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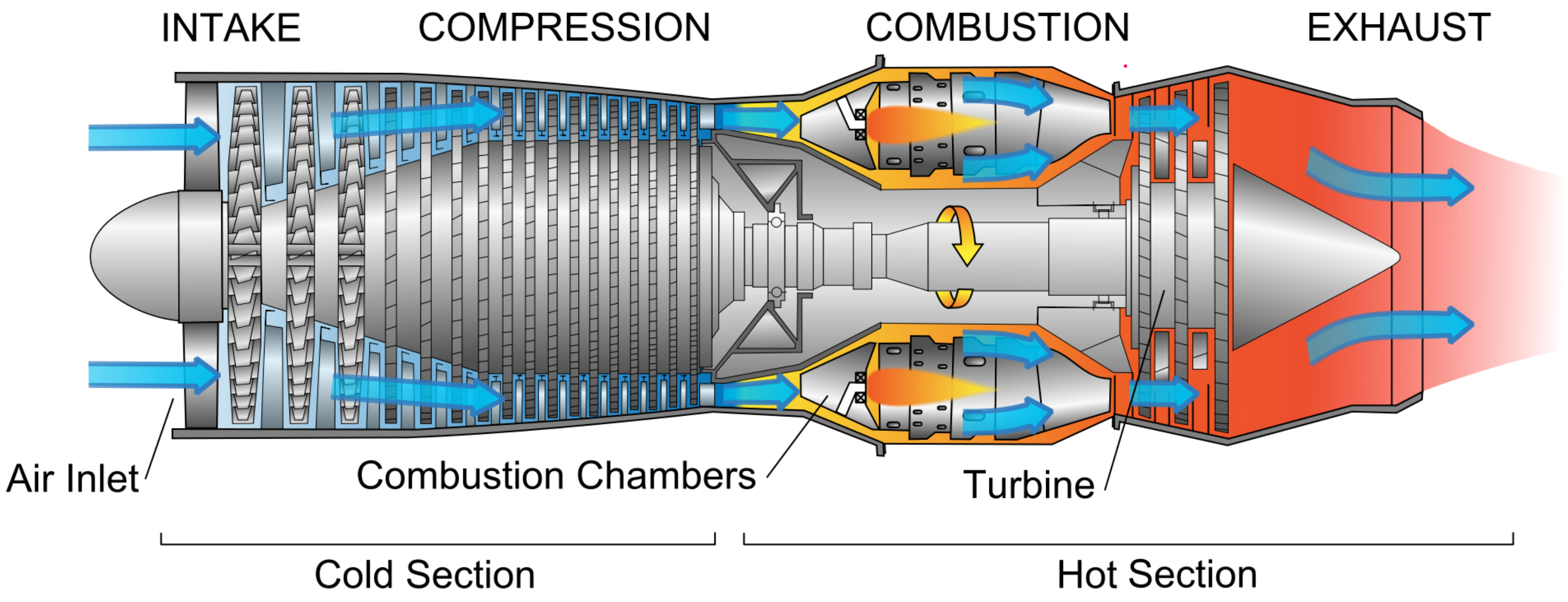
AA103
Air and Space Propulsion

Topic 12 - The Turbojet Cycle

Suggested reading – AA283 Course reader Chapter 4

Turbojet cutaway schematic



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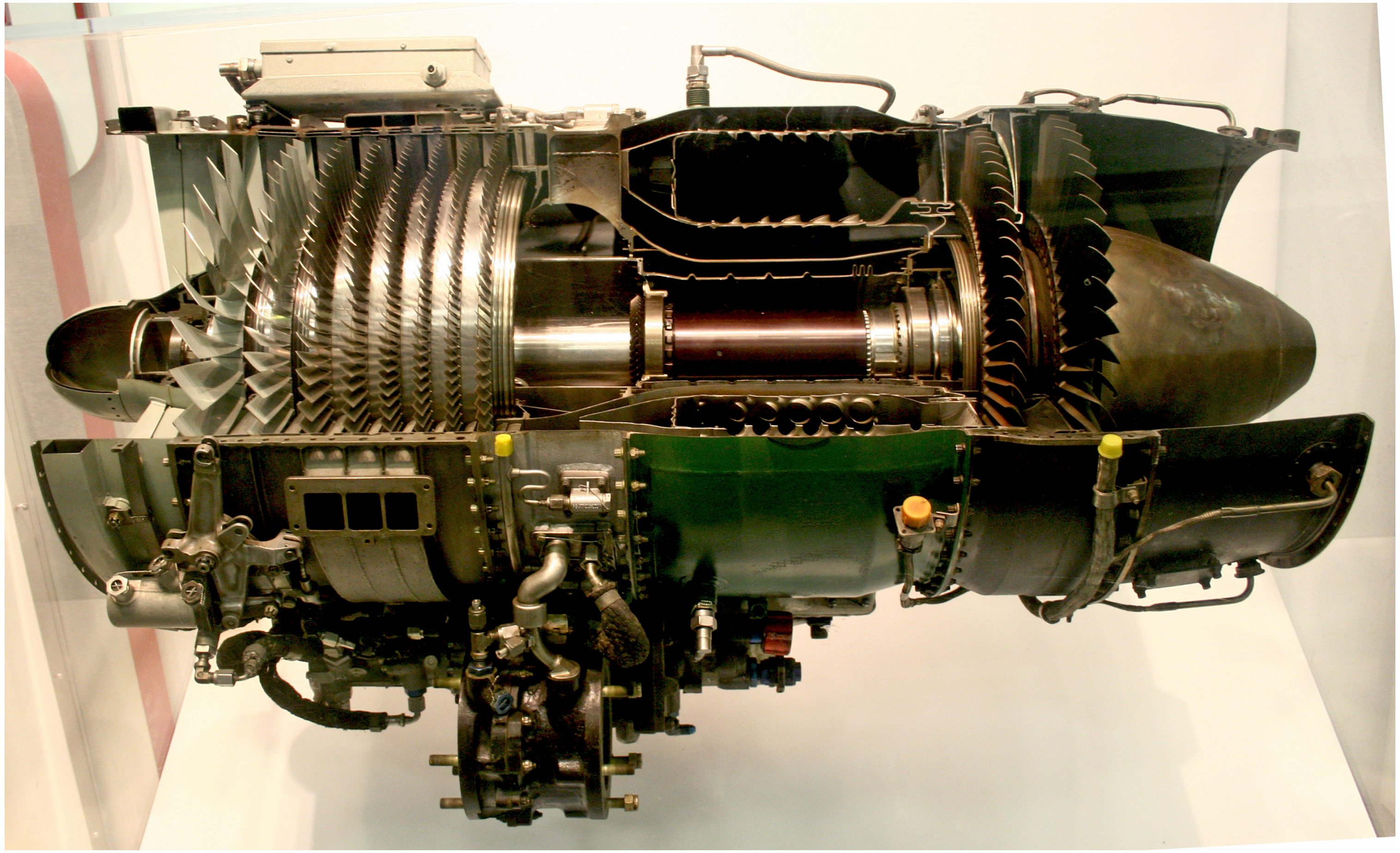
Cutaway of a GE J85 turbojet



