# Joint Optimization of Communication Rates and Linear Systems

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### Abstract

We consider a linear system, such as a controller or estimator, in which several signals are transmitted over communication channels with bit rate limitations. We model the effect of bit rate limited wireless channels by conventional uniform quantization, and use a standard white-noise model for quantization errors. We focus on finding the allocation of communication resources such as transmission powers, bandwidths, or time-slot fractions, that yields optimal system performance. We show that if the linear system is fixed, the problem of allocating communication resources is often convex. We discuss optimization algorithms that exploit the problem structure, and present efficient heuristics for obtaining integer-valued solutions. The problem of jointly allocating communication resources and designing the linear system is in general not convex, but can be solved heuristically in a way that exploits the problem structure and appears to work well in practice.

## 1 Introduction

We consider a linear system in which several signals are transmitted over wireless communication links, as shown in figure 1 (left). Here, $w$ is a vector of exogenous signals (such as disturbances or noises acting on the system), $z$ is a vector of performance signals (including error and actuator signals) and $y$ and $y_r$ are the signals transmitted over the communication network, and received, respectively. This arrangement can represent a variety of systems, such as distributed controllers or estimators in which sensor, actuator, or command signals are sent over wireless communication networks.

Many issues arise in the design of networked controllers and the associated communication systems, including bit rate limitations, communication delays, packet loss, transmission errors, and asynchronicity. In this paper we consider only the first issue, *i.e.*, bit rate limitations. Hence, we assume that each communication link has a fixed and known delay (which we model as part of the LTI system), does not drop packets, transfers bits without error, and operates (at least for purposes of analysis) synchronously with the linear system.

Our focus is the joint optimization of the linear system and the resource allocation in the communication network. We assume that the coding scheme and medium

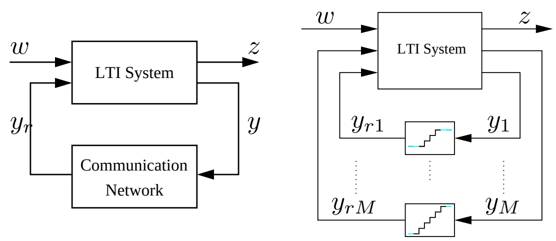


**Figure 1:** System set-up and uniform quantization model.

access control of the communication system is fixed, and concentrate on the selection of certain critical communication parameters, such as transmission powers and bandwidths allocated to the channels (or groups of channels), which in turn limit the achievable bit rates.

For a fixed sampling frequency $f_s$ of the linear system, the bit rate limit translates into a constraint on the number of bits that can be transmitted over each communication channel during one sampling period. We will assume that the individual signals $y_i$ are coded using conventional memoryless uniform quantizers; see figure 1 (right). This coding scheme is certainly not optimal (see, *e.g.*, [8]), but it is conventional, easily implemented, and allows us to use a simple and standard model for the loss of system performance due to the network communication constraints. Similar models have been used to analyze and design fixed-point implementations of filters and controllers, see, *e.g.*, [9].

We consider two specific problems in this paper. First, we assume the linear system is fixed and consider the problem of choosing the communication variables to optimize the overall system performance. We show that this problem is convex, and can be solved globally and efficiently using a dual decomposition method. The second problem we consider is the problem of jointly allocating communication resources and designing the linear system in order to optimize performance. This problem is in general not convex. We propose a heuristic approach that exploits the problem structure and appears to work well in practice.

## 2 Linear system and quantizer model

**Linear system model** We assume a synchronous, single-rate discrete-time system, described as

$$z = G_{11}(\varphi)w + G_{12}(\varphi)y_r, \quad y = G_{21}(\varphi)w + G_{22}(\varphi)y_r$$

where $G_{ij}$ are LTI operators and $\varphi \in \mathbf{R}^q$ is the vector of design parameters in the linear system that can be tuned to optimize performance. To give lighter notation, we suppress the dependence of $G_{ij}$ on $\varphi$ except when necessary. We assume that $y(t),\ y_r(t) \in \mathbf{R}^M$, *i.e.*, that $M$ scalar signals $y_1, \ldots, y_M$ are transmitted over the network during each sampling period. We also assume that the signals sent ($y$) and received ($y_r$) over the communication links are related by memoryless scalar quantization, which we describe in detail below. All communication delays are assumed constant and known and included in the LTI system.

