and much lower payoff for the seller ($360–370) than under the current mechanism. The probability of trade, however, has the potential to drop as low as 0.266 under the buyer-optimal mechanism (with an upper bound of 0.609). Some of these changes are due to the fact that in this buyer-optimal bargaining the buyer is no longer forced to treat the auction price as a lower bound on the available bargaining prices. The seller-optimal mechanism would potentially yield improvements for the seller, with the seller's expected gains from trade lying in a range $1,870–4,349, and the buyer's gains in this mechanism dropping to $474–649. The probability of trade under the seller-optimal mechanism can be as low as 0.634, not nearly as low as in the buyer-optimal mechanism, and close to that of the real-world bargaining. The bounds on the total expected gains from trade in the seller-optimal mechanism are similar to those in the second-best mechanism ($2,344–4,999).

The qualitative and quantitative findings are similar in the fleet/lease sample results, discussed in Appendix C: the real-world bargaining falls short of the ex-post efficient gains by $223–834 (a 12–20% loss) and falls short of the ex-post efficient trade probability by 0.199–0.235, implying that 58.2–68.7% of failed trades are cases where the buyer actually values the car *more* than the seller.

[29] This highlights an interesting distinction between a *secret* reserve price and a *public* reserve price. In an independent private values environment, a public reserve auction is equivalent to an auction followed by the seller-optimal bargaining mechanism, and is optimal for the *seller* in most standard auction environments. A secret reserve auction with post-auction bargaining, however, may be preferred by the *intermediating platform*—in this case, the auction house—because the auction house facilitates trade, and may wish to offer a mechanism that is attractive to *both* buyers and sellers. Appendix B.5 contains further discussion of secret vs. public reserve prices.

6.4 Bargaining Between a Random Buyer and Seller

I now analyze the estimated expected gains from trade in a setting where bargaining takes place between the seller and a random buyer rather than the high-value bidder. Note that this alters the support of the types in the bargaining game for both the buyer *and* the seller, and this shifts not only the outcomes achieved in the real-world bargaining, but also the ex-ante and ex-post efficient frontiers. The allocation function for the real-world bargaining in this setting is possible to simulate by evaluating the estimated $g(r, p^A)$ function at a very small realized value of p^A (I choose the 0.001 quantile). At this small realization, the value of the buyer who bargains approximately represents a draw from the full support of buyer values, $[\underline{b}, \overline{b}]$ rather than from the truncated support used in the main analysis above, $[\underline{b}(p^A), \overline{b}]$. Similarly, the value of the seller who bargains is a draw from the full support of seller values, $[\underline{s}, \overline{s}]$ rather than from $[\underline{s}(p^A), \overline{s}]$.

Panel C of Table 3 displays the results. The range for the gains from trade under ex-post efficiency is slightly tighter than in the main results in panels A and B, and the estimated bounds on the gains in the real-world bargaining lie closer to zero than in the main results. The overall loss in efficiency between the real bargaining and ex-post efficiency is $976–2,204, which corresponds to a loss of 39–45% of the ex-post efficient surplus. This loss is much larger than the percentage loss in the main results in panels A and B, suggesting that bargaining between a random buyer and seller is more inefficient than bargaining between the high bidder and the seller. This suggests that the presence of the auction (through its roles of truncating the support of the types who arrive at bargaining and constraining from below the level of the final offer) does indeed improve the efficiency of the bargaining. The main findings of the paper still hold in this analysis: the real-world bargaining is inefficient, and nearly all of this inefficiency is due to factors beyond the information constraints highlighted in Myerson and Satterthwaite (1983), as the second-best mechanism yields outcomes that are nearly ex-post efficient.

7 Discussion and Conclusion

The finding of this paper that the ex-ante and ex-post efficient frontiers are close to one another in this market stands in stark contrast to the result in the most popularly studied theoretical example of bilateral bargaining, that of symmetric uniform values (where both buyer and seller values are uniformly distributed on the interval $[0, 1]$).[30] This case is known to yield a gap between the ex-post efficient and second-best probabilities of trade. The large gap in this special case, however, may have little bearing on the gap to be expected in real-world settings, where the features of the distribution and the extent of asymmetries may diverge far from uniformity and symmetry. Also, as the results above highlight, even in situations where some efficient trades fail to occur, many of

[30] This case is given as an example in Chatterjee and Samuelson (1983) and in Myerson and Satterthwaite (1983).

these failed trades may be cases where only small gains from trade exist (i.e. where the buyer's value is close to the seller's), and thus the loss in efficiency due to information constraints need not be large.