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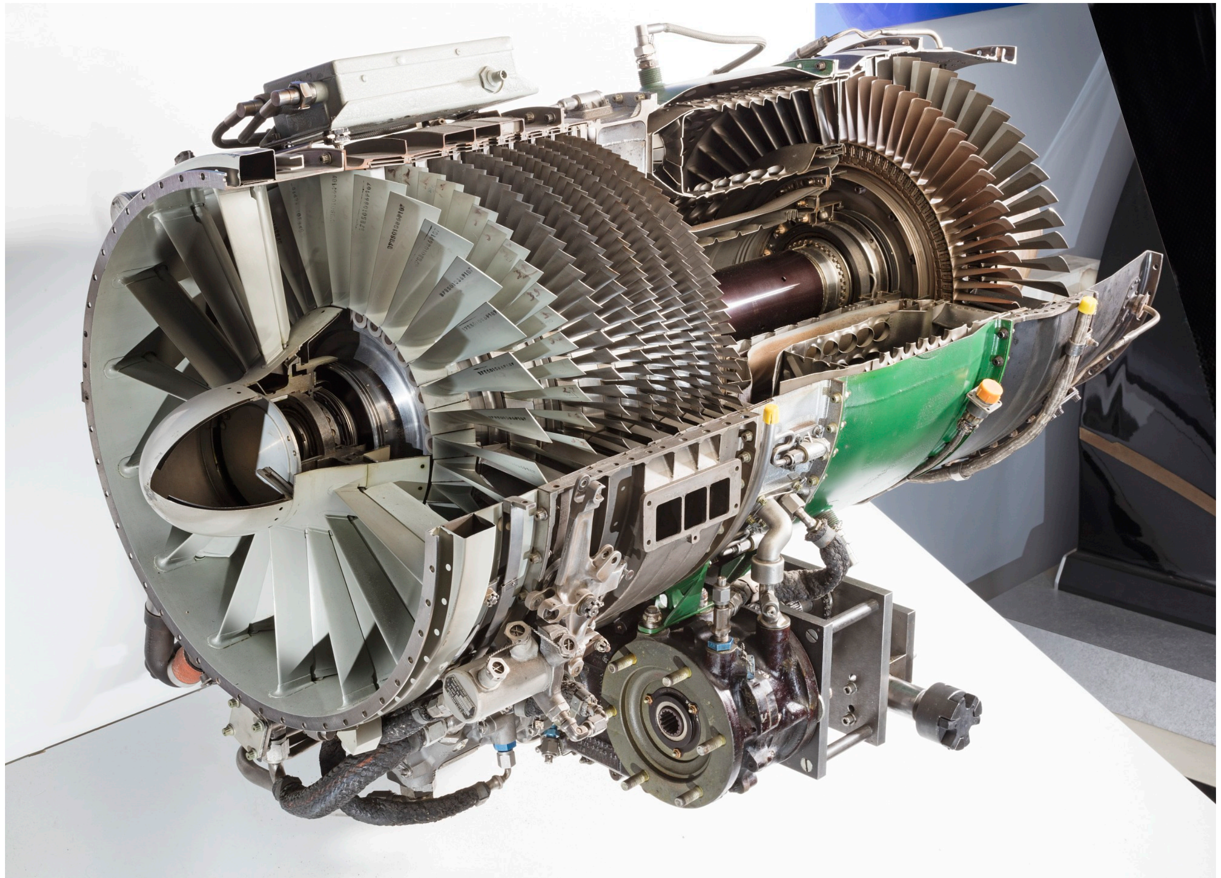
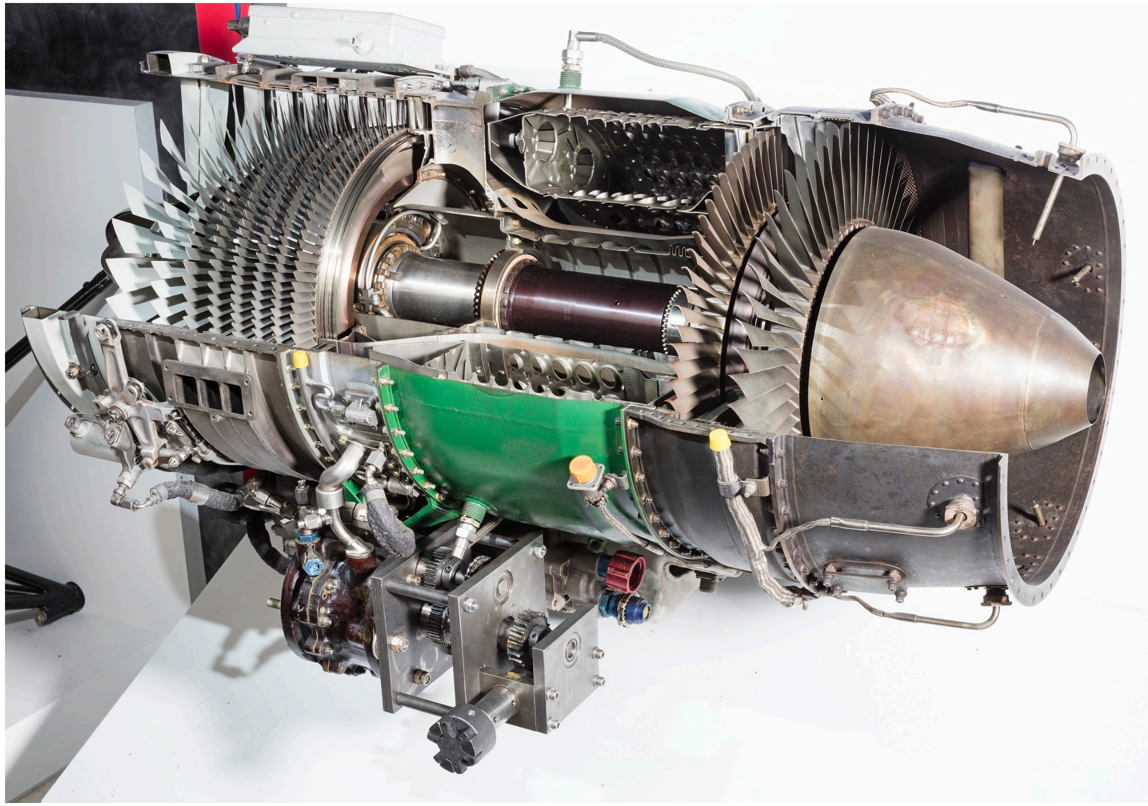
Cutaway of a GE J85 turbojet