**Unit uniform quantizer** The unit range uniform $b$-bit quantizer partitions the range $[-1, 1]$ into $2^b$ intervals of uniform width $2^{1-b}$. To each quantization interval a codeword of $b$ bits is assigned. Given the associated codeword, a numerical value $u_r$ is reconstructed by taking the midpoint of the interval corresponding to the codeword. The relationship between the original and reconstructed values can be expressed as

$$Q_b(u) = \mathbf{round}(2^{b-1}u)/2^{b-1} \tag{1}$$

for $|u| < 1$. Here, $\mathbf{round}(z)$ is the integer nearest to $z$ (with ties rounded down). The behavior of the quantizer for $|u| \geq 1$ is not specified. The details of the overflow behavior will not affect our analysis or design, since we assume by appropriate scaling (described below) that overflow does not occur, or occurs rarely enough to not affect the system performance. The associated quantization error can be expressed as

$$E_b(u) = u_r - u = \left(\mathbf{round}(2^{b-1}u) - 2^{b-1}u\right)/2^{b-1}.$$

As long as the quantizer does not overflow, the quantization error $E_b(u)$ lies in the interval $\pm 2^{-b}$.

**Scaling** To avoid overflow, each signal $y_i(t)$ is scaled by the factor $s_i^{-1} > 0$ prior to encoding with a unit uniform $b_i$-bit quantizer, and re-scaled by the factor $s_i$ after decoding (figure 2), so that

$$y_{ri}(t) = s_i Q_{b_i}(y_i(t)/s_i).$$

The associated quantization error is given by

$$q_i(t) = y_{ri}(t) - y_i(t) = s_i E_{b_i}(y_i(t)/s_i),$$

which lies in the interval $\pm s_i 2^{-b_i}$, provided $|y_i(t)| < s_i$. To minimize quantization error while ensuring no overflow (or ensuring that overflow is rare) the scale factors $s_i$ should be chosen as the maximum possible value of $|y_i(t)|$, or as a value that with very high probability is larger than $|y_i(t)|$. We will use the so-called $3\sigma$*-rule*,

$$s_i = 3\,\mathbf{rms}(y_i),$$

where $\mathbf{rms}(y_i)$ is the root-mean-square value of $y_i$. If $y_i$ has a Gaussian distribution, this scaling ensures that overflow occurs only about 0.3% of the time.

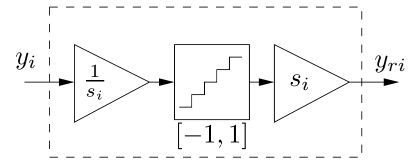


**Figure 2:** Scaling before and after the quantizer.

**White-noise quantization error model** Assuming that overflow is rare, we model the quantization errors $q_i(t)$ as independent random variables, uniformly distributed on the interval (cf. [5, Chapter 10])

$$s_i[-2^{-b_i}, 2^{-b_i}].$$

In other words, we model the effect of quantizing $y_i(t)$ as an additive white noise source $q_i(t)$, so that

$$y_{ri}(t) = y_i(t) + q_i(t)$$

where $\mathbf{E}\, q_i = 0$ and $\mathbf{E}\, q_i(t)^2 = (1/3)s_i^2 2^{-2b_i}$.