Overall, it is not obvious whether the results of this paper should be interpreted as implying that the real-world bargaining in this market is relatively efficient or relatively inefficient compared to other markets, particularly given that there are no existing empirical studies of bargaining with two-sided uncertainty to which these results may be compared. Estimating a structural model of one-sided uncertainty, Ambrus et al. (2018) found an efficiency loss of 14% in studying ransom negotiations, and the losses I find are similar to these (17–23% in the dealers sample and 12–20% in the fleet/lease sample). Several papers in the experimental literature can also provide an interesting comparison. Bazerman et al. (1998) argued that real-world bargaining can potentially yield *more* efficient outcomes than the theoretical ex-ante efficient frontier due to non-traditional utility functions (where one player's utility nests the other's), limits on players' abilities to mimic other types, and other features of bounded rationality; and Valley et al. (2002) document experimental evidence that communication can allow agents to outperform the ex-ante efficient frontier. In light of these arguments, the bargaining at wholesale auto auctions might be seen as relatively *inefficient* given that it falls short of that frontier at all.[31]

As discussed in the introduction, a gap between the outcome of actual bargaining and the ex-ante efficient frontier can occur for a number of reasons. First, real-world bargaining mechanisms can have multiple equilibria, many of which may be inefficient (Ausubel and Deneckere 1993), and the actions I observe in the data may correspond to one of these inefficient equilibria. Second, it may be that this particular bargaining protocol, even in its most efficient equilibria, falls short of the frontier.[32] Third, it may be the case that a gap exists because of a Wilson-doctrine-like argument: ex-ante efficient mechanisms can be unwieldy to implement in practice (in particular when $\eta \in (0,1)$). These mechanisms require that players and the mechanism designer all have knowledge of buyer and seller distributions, and furthermore that the players comprehend that it is indeed incentive compatible for them to truthfully reveal their values. The implementation of alternating-offer bargaining, on the other hand, does not require such assumptions and the rules can be easily explained to both the players and the market designer, unlike the black box that theoretical mechanisms may appear to be from a player's perspective. It may be the case that a practical,

[31]Importantly, nothing in my estimation procedure forces the real-world outcome to lie within the ex-ante efficient frontier; this is a finding implied by my estimates, not a constraint placed on the model.

[32]Ausubel and Deneckere (1993) explain one aspect of this potential efficiency loss: "In [the second-best mechanism], each player reveals his private information before hearing his opponent's report. By way of contrast, in sequential bargaining, the player who reveals second may be less apt to report truthfully than if he were still ignorant of his opponent's report. To the extent that information revelation is inhibited, this might further contribute to waste." Ausubel and Deneckere (1993) demonstrated that this loss need not occur in sequential bargaining with one-sided incomplete information when the value distribution has a monotone hazard rate (unlike the distributions I estimate). Ausubel, Cramton, and Deneckere (2002) argued that these results can extend to alternating-offer games, but no general exposition exists.

more efficient alternative to alternating-offer bargaining exists, or it may be—as hypothesized by Wilson (1986) and Ausubel and Deneckere (1993)—that "[real-world bargaining mechanisms] survive because they employ trading rules that are efficient for a wide class of environments."[33]

Actual quantitative estimates of real-world bargaining efficiency from other studies will be a welcome addition to the literature in the future for comparison to the estimates in this paper. A fruitful avenue for future empirical research would be to apply the bounding methodology developed in this paper to study the efficiency of bargaining in other settings, potentially exploiting more fully all of the offers observed in alternating-offer bargaining data—a form of data that is becoming increasingly available.[34]

My consistent finding is that the ex-ante and ex-post efficient frontiers lie close together in this market, while the real-world bargaining falls short of the ex-ante efficient frontier. This suggests that efficiency loss in this market may not be due to incomplete information alone, but to the other aspects of the real-world bargaining described above. It is important to note, however, that these other aspects all have roots in incomplete information; if players were to have complete information, many of these other barriers to efficiency might also disappear.

[33] An interesting avenue for future theory research would be to apply recent robust mechanism design techniques (surveyed in Carroll 2018) to the analysis of alternating-offer bargaining to determine whether there is a designer's objective function or information structure under which alternating-offer bargaining is preferable to other mechanisms for bilateral trade.

[34] Such data is found in Merlo and Ortalo-Magne (2004), Keniston (2011), Bagwell et al. (2017), Backus et al. (2018), and Hernandez-Arenaz and Iriberri (2018).

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