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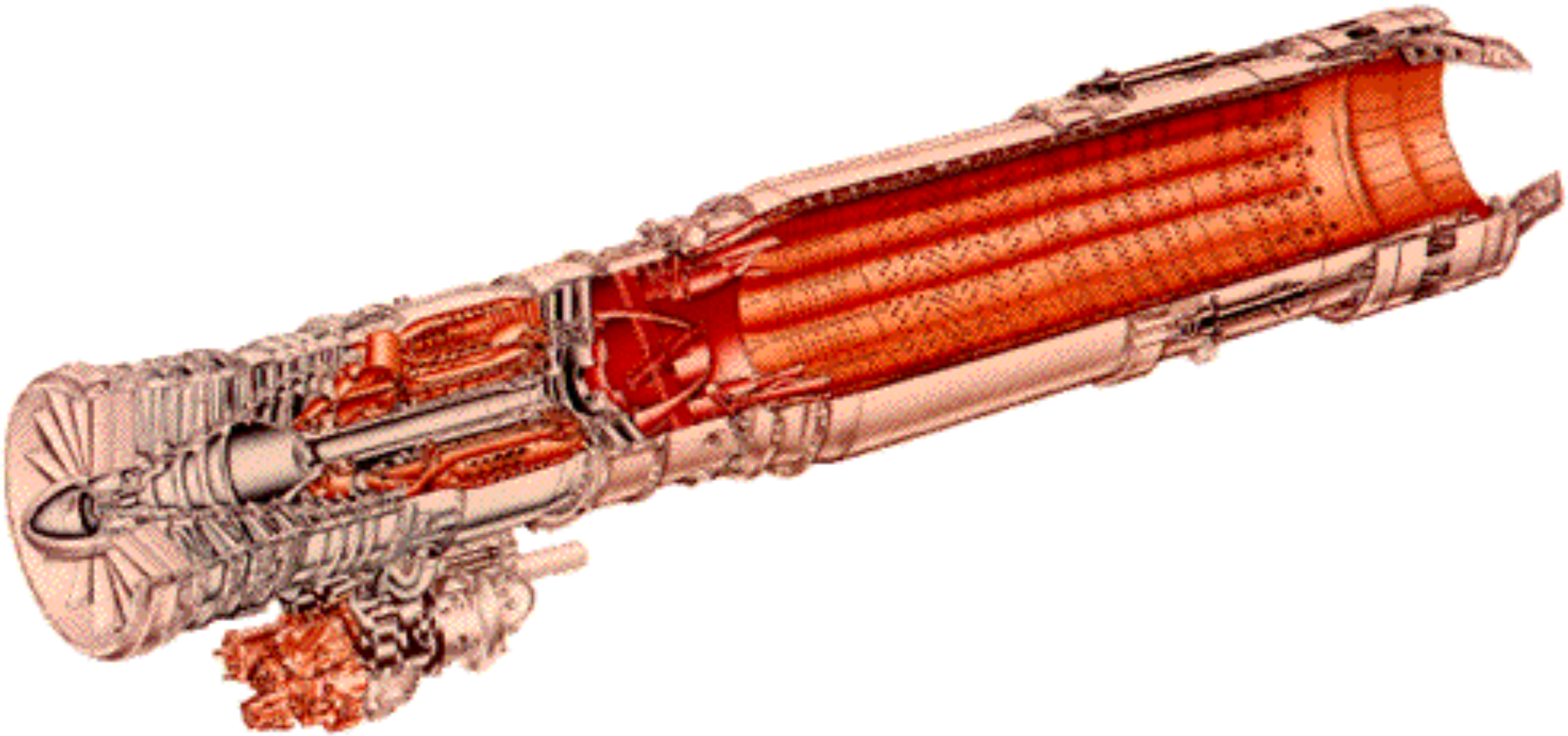
GE J85 turbojet with afterburner



J85 applications

F - 5, T - 38

White Knight

Thermal efficiency of the ideal turbojet

Recall the definition from Chapter 2

$$\eta_{th} = \frac{Power\ to\ the\ vehicle + \dfrac{\Delta\ kinetic\ energy\ of\ air}{second} + \dfrac{\Delta\ kinetic\ energy\ of\ fuel}{second}}{\dot{m}_f h_f} \qquad (4.1)$$

$$\eta_{th} = \frac{TU_0 + \left[\dfrac{\dot{m}_a(U_e - U_0)^2}{2} - \dfrac{\dot{m}_a(0)^2}{2}\right] + \left[\dfrac{\dot{m}_f(U_e - U_0)^2}{2} - \dfrac{\dot{m}_f(U_0)^2}{2}\right]}{\dot{m}_f h_f}. \qquad (4.2)$$

If the nozzle is fully expanded

$$\eta_{th} = \frac{(\dot{m}_a + \dot{m}_f)\dfrac{U_e^{\ 2}}{2} - \dot{m}_a\dfrac{U_0^{\ 2}}{2}}{\dot{m}_f h_f} \qquad (4.3)$$

Rearrange

$$\eta_{th} = \frac{(\dot{m}_a + \dot{m}_f)\frac{U_e^2}{2} - \dot{m}_a\frac{U_0^2}{2}}{\dot{m}_f h_f} = \frac{(\dot{m}_a + \dot{m}_f)(h_{te} - h_e) - \dot{m}_a(h_{t0} - h_0)}{(\dot{m}_a + \dot{m}_f)h_{te} - \dot{m}_a h_{t0}}$$

$$\eta_{th} = 1 - \frac{Q_{rejected\ during\ the\ cycle}}{Q_{input\ during\ the\ cycle}} = 1 - \frac{(\dot{m}_a + \dot{m}_f)h_e - \dot{m}_a h_0}{(\dot{m}_a + \dot{m}_f)h_{te} - \dot{m}_a h_{t0}} \qquad (4.4)$$

$$\eta_{th} = 1 - \frac{T_0}{T_{t0}}\left\{\frac{(1+f)\frac{T_e}{T_0} - 1}{(1+f)\frac{T_{te}}{T_{t0}} - 1}\right\}$$

For the ideal ramjet the factor in brackets is one.

$$\eta_{th} \cong 1 - \frac{T_0}{T_{t0}} = 1 - \frac{1}{\tau_r} = \frac{\left(\frac{\gamma - 1}{2}\right)M_0^2}{1 + \left(\frac{\gamma - 1}{2}\right)M_0^2}. \qquad (4.5)$$

For the ideal ramjet the thermal efficiency is determined by the flight Mach number.

If the Mach number is zero then the thermal efficiency is Zero and there is no thrust!

The turbojet uses a compressor to produce compression at zero Mach number.