**Performance of the closed-loop system** We can express $z$ and $y$ in terms of the inputs $w$ and $q$ as

$$z = G_{zw}w + G_{zq}q, \qquad y = G_{yw}w + G_{yq}q,$$

where $G_{zw}, G_{zq}, G_{yw}$ and $G_{yq}$ are the closed-loop transfer matrices from $w$ and $q$ to $z$ and $y$, respectively. The overall variance of $z$ is given by

$$V = \mathbf{E}\,\|z\|^2 = V_q + \mathbf{E}\,\|G_{zw}w\|^2. \tag{2}$$

The variance of $z$ induced by quantization is given by

$$V_q = \mathbf{E}\,\|G_{zq}q\|^2 = \sum_{i=1}^{M} \|G_{zqi}\|^2 (1/3) s_i^2 2^{-2b_i},$$

where $G_{zqi}$ is the $i$th column of the transfer matrix $G_{zq}$, and $\|\cdot\|$ denotes the $\mathbf{L}^2$ norm (see [2, §5.2.3]). We can use $V_q$ as a measure of the effect of quantization on the overall system performance. We express $V_q$ as

$$V_q = \sum_{i=1}^{M} a_i 2^{-2b_i}, \tag{3}$$

where $a_i = (1/3)\|G_{zqi}\|^2 s_i^2$. This expression shows how $V_q$ depends on the allocation of quantizer bits $b_1, \ldots, b_M$, as well as the scalings $s_1, \ldots, s_M$ and LTI system parameters (which affect the $a_i$'s). While the formula (3) was derived assuming that $b_i$ are integers, it makes sense for $b_i \in \mathbf{R}$.

## 3 Communications model and assumptions

The theoretical capacity of wireless communication links depends on the media access scheme and the selection of certain critical parameters, such as transmission powers and bandwidths or time-slot fractions allocated to individual channels (or groups of channels). We refer to these parameters collectively as *communication variables*, and denote the vector of communication variables by $\theta$. The communication variables are themselves limited by various resource constraints, such as limits on the total power or bandwidth available. We will assume that the medium access methods and coding and modulation schemes are fixed, but that we can optimize over the communication variables $\theta$.

We let $b \in \mathbf{R}^M$ denote the vector of bits allocated to each quantized signal, and $r \in \mathbf{R}^M$ the vector of associated communication rates (in bits per second). Then $b_i = \alpha r_i$, where $\alpha = c_s/f_s$, $f_s$ is the sample frequency, and $c_s$ is the channel coding efficiency in source bits per transmission bit. This allows us to express capacity constraints in terms of bit allocations rather than communication rates.

We will use the following general model to relate the bit allocations $b$ and the communication variables $\theta$:

$$\begin{array}{rcll} f_i(b,\theta) & \leq & 0, & i=1,\ldots,m_f \\ h_i^T\theta & \leq & d_i, & i=1,\ldots,m_h \\ \theta_i & \geq & 0, & i=1,\ldots,m_\theta \\ \underline{b}_i \;\leq\; b_i & \leq & \overline{b}_i, & i=1,\ldots,M \end{array} \tag{4}$$

We make the following assumptions:

- The functions $f_i$ are convex functions of $(b,\theta)$, monotone increasing in $b$ and monotone decreasing in $\theta$. These inequalities describe capacity constraints on individual links or groups of links.
- The second set of constraints describes resource limitations, such as a total available power or bandwidth. We assume the vectors $h_i$ have nonnegative entries and that $d_i$, which represent resource limits, are positive.
- The third constraint specifies that the communication resource variables are nonnegative.
- The last group of inequalities specify lower and upper bounds for each bit allocation. We assume that $\underline{b}_i$ and $\overline{b}_i$ are nonnegative integers. The lower bounds ensure that the white noise model for quantization errors is reasonable while the upper bounds may arise from hardware limitations.

This generic model will allow us to formulate the communication resource allocation problem, *i.e.*, the problem of choosing $\theta$ to optimize overall system performance, as a convex optimization problem. The important additional constraint that $b$ should be integer valued will be addressed in §4.1.

### 3.1 Capacity constraints

In this section, we describe some simple channel models, showing how they fit the generic model (4). More detailed descriptions of these channel models, as well as derivations, can be found in, *e.g.*, [4, 6].

