Design of Robust Global Power and Ground Networks

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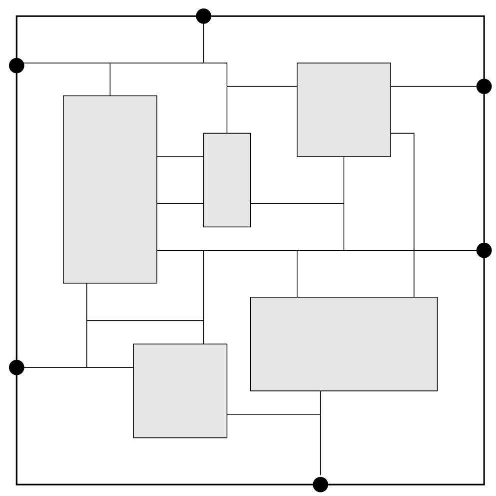
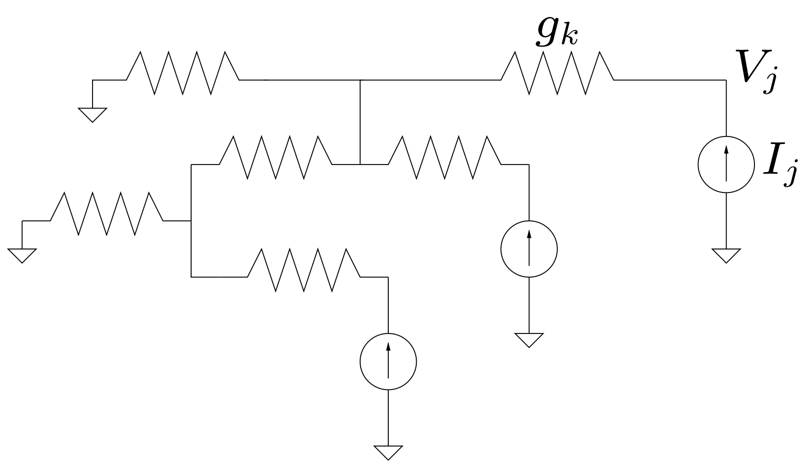
Global power & ground network design



Problem: size wires (choose topology)

- minimize wire area subject to node voltage, current density constraints
- don't consider fast dynamics (C,L)
- **do consider (slow) variation in block currents**

(Quasi-)static model



- segment conductance $g_k = w_k/(\rho l_k)$; current density $j_k = i_k/w_k$
- conductance matrix $G(w) = \sum_k w_k a_k a_k^T$; node voltages $V = G(w)^{-1} I$
- **statistical model for block currents:** $\mathbf{E}\, I I^T = \Gamma$
  - Γ is block current correlation matrix
  - $\Gamma_{jj}^{1/2} = \mathrm{RMS}(I_j)$; Γ_{ij} gives correlation between I_i, I_j

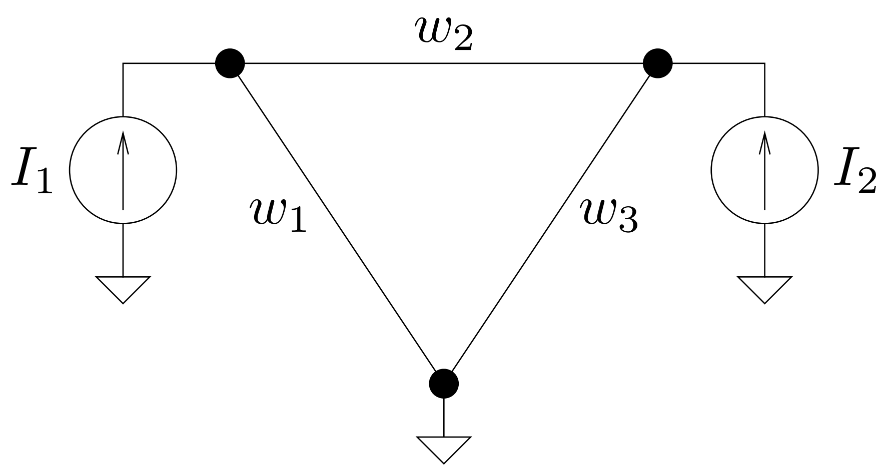
Sizing problem

$$
\begin{array}{lll}
\text{minimize} & A = \sum_k l_k w_k & \text{(area)} \\
\text{subject to} & V_j \leq V_{\max} & \text{(node voltage limit)} \\
& \mathbf{E}\, j_k^2 \leq j_{\max}^2 & \text{(RMS current density limit)} \\
& w_k \geq 0 & \text{(nonneg. wire widths)}
\end{array}
$$

can't solve, except special case I constant

- (Erhard & Johannes) can improve any mesh design by pruning to a tree
- (Chowdhury & Breuer) can size P&G trees via geometric programming

Meshes, trees and current variation



- I_1, I_2 constant (or highly correlated): set $w_2 = 0$ (yields tree)
- I_1, I_2 anti-correlated: better to use $w_2 > 0$ (yields mesh)

Average power formulation

- power dissipated in wires: $P = V^T I = I^T G(w)^{-1} I$
- average power: $\mathbf{E}\, P = \mathbf{E}\, I^T G(w)^{-1} I = \mathbf{Tr}\, G(w)^{-1} \Gamma$

$$\begin{array}{ll} \text{minimize} & \mathbf{Tr}\, G(w)^{-1}\Gamma + \mu \sum_k l_k w_k \quad \text{(average power } +\mu\cdot\text{area)} \\ \text{subject to} & w_k \geq 0 \end{array}$$