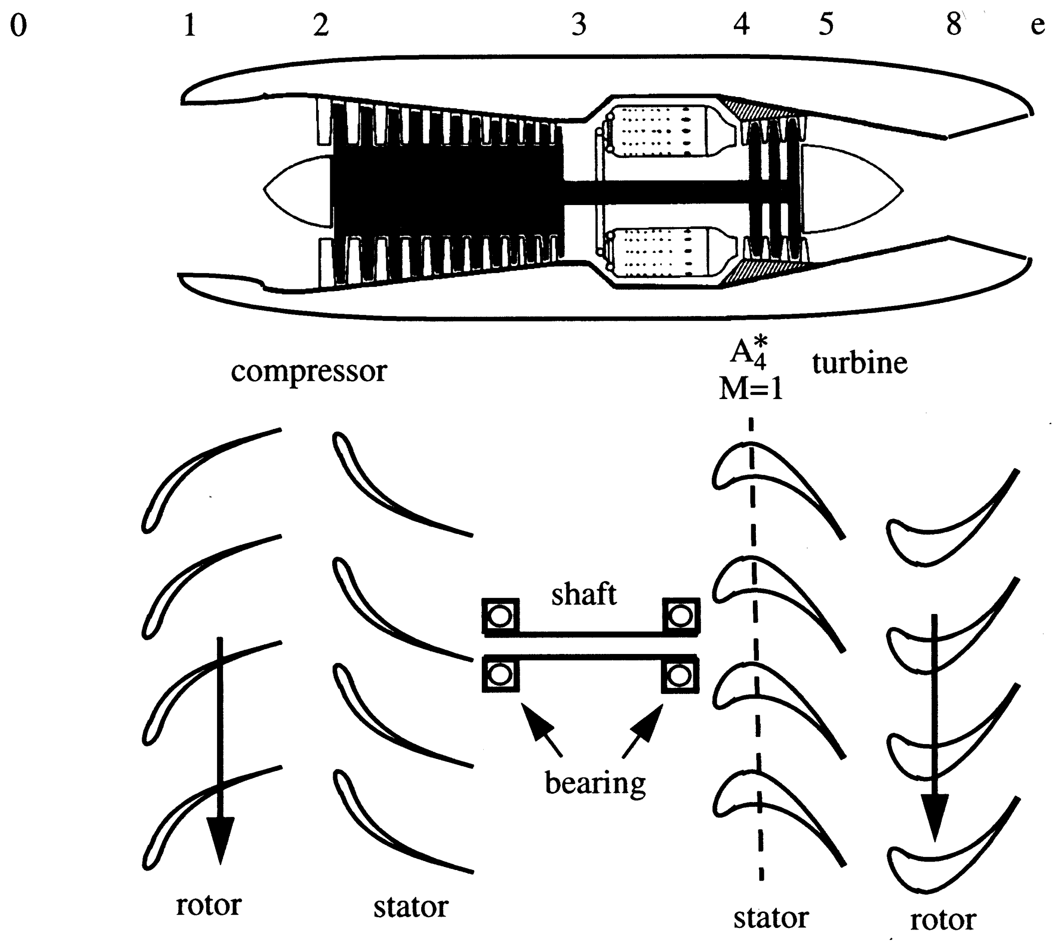
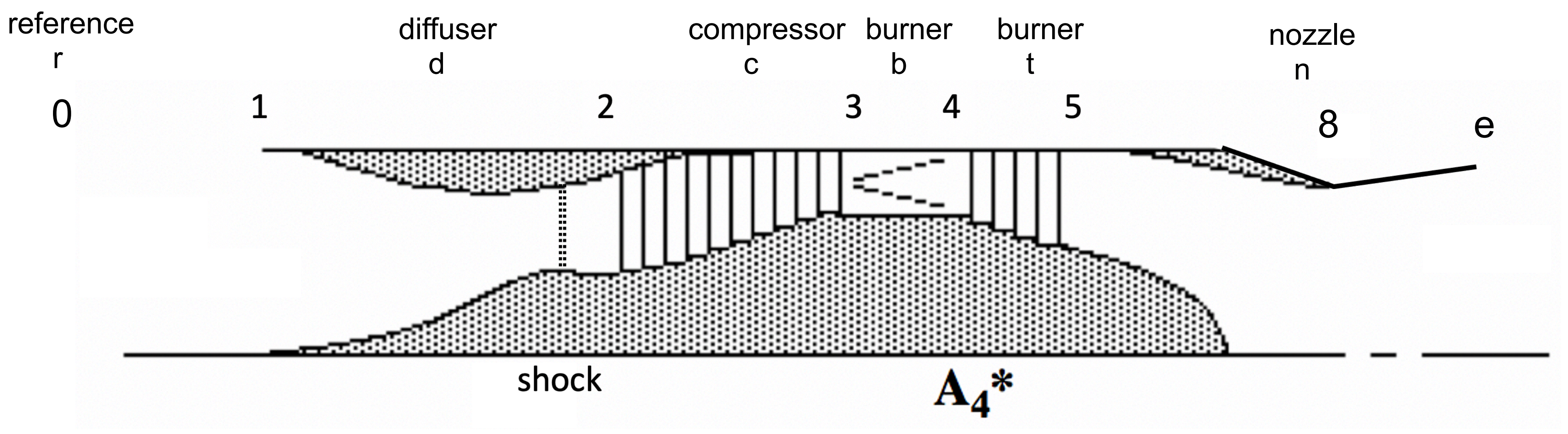


Figure 4.1 Turbojet engine and compressor-turbine schematic

Engine notation



$$\tau = \frac{\textit{The stagnation temperature leaving the component}}{\textit{The stagnation temperature entering the component}}$$

$$\pi = \frac{\textit{The stagnation pressure leaving the component}}{\textit{The stagnation pressure entering the component}}$$

Station 0 – This is the reference state of the gas well upstream of the engine entrance. The temperature and pressure parameters are

$$\tau_r = \frac{T_{t0}}{T_0} = 1 + \left(\frac{\gamma - 1}{2}\right) M_0^2$$

$$\pi_r = \frac{P_{t0}}{P_0} = \left(1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right)^{\frac{\gamma}{\gamma - 1}}$$

Note that these definitions are exceptional in that the denomintor is the static temperature and pressure of the free stream.

Station 1 – This is the entrance to the engine inlet. The purpose of the inlet is to reduce the Mach number of the incoming flow to a low subsonic value with as small a stagnation pressure loss as possible. From the entrance to the end of the inlet there is generally an increase in area and so the component is appropriately called a diffuser.

Station 1.5 – The inlet throat

Station 2 – The compressor face. The temperature/pressure parameters across the diffuser are

$$\tau_d = \frac{T_{t2}}{T_{t1}} \qquad \pi_d = \frac{P_{t2}}{P_{t1}}$$

Station 3 – The exit of the high pressure compressor. The temperature/pressure parameters across the compressor are

$$\tau_c = \frac{T_{t3}}{T_{t2}} \qquad \pi_c = \frac{P_{t3}}{P_{t2}}$$

Station 4 – The exit of the burner. The temperature and pressure parameters across the burner are

$$\tau_b = \frac{T_{t4}}{T_{t3}} \qquad \pi_b = \frac{P_{t4}}{P_{t3}} \tag{2.56}$$

The temperature at the exit of the burner is regarded as the highest temperature in the Brayton cycle although generally higher temperatures do occur at the upstream end of the burner where combustion takes place. The burner is designed to allow an influx of cooler compressor air to mix with the combustion gases bringing the temperature down to a level that the high pressure turbine can tolerate. Modern engines operate at values of T_{t4} that approach 3700°R (2050°K).

Station 5 - The exit of the turbine. The temperature/pressure parameters across the turbine are

$$\tau_t = \frac{T_{t5}}{T_{t4}} \qquad \pi_t = \frac{P_{t5}}{P_{t4}} \tag{2.57}$$

Station 6 - The exit of the afterburner if there is one. The temperature/pressure parameters across the afterburner are

$$\tau_a = \frac{T_{t6}}{T_{t5}} \qquad \pi_a = \frac{P_{t6}}{P_{t5}} \tag{2.58}$$

Station 7 - The entrance to the nozzle.

Station 8 - The nozzle throat.

Station e - The nozzle exit. The temperature/pressure parameters across the nozzle are

$$\tau_n = \frac{T_{te}}{T_{t7}} \qquad \pi_n = \frac{P_{te}}{P_{t7}} \tag{2.59}$$

In the absence of an afterburner

$$\tau_n = \frac{T_{te}}{T_{t5}} \qquad \pi_n = \frac{P_{te}}{P_{t5}} \tag{2.60}$$

Two additional parameters

$$\tau_f = \frac{h_f}{C_p T_0} \tag{2.61}$$

$$\tau_\lambda = \frac{T_{t4}}{T_0} \tag{2.62}$$

Compressor - turbine power balance.

$$(\dot{m}_a + \dot{m}_f)(h_{t4} - h_{t5}) = \dot{m}_a(h_{t3} - h_{t2}) \tag{4.6}$$

Across the burner.

$$(\dot{m}_a + \dot{m}_f)h_{t4} = \dot{m}_a h_{t3} + \dot{m}_f h_f . \tag{4.7}$$

Subtract (4.6) from (4.7). Across the engine

$$(\dot{m}_a + \dot{m}_f)h_{t5} = \dot{m}_a h_{t2} + \dot{m}_f h_f \tag{4.8}$$

The inlet and nozzle flow are assumed to be adiabatic.
Therefore (4.8) is equivalent to:

$$(\dot{m}_a + \dot{m}_f)h_{te} = \dot{m}_a h_{t0} + \dot{m}_f h_f . \tag{4.9}$$

Recall the thermal efficiency.

$$\eta_{th} = \frac{(\dot m_a + \dot m_f)(h_{te} - h_e) - \dot m_a(h_{t0} - h_0)}{(\dot m_a + \dot m_f)h_{t4} - \dot m_a h_{t3}} \tag{4.10}$$

Use (4.9) and (4.7) to rewrite (4.10).