**Gaussian channel** We start by considering a single Gaussian channel. The communication variables are the bandwidth $W > 0$ and transmission power $P > 0$. Let $N$ be the power spectral density of the additive white Gaussian noise at the front-end of the receiver. The Shannon capacity is given by ([4])

$$R = W \log_2\left(1 + \frac{P}{NW}\right)$$

(in bits per second). The achievable communication rate $r$ is bounded by this channel capacity, *i.e.*, we must have $r \leq R$. In terms of $b, P$ and $W$, we have

$$f(b,W,P) = b - \alpha W \log_2\left(1 + \frac{P}{NW}\right) \leq 0,$$

which fits the generic form (4). It can be verified that $f$ is convex and monotone increasing in $b$, and monotone decreasing in $W$ and $P$.

**Gaussian broadcast channel with FDMA** In the Gaussian broadcast channel with frequency-domain multiple access (FDMA), a transmitter sends information to $n$ receivers over disjoint frequency bands with bandwidths $W_i > 0$. The communication parameters are the bandwidths $W_i$ and the transmit powers $P_i > 0$ for each individual channel. The communication variables are constrained by a total power limit

$$P_1 + \cdots + P_n \leq P_{\mathrm{tot}}$$

and a total available bandwidth limit

$$W_1 + \cdots + W_n \leq W_{\mathrm{tot}},$$

which have the generic form for resource limits. The receivers are subject to independent white Gaussian noises with power spectral densities $N_i$. The transmitter assigns power $P_i$ and bandwidth $W_i$ to the $i$th receiver. The achievable bit rates $b$ are constrained by the Shannon capacity, *i.e.*,

$$b_i \leq \alpha W_i \log_2\left(1 + \frac{P_i}{N_i W_i}\right), \qquad i=1,\ldots,n. \tag{5}$$

Again, the constraints relating $b$ and $\theta = (P, W)$ have the generic form (4).

**Gaussian multiple access channel with CDMA** In this model, $n$ transmitters, each with power $P_i$, send information to a common receiver which is corrupted by additive white Gaussian noise of power density $N$. For code-division multiplexing, the achievable rates $b$ satisfy the set of constraints

$$\sum_{i\in Z} b_i \leq \alpha W \log_2\left(1 + \frac{\sum_{i\in Z} P_i}{NW}\right), \ \forall\ Z \subseteq \{1,\ldots,n\}.$$

The communication variables here are the transmission powers $P_i$, which satisfy $0 \leq P_i \leq \bar{P}_i$ where $\bar{P}_i$ is the upper bound for $P_i$, or a total power limit. These inequalities also have the generic convex form (4).

**Gaussian multiple access channel with FDMA** In a Gaussian multiple access channel with FDMA, each transmitter sends information in disjoint frequency bands with bandwidth $W_i$. The achievable bit rates are determined by the constraints

$$b_i \leq \alpha W_i \log_2 \left(1 + \frac{P_i}{NW_i}\right), \qquad i = 1, \dots, n.$$

Here the communication variables are the powers $P_i$ and bandwidths $W_i$, limited by separate or total power constraints, and a total bandwidth constraint.

**Variations and extensions** Channels with time-varying gain variations (fading) as well as rate constraints based on bit error rates (with or without coding) can be formulated in a similar manner; see, *e.g.*, [7]. We can also combine the channel models described above to model more complex communication systems, where different groups of channels may have separate or total power and bandwidth constraints.

## 4 Resource allocation for fixed linear system

In this section, we assume that the linear system is fixed and consider the problem of choosing the communication variables to optimize the system performance. We take as the objective (to be minimized) the variance of the performance signal $z$, given by (2). Since the variance induced by $w$ is independent of the communication variables, we can just as well minimize the quantization-induced variance of $z$, *i.e.*, the quantity $V_q$ defined in (3). This leads to the optimization problem

$$\begin{array}{lll} \text{minimize} & \sum_{i=1}^{M} a_i 2^{-2b_i} & \\ \text{subject to} & f_i(b,\theta) \leq 0, & i = 1, \dots, m_f \\ & h_i^T \theta \leq d_i, & i = 1, \dots, m_h \\ & \theta_i \geq 0, & i = 1, \dots, m_\theta \\ & \underline{b}_i \leq b_i \leq \overline{b}_i, & i = 1, \dots, M \end{array} \qquad (6)$$

where the optimization variables are $\theta$ and $b$. Since the objective function, and each constraint function in the problem (6) is convex, this is a convex optimization problem (ignoring the integrality constraint on $b$). This means that it can be solved globally and efficiently using a variety of methods (see, *e.g.*, [3]).