- parameter $\mu > 0$ trades off average power, area
- nonlinear but **convex problem**, readily (globally) solved
- indirectly limits $\mathbf{E}\, j_k^2$, V_j

Properties of solution

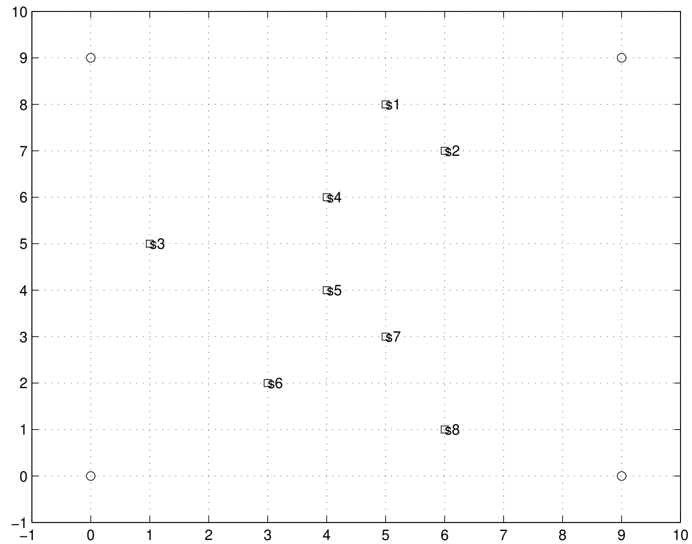
observation: many w_k's are zero, $i.e.$, many wires aren't used
average power formulation can be used for **P&G topology selection:**

- start with lots of (potential) wires
- let average power formulation choose among them
- topology (given by nonzero w_k) independent of μ

resulting current density and node voltages:

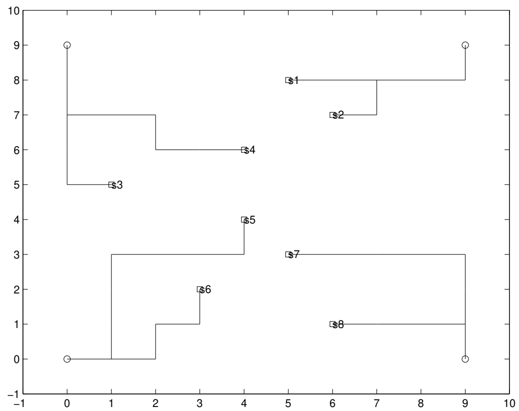
- RMS current density is equal in all (nonzero) segments
  in fact $\mu = \rho j_{\max}^2$ yields $\mathbf{E}\, j_k^2 = j_{\max}^2$ in all (nonzero) segments
- observation: V_j are small

Example



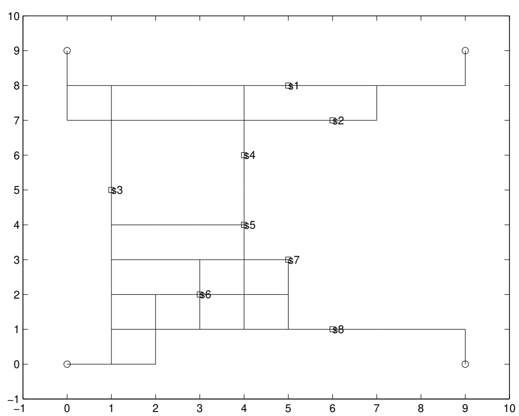
- 10×10 grid, each node connected to neighbors (180 segments)
- 8 current sources, $I \in \mathbf{R}^8$ is random with three possible values
- 4 ground pins on the perimeter (at corner points)

design for constant currents (with same RMS values)



- a tree; each source connected to nearest ground pin
- RMS current density 1, area = 448, max. voltage = 7.7

design via average power formulation



- mesh, not a tree
- RMS current density 1, area = 347, max. voltage = 5.7

Barrier method

use Newton's method to minimize

$$\mathbf{Tr}\, G(w)^{-1}\Gamma + \mu l^T w - \beta^{(i)} \sum_k \log w_k$$

- barrier term $-\beta \sum_k \log w_k$ ensures $w_k > 0$
- solve for decreasing sequence of $\beta^{(i)}$
- can show $w^{(i)}$ is at most $n\beta^{(i)}$ suboptimal
- $O(n^3)$ cost per Newton step

works very well for $n < 1000$ or so; easy to add other convex constraints

Pruning

- often clear in few iterations which w_k are converging to 0
- removing these w_k early greatly speeds up convergence
- sizes 1000s of w_ks in minutes

Where Γ comes from

- from simulation: $\Gamma = \dfrac{1}{T_{\text{sim}}} \displaystyle\int_0^{T_{\text{sim}}} I(t) I(t)^T \, dt$

- or, from block RMS currents and estimates of correlation:

$$\Gamma_{ij} = \mathsf{RMS}(I_i)\ \mathsf{RMS}(I_j)\ \rho_{ij}$$

- can use eigenvalue decomposition to simplify Γ

$$\Gamma = \sum_i \lambda_i q_i q_i^T, \qquad \hat{\Gamma} = \sum_{i=1}^{r} \lambda_i q_i q_i^T$$

(reduced rank approximation speeds up avg. pwr. solution)

Conclusion

- P&G meshes outperform trees when current variation taken into account
- Average power formulation
  - yields tractable convex optimization problem
  - chooses topology
  - guarantees RMS current density limit
  - indirectly limits node voltages