$$\eta_{th} = \frac{(\dot m_a + \dot m_f)h_{t4} - \dot m_a h_{t3} - (\dot m_a + \dot m_f)h_e + \dot m_a h_0}{(\dot m_a + \dot m_f)h_{t4} - \dot m_a h_{t3}} \tag{4.11}$$

or

$$\eta_{th} = 1 - \left\{\frac{(\dot m_a + \dot m_f)h_e - \dot m_a h_0}{(\dot m_a + \dot m_f)h_{t4} - \dot m_a h_{t3}}\right\} = 1 - \frac{h_0}{h_{t3}}\left\{\frac{(1+f)\frac{h_e}{h_0} - 1}{(1+f)\frac{h_{t4}}{h_{t3}} - 1}\right\}. \tag{4.12}$$

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For constant heat capacity

$$\eta_{th} = 1 - \frac{T_0}{T_{t3}} \left\{ \frac{(1+f)\dfrac{T_e}{T_0} - 1}{(1+f)\dfrac{T_{t4}}{T_{t3}} - 1} \right\} \quad (4.13)$$

Ideal Brayton cycle

$$\frac{T_{t3}}{T_0} = \left(\frac{P_{t3}}{P_0}\right)^{\frac{\gamma-1}{\gamma}} \qquad \frac{T_{t4}}{T_e} = \left(\frac{P_{t4}}{P_e}\right)^{\frac{\gamma-1}{\gamma}} . \quad (4.14)$$

The heat interaction occurs at constant pressure.

$$\frac{T_{t4}}{T_e} = \frac{T_{t3}}{T_0} . \quad (4.15)$$

Thermal efficiency of the ideal turbojet.

$$\eta_{th_{ideal\ turbojet}} = 1 - \frac{T_0}{T_{t3}} = 1 - \frac{1}{\tau_r \tau_c}. \qquad (4.16)$$

Turbojet cycle P-V diagram.

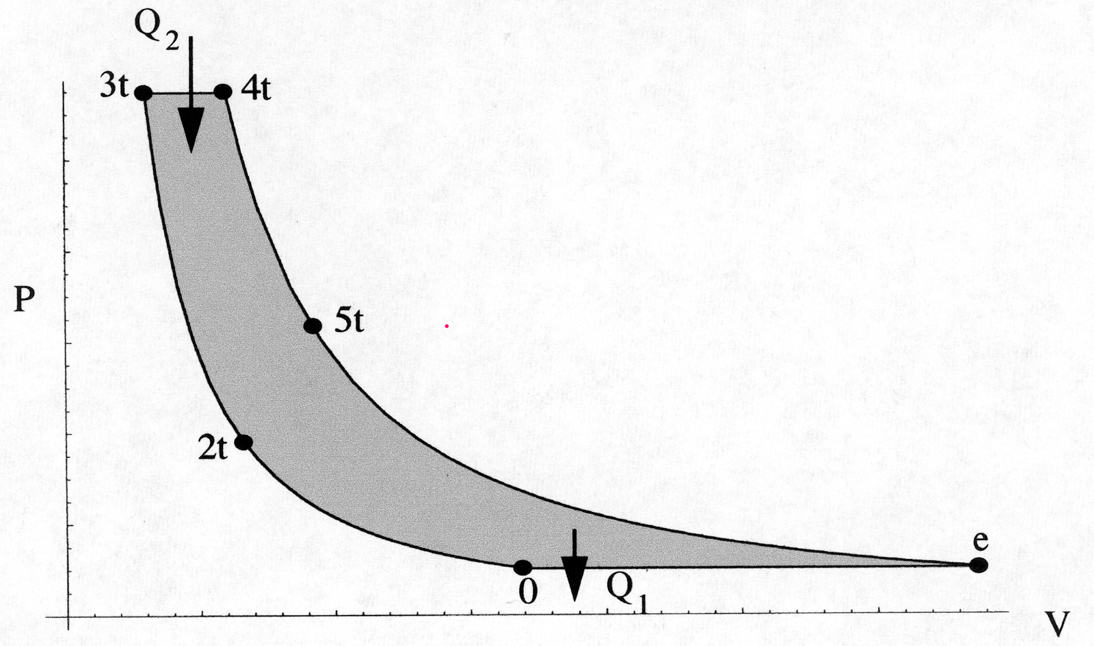

Figure 4.2 P-V diagram of the ideal turbojet cycle. Station number with a "t" refers to the stagnation state of the gas at that point.

Turbojet cycle T-S diagram.

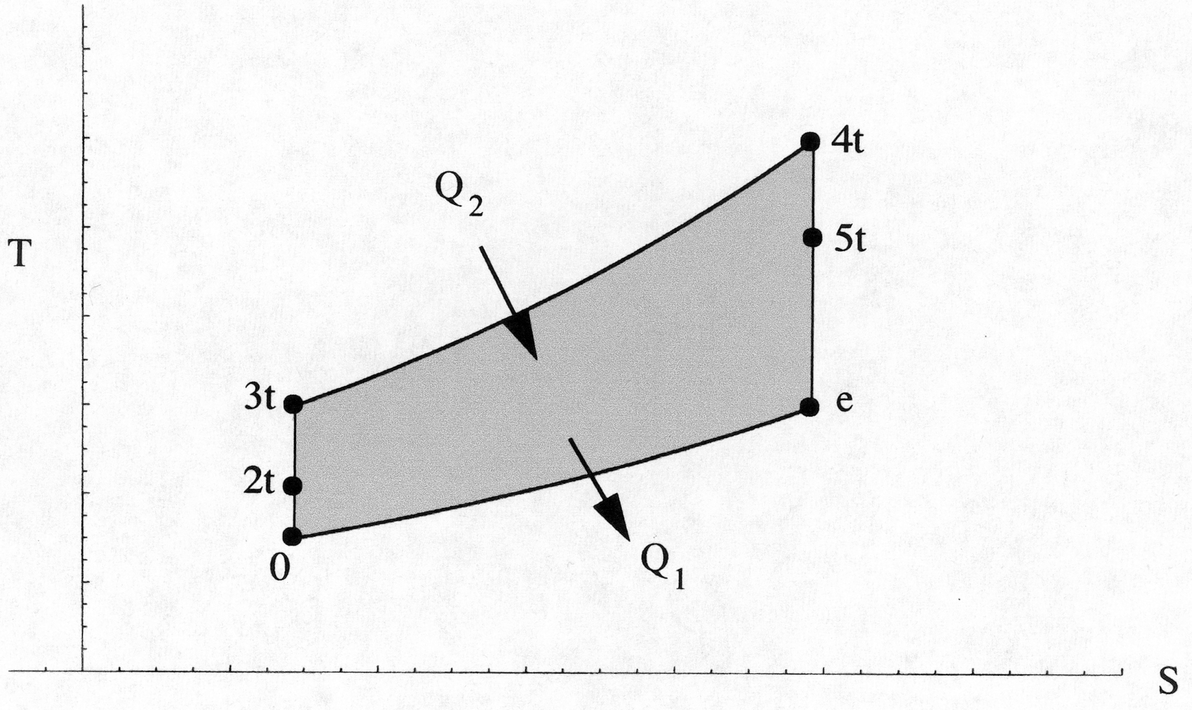

Figure 4.3 T-S diagram of the ideal turbojet cycle.