In many cases, we can solve the problem (6) more efficiently by exploiting its special structure. The objective function in the communication resource allocation problem (6) is separable, *i.e.*, a sum of functions of individual $b_i$'s. In addition, the constraint functions $f_k(b, \theta)$ usually involve only one $b_i$ and a few components of $\theta$ (since the channel capacity is determined by the bandwidth, power, or time-slot fraction, for example, allocated to that channel). Thus, the resource allocation problem (6) is almost separable; small groups of variables are coupled through the constraints $h_i^T \theta \leq d_i$ that limit the total power, total bandwidth, or total time-slot fractions. This structure can be efficiently exploited using a technique called dual decomposition (see, *e.g.*, [3, 1]). The dual problem can be solved in time linear in $M$, which is far better than the standard convex optimization methods applied to the primal problem, which require time proportional to $M^3$. More details are given in the full length version of this paper [10].

### 4.1 Integrality of bit allocations

We now return to the requirement that the bit allocations must be integers. The first thing we observe is that we can always round down the bit allocations found by solving the convex problem to the nearest integers. Let $b_i$ denote the optimal solution of the convex resource allocation problem (6) and define $\tilde{b}_i = \lfloor b_i \rfloor$. Here, $\lfloor b_i \rfloor$ denotes the *floor* of $b_i$, *i.e.*, the largest integer smaller than or equal to $b_i$. Since $f_k$ and $h_k$ are monotone increasing in $b$, and $\tilde{b} \leq b$, the rounded solution $\tilde{b}$ is always feasible.

We can also obtain a crude performance bound for $\tilde{b}$. Clearly the objective value $J_{\text{cvx}}$ obtained by ignoring the integer constraint is a lower bound on $J_{\text{opt}}$. Let the objective value of the rounded-down feasible bit allocation $\tilde{b}$ be $J_{\text{rnd}}$. It is easily shown that

$$J_{\text{cvx}} \leq J_{\text{opt}} \leq J_{\text{rnd}} \leq 4 J_{\text{opt}},$$

*i.e.*, the performance of the suboptimal integer allocation obtained by rounding down is never more than a factor of four worse than the optimal solution.

**Variable threshold rounding** Of course, far better heuristics can be used to obtain better integer solutions. Here we give a simple method based on a variable rounding threshold. Let $0 < t \leq 1$ be a threshold parameter, and round $b_i$ as follows:

$$\tilde{b}_i = \begin{cases} \lfloor b_i \rfloor, & \text{if } b_i - \lfloor b_i \rfloor \leq t, \\ \lceil b_i \rceil, & \text{otherwise.} \end{cases} \qquad (7)$$

Here, $\lceil b_i \rceil$ denotes the *ceiling* of $b_i$, *i.e.*, the smallest integer larger than or equal to $b_i$. In other words, we round $b_i$ down if its remainder is smaller than or equal to the threshold $t$, and round up otherwise.

For a given fixed threshold $t$, we can round the $b_i$'s as in (7), and then solve a convex feasibility problem over the remaining continuous variables $\theta$:

$$\begin{array}{rcl} f_i(\tilde{b}, \theta) & \leq & 0 \\ h_i^T \theta & \leq & d_i \\ \theta_i & \geq & 0 \end{array} \qquad (8)$$

The upper and lower bound constraints $\underline{b}_i \leq \tilde{b}_i \leq \overline{b}_i$ are automatically satisfied because $\underline{b}_i$ and $\overline{b}_i$ are integers. If this problem is feasible, then the rounded $\tilde{b}_i$'s and the corresponding $\theta$ are suboptimal solutions to the integer constrained bit allocation problem. Since $f_i$ is monotone increasing in $b$, hence in $t$, and monotone decreasing in $\theta$, there exists a $t^\star$ such that (8) is feasible if $t \geq t^\star$ and infeasible if $t < t^\star$. The optimal rounding threshold $t^\star$ can be found by bisection.