4.2 Thrust of an ideal turbojet

Thrust equation

$$\frac{T}{P_0 A_0} = \gamma M_0^2 \left((1 + f)\frac{U_e}{U_0} - 1 \right). \qquad (4.17)$$

We need to work out the velocity ratio.

$$\frac{U_e}{U_0} = \frac{M_e}{M_0}\sqrt{\frac{T_e}{T_0}} . \qquad (4.18)$$

Begin with the pressure

$$P_{te} = P_0 \left(\frac{P_{t0}}{P_0}\right)\left(\frac{P_{t2}}{P_{t0}}\right)\left(\frac{P_{t3}}{P_{t2}}\right)\left(\frac{P_{t4}}{P_{t3}}\right)\left(\frac{P_{t5}}{P_{t4}}\right)\left(\frac{P_{te}}{P_{t5}}\right) \qquad (4.19)$$

Express (4.19) in terms of engine parameters

$$P_{te} = P_0 \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n . \qquad (4.20)$$

Assume

$$\pi_d = 1 \qquad \pi_n = 1 \qquad (4.21)$$

$$\pi_b = 1 \qquad (4.22)$$

Now

$$P_{te} = P_0\pi_r\pi_c\pi_t = P_e\left(1 + \frac{\gamma - 1}{2}M_e^2\right)^{\frac{\gamma}{\gamma - 1}}. \quad (4.23)$$

Assume the nozzle is fully expanded

$$\pi_r\pi_c\pi_t = \left(1 + \frac{\gamma - 1}{2}M_e^2\right)^{\frac{\gamma}{\gamma - 1}}. \quad (4.24)$$

Assume the compressor and turbine behave isentropically

$$\pi_c = \tau_c^{\frac{\gamma}{\gamma - 1}} \qquad \pi_t = \tau_t^{\frac{\gamma}{\gamma - 1}} \quad (4.25)$$

Exit Mach number

$$M_e^2 = \frac{2}{\gamma - 1}(\tau_r\tau_c\tau_t - 1) \quad (4.26)$$

$$\frac{M_e^2}{M_0^2} = \left(\frac{\tau_r\tau_c\tau_t - 1}{\tau_r - 1}\right). \quad (4.27)$$

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Now the temperature

$$T_{te} = T_0\left(\frac{T_{t0}}{T_0}\right)\left(\frac{T_{t2}}{T_{t0}}\right)\left(\frac{T_{t3}}{T_{t2}}\right)\left(\frac{T_{t4}}{T_{t3}}\right)\left(\frac{T_{t5}}{T_{t4}}\right)\left(\frac{T_{te}}{T_{t5}}\right) \quad (4.28)$$

Express (4.28) in terms of engine parameters

$$T_{te} = T_0\tau_r\tau_d\tau_c\tau_b\tau_t\tau_n . \quad (4.29)$$

Assume the inlet and nozzle flows are adiabatic

$$T_{te} = T_0\tau_r\tau_c\tau_b\tau_t = T_e\left(1 + \frac{\gamma - 1}{2}M_e^2\right) = T_e\tau_r\tau_c\tau_t . \quad (4.30)$$

From (4.30)

$$\frac{T_e}{T_0} = \tau_b = \frac{T_{t4}}{T_{t3}} . \quad (4.31)$$

It is convenient to express the temperature ratio as

$$\frac{T_e}{T_0} = \frac{\tau_\lambda}{\tau_r \tau_c} \qquad (4.32)$$

Thrust formula

$$\frac{T}{P_0 A_0} = \frac{2\gamma}{\gamma - 1}(\tau_r - 1)\left((1 + f)\left(\left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right)\frac{\tau_\lambda}{\tau_r \tau_c}\right)^{1/2} - 1\right) \qquad (4.33)$$

Fuel/air ratio

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{\tau_f - \tau_\lambda}. \qquad (4.34)$$

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Apparently the turbojet thrust is a function of four variables.

$$\frac{T}{P_0 A_0} = F(\tau_r, \tau_c, \tau_\lambda, \tau_t) \qquad (4.35)$$

But the turbine and compressor are not independent variables.
They are coupled together by a shaft that transfers the work
Done on the turbine to the compressor.

$$(\dot{m}_a + \dot{m}_f)(T_{t4} - T_{t5}) = \dot{m}_a(T_{t3} - T_{t2}) \qquad (4.36)$$

$$(1 + f)\tau_\lambda(1 - \tau_t) = \tau_r(\tau_c - 1) \qquad (4.37)$$

or

$$\boxed{\tau_t = 1 - \frac{\tau_r(\tau_c - 1)}{(1 + f)\tau_\lambda}} \qquad (4.38)$$

Finally the velocity ratio is

$$\left(\frac{U_e}{U_0}\right)^2 = \frac{1}{(\tau_r - 1)}\left(\tau_\lambda - \tau_r(\tau_c - 1) - \frac{\tau_\lambda}{\tau_r \tau_c}\right) \qquad (4.39)$$

where the fuel/air ratio in (4.38) has been neglected

Specific impulse

$$\frac{I_{sp} g}{a_0} = \left(\frac{1}{f}\right)\left(\frac{1}{\gamma M_0}\right)\frac{T}{P_0 A_0} \quad . \qquad (4.40)$$

4.3 Maximum thrust ideal turbojet

How much compression is optimal ?

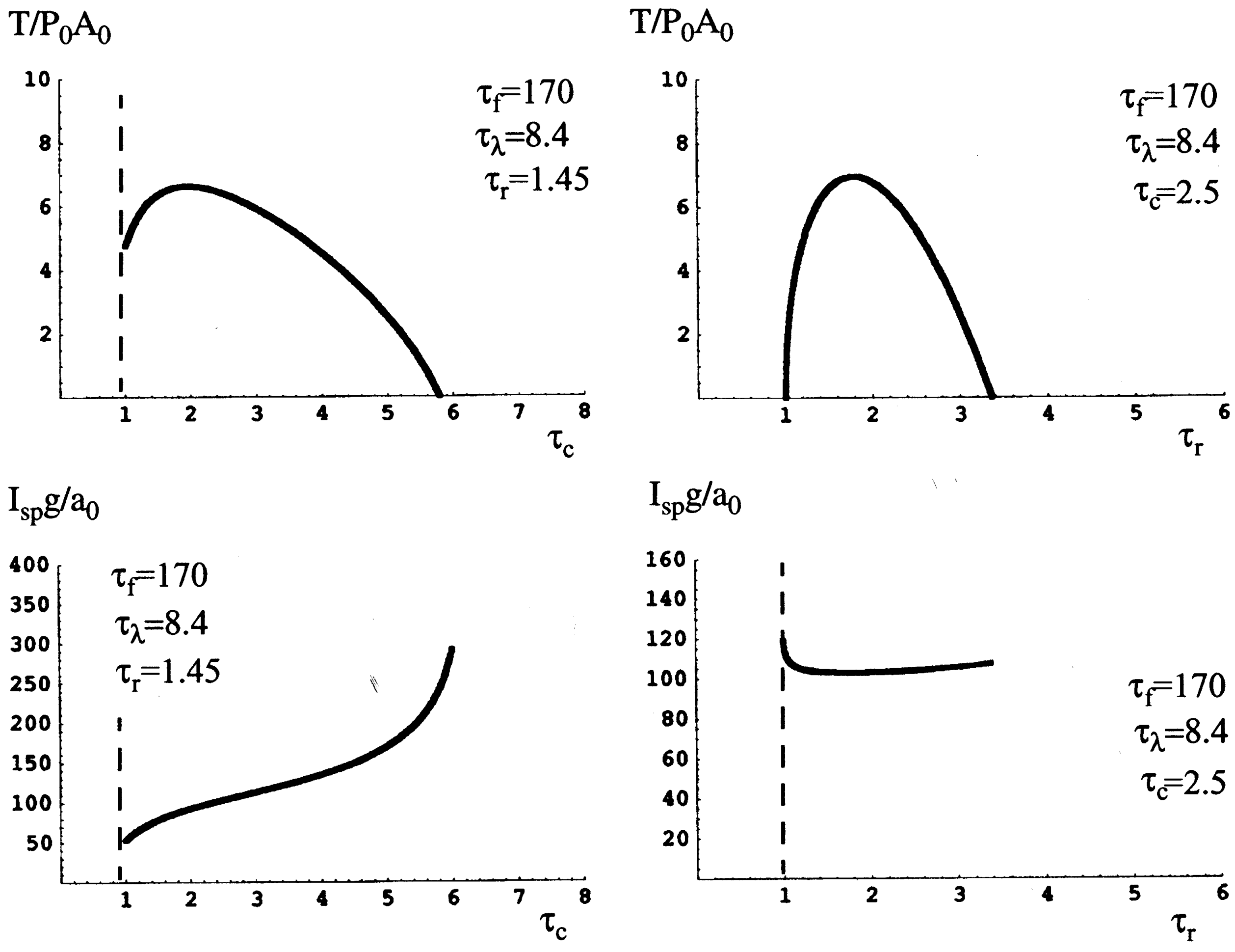

Figure 4.4 Thrust and specific impulse curves for an ideal turbojet.

Maximize the velocity ratio.

$$\frac{\partial}{\partial \tau_c}\left(\frac{U_e^2}{U_0^2}\right) = \frac{1}{(\tau_r - 1)}\left(-\tau_r + \frac{\tau_\lambda}{\tau_r \tau_c^2}\right) = 0 \tag{4.41}$$

The maximum velocity ratio occurs when

$$\tau_{c_{max\ thrust}} = \frac{\sqrt{\tau_\lambda}}{\tau_r}. \tag{4.42}$$

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4.4 Turbine - nozzle mass flow matching

$$\dot{m}_4 = \dot{m}_e$$

$$(P_{t4}A_4/\sqrt{\gamma RT_{t4}})f(M_4) = (P_{t8}A_8/\sqrt{\gamma RT_{t8}})f(M_8) \tag{4.43}$$

The turbine inlet is choked and the nozzle throat is choked.

$$P_{t4}A_4^*/\sqrt{T_{t4}} = P_{t8}A_8/\sqrt{T_{t8}} \tag{4.44}$$

Assume the turbine operates isentropically.

$$P_{t5}/P_{t4} = (T_{t5}/T_{t4})^{\frac{\gamma}{\gamma-1}} \tag{4.45}$$

Thus

$$\boxed{\tau_t = T_{t5}/T_{t4} = (A_4^*/A_8)^{(2(\gamma-1))/(\gamma+1)}} \tag{4.46}$$

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4.5 Free stream - compressor inlet mass flow matching

$$\dot{m}_a = \dot{m}_2$$

$$(P_{t0}A_0/\sqrt{T_{t0}})f(M_0) = (P_{t2}A_2/\sqrt{T_{t2}})f(M_2) \qquad (4.47)$$

$$P_{t0}A_0 f(M_0) = P_{t2}A_2 f(M_2) \qquad (4.48)$$

$$\frac{P_{t0}A_0 f(M_0)}{P_{t2}A_2} = f(M_2). \qquad (4.49)$$

$$\boxed{(1/\pi_d)(A_0/A_2)f(M_0) = f(M_2)} \qquad (4.50)$$

4.6 Compressor - turbine mass flow matching

$$\dot{m}_2(1+f) = \dot{m}_4 \tag{4.51}$$

$$(1+f)(P_{t2}A_2/\sqrt{T_{t2}})f(M_2) = (P_{t4}A_4^*/\sqrt{T_{t4}})$$

$$f(M_2) = \left(\frac{1}{1+f}\right)\frac{\pi_c\pi_b}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right) \tag{4.52}$$

$$\boxed{f(M_2) = \frac{\pi_c}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right)} \tag{4.53}$$

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4.7 Summary - engine matching conditions f<<1

$$\tau_t = \left(A_4^*/A_8\right)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad (4.54)$$

$$\tau_c - 1 = \frac{\tau_\lambda}{\tau_r}(1-\tau_t) \qquad (4.55)$$

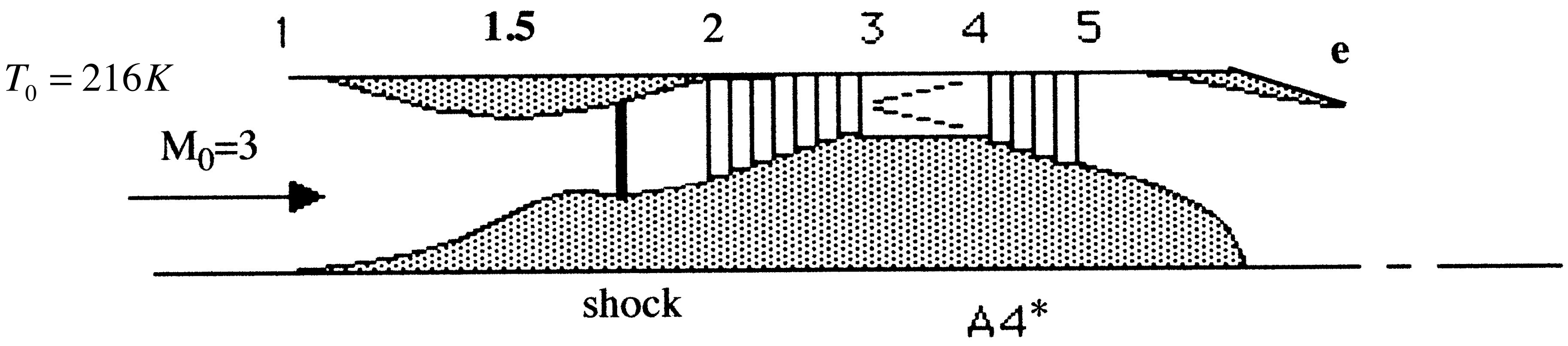
$$f(M_2) = \frac{\pi_c}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right) \qquad (4.56)$$

$$\left(\frac{1}{\pi_d}\right)\left(\frac{A_0}{A_2}\right)f(M_0) = f(M_2) \qquad (4.57)$$

4.7.1 EXAMPLE - TURBOJET IN SUPERSONIC FLOW WITH AN INLET SHOCK

A turbojet operates supersonically at $M_0 = 3$ and $T_{t4} = 1944K$. The compressor and turbine polytropic efficiencies are $\eta_{pc} = \eta_{pt} = 1$. At the condition shown, the engine operates semi-ideally with $\pi_b = \pi_n = 1$ but $\pi_d \neq 1$ and with a simple convergent nozzle.

The relevant areas are $A_1/A_2 = 2$, $A_2/A_{4*} = 14$ and $A_e/A_{4*} = 4$. Supersonic flow is established at the entrance to the inlet with a normal shock downstream of the inlet throat. This type of inlet operation is called supercritical and will be discussed further in Section 4.11.



1) Sketch the distribution of stagnation pressure, P_t/P_{t0} and stagnation temperature, T_t/T_{t0} through the engine. Assign numerical values at each station.