### 4.2 Example: a networked linear estimator

To illustrate the ideas, we consider the problem of designing a networked linear estimator with the structure shown in figure 3. We want to estimate an unknown point $x \in \mathbf{R}^{20}$ using $M = 200$ linear sensors

$$y_i = c_i^T x + v_i, \qquad i = 1, \ldots, M.$$

Each sensor uses $b_i$ bits to code its measurement, and transmits the coded signal to the central estimator over a Gaussian multiple access channel with FDMA. The performance of the estimator is evaluated by the estimation error variance $J_K = \mathbf{E}\,\|\widehat{x} - x\|^2$.

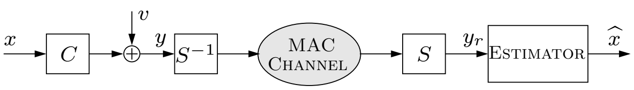


**Figure 3:** Networked estimator over MAC channel.

We assume that $\|x\| \leq 1$ and that the sensor noises $v_i$ are IID with $\mathbf{E}\, v_i = 0$, $\mathbf{E}\, v_i^2 = 10^{-6}$. In this example, the sensor coefficients $c_i$ are uniformly distributed in direction with $\|c_i\|$ uniformly distributed on $[0, 5]$. Since $\|x\| \leq 1$, we choose scaling factors $s_i = \|c_i\|$.

The noise power density of the Gaussian multiple access channel is $N = 0.1$, the coding constant is $\alpha = 2$, and the upper and lower bounds for bit allocations are $\underline{b} = 5$ and $\overline{b} = 12$. The total available power is $P = 300$ and the total available bandwidth is $W = 200$.

The estimator is a linear unbiased estimator

$$\widehat{x} = K y_r,$$

where $KC = I$, with $C = [c_1, \cdots, c_M]^T$. In particular, the minimum variance estimator is given by

$$K = (C^T (R_v + R_q)^{-1} C)^{-1} C^T (R_v + R_q)^{-1} \qquad (9)$$

where $R_v$ and $R_q$ are the covariance matrices for the sensor noises and quantization noises, respectively. (Note that the estimator gain depends on the bit allocations) The associated estimation error variance is

$$J_K(b) = \frac{1}{3} \sum_{i=1}^{M} s_i^2 \|k_i\|^2 2^{-2b_i} + \mathbf{Tr}(K R_v K^T).$$

where $k_i$ is the $i$th column of the matrix $K$. Clearly, $J_K(b)$ is in the form of (2), and will serve as the objective function for the resource allocation problem (6).

First we allocate power and bandwidth evenly to all sensors, which results in $b_i = 8$ for each sensor. Based on this allocation, we compute the quantization noice variances $\mathbf{E}\, q_i^2 = (1/3) s_i^2 2^{-2b_i}$ and design a least-squares estimator as in (9). The resulting RMS estimation error is $3.676 \times 10^{-3}$. Then we fix the estimator gain $K$, and solve the relaxed optimization problem (6) to find the resource allocation that minimizes the estimation error variance. The resulting RMS value is $3.1438 \times 10^{-3}$. Finally, we perform a variable threshold rounding with $t^\star = 0.4211$. Figure 4 shows the distribution of rounded bit allocation. The resulting RMS estimation error is $3.2916 \times 10^{-3}$. Thus, the allocation obtained from optimization and variable threshold rounding give a 10% improved performance compared to the uniform resource allocation, which is not very far from the performance bound given by the relaxed convex optimization problem.

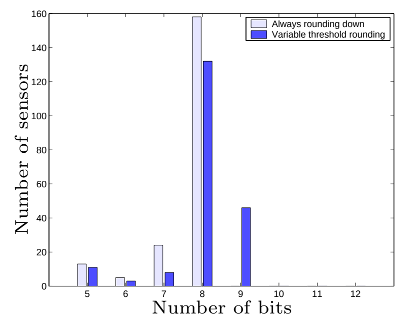


**Figure 4:** Bit allocation for networked estimator.

## 5 Joint optimization of communication and linear system variables

When the linear system is fixed, the problem of optimally allocating communication resources (ignoring integrality of bit allocation) is convex and can be efficiently solved. In order to achieve the optimal system performance, however, one should optimize the parameters of the linear system *and* the communication system *jointly*. Unfortunately, this joint design problem is in general not convex.

In some cases, however, the joint design problem is convex in subsets of the variables. For example (and ignoring the integrality constraints) the globally optimal communication variables can be computed very efficiently, sometimes even semi-analytically, when the linear system is fixed. Similarly, when the communication variables are fixed, we can (sometimes) compute the globally optimal variables for the linear system. Finally, when the linear system variables and the communication variables are fixed, it is straightforward to compute the quantizer scalings using the $3\sigma$-rule. This makes it natural to apply an approach where we sequentially fix one set of variables and optimize over the others:

**given** initial linear system variables $\phi^{(0)}$, communication variables $\theta^{(0)}$, scalings $s^{(0)}$
**repeat**

1. Fix $\phi^{(k)}$, $s^{(k)}$, and optimize over $\theta$. Let $\theta^{(k+1)}$ be the optimal value.
2. Fix $\theta^{(k+1)}$, $s^{(k)}$, and optimize over $\phi$. Let $\phi^{(k+1)}$ be the optimal value.
3. Fix $\phi^{(k+1)}$, $\theta^{(k+1)}$. Let $s^{(k+1)}$ be appropriate scaling factors.

**until** convergence

Since the joint problem is not convex, there is no guarantee that this heuristic converges to the global optimum. On the other hand the method appears to work well in practice.

### 5.1 Example: networked linear estimator

To demonstrate the heuristic method for joint optimization described above, we apply it to the networked linear estimator described in §4.2. The algorithm converges in six iterations, and we obtain very different resource allocation results from before. Figure 5 shows the distribution of the rounded bit allocations. This result is intuitive: try to assign as much resources as possible to the best sensors, and give the bad sensors the minimum number of bits. The RMS estimation error of the joint design is $0.721 \times 10^{-3}$, which is an 80% reduction compared to the result in §4.2. After applying the variable threshold rounding, we find an integer-valued bit allocation with RMS estimation error of $0.7221 \times 10^{-3}$, which is very close to the objective value of the relaxed problem.

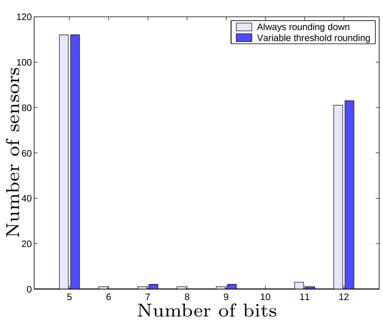


**Figure 5:** Joint optimization of network and estimator.

## 6 Conclusions

We have considered the problem of jointly optimizing the parameters of a linear system and the resource allocation in the associated communication system. To model the influence of communication rate allocations on the performance of the linear system, we assumed conventional uniform quantization and used a simple white noise model of the quantization errors. First, we assumed the linear system to be fixed and considered the problem of choosing the communication variables to optimize the overall system performance. We observed that this problem is often convex (ignoring the integrality constraint) hence readily solved. Moreover, for many important channel models, the communication resource allocation problem is separable except for a small number of constraints on the total communication resources. We described how dual decomposition can be used to solve this class of problems efficiently, and gave a variable threshold rounding method to deal with the integrality of bit allocations. Finally, we considered joint allocation of communication resources and design of the linear system. This problem is in general not convex. However, it is often convex in subsets of variables. We gave an iterative heuristic method for the joint design problem that exploits this special structure.

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