Solution - Note that $f(3) = 0.236$, $T_{t0} = 605K$ and

$$\frac{A_e}{A_1} = \left(\frac{A_e}{A_{4*}}\right)\left(\frac{A_{4*}}{A_2}\right)\left(\frac{A_2}{A_1}\right) = \frac{4}{14}\left(\frac{1}{2}\right) = 0.143 \quad (4.58)$$

We need to determine π_c, $f(M_2)$ and π_d. The analysis begins at the nozzle where the flow is choked. Choking at the turbine inlet and nozzle determines the turbine temperature and pressure ratio.

$$\tau_t = \left(\frac{A_{4*}}{A_e}\right)^{\frac{2(\gamma-1)}{\gamma+1}} = \left(\frac{1}{4}\right)^{1/3} = 0.63 \quad (4.59)$$

$$\pi_t = \tau_t^{\frac{\gamma}{\gamma-1}} = 0.63^{3.5} = 0.198. \quad (4.60)$$

Matching turbine and compressor work gives the compressor temperature and pressure ratio.

$$\tau_c = 1 + \frac{\tau_\lambda}{\tau_r}(1-\tau_t) = 1 + \frac{1944}{605}(1-0.63) = 2.19 \quad (4.61)$$

$$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}} = 2.19^{3.5} = 15.54. \quad (4.62)$$

Now the Mach number at the compressor face is determined.

$$f(M_2) = \frac{A_{4*}}{A_2}\left(\frac{\tau_r}{\tau_\lambda}\right)^{1/2}\pi_c\pi_b = \frac{A_{4*}}{A_2}\left(\frac{605}{1944}\right)^{1/2}15.54 = \frac{A_{4*}}{A_2}8.67 = \frac{8.67}{14} = 0.62$$

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Use free-stream-compressor-mass-flow matching to determine the stagnation pressure loss across the inlet.

$$\pi_d = \frac{A_0 f(M_0)}{A_2 f(M_2)} = \frac{0.236}{0.62}2 = 0.76 \tag{4.64}$$

Now determine the stagnation pressure ratio across the engine.

$$\frac{P_{te}}{P_{t0}} = \pi_d \pi_c \pi_t = 0.76(15.54)(0.198) = 2.34 \tag{4.65}$$

Now the exit static pressure ratio is determined.

$$\frac{P_e}{P_0} = \frac{P_{te}}{P_{t0}} \left(\frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_6^2} \right)^{\frac{\gamma}{\gamma - 1}} = (2.34)\left(\frac{2.8}{1.2}\right)^{3.5} = 45.4 \tag{4.66}$$

As is the static temperature ratio.

$$\frac{T_{te}}{T_{t0}} = \frac{\tau_\lambda}{\tau_r}\tau_t = \frac{1944}{605}0.63 = 2.02 \tag{4.67}$$

$$\frac{T_e}{T_0} = \frac{T_{te}}{T_{t0}} \left(\frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M_e^2} \right) = 2.02\left(\frac{2.8}{1.2}\right) = 4.71 \tag{4.68}$$

velocity ratio

$$\frac{U_e}{U_0} = \frac{M_e}{M_0}\left(\frac{T_e}{T_0}\right)^{1/2} = \frac{1}{3}(4.71)^{1/2} = 0.723 \tag{4.69}$$

and thrust

$$\frac{T}{P_0 A_0} = \gamma M_0^2 \left(\frac{U_e}{U_0} - 1\right) + \frac{A_e}{A_0}\left(\frac{P_e}{P_0} - 1\right) \tag{4.70}$$

$$\frac{T}{P_0 A_0} = 1.4(9)(0.723 - 1) + 0.143(45.4 - 1) = -3.49 + 6.35 = 2.86$$

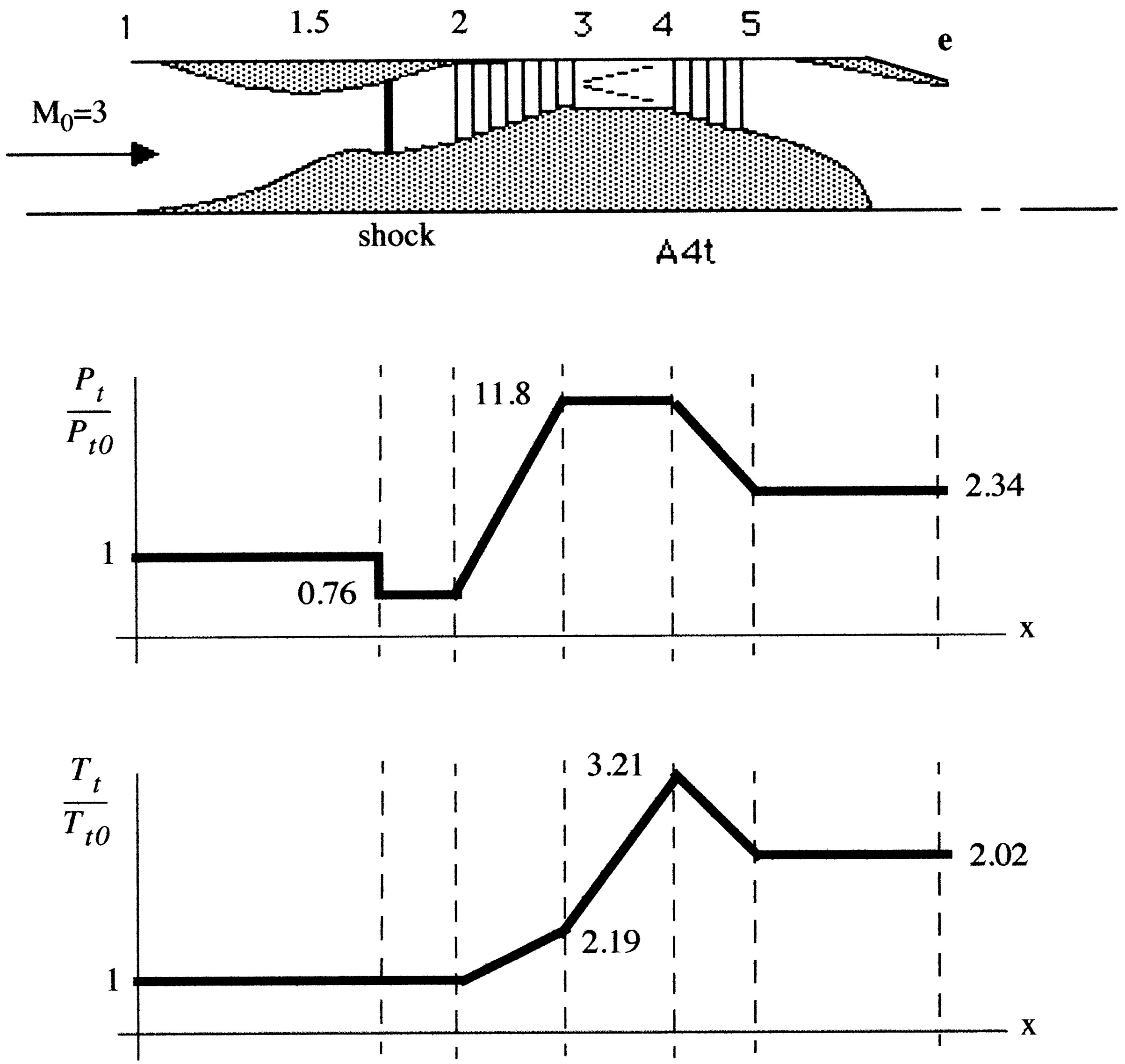
At this point we have all the information we need (and then some) to answer the problem. The pressure ratios are:

$$\frac{P_{t2}}{P_{t0}} = \pi_d = 0.76; \quad \frac{P_{t3}}{P_{t0}} = \pi_d \pi_c = 11.8$$
$$\frac{P_{t4}}{P_{t0}} = \pi_d \pi_c \pi_b = 11.8; \quad \frac{P_{t5}}{P_{t0}} = \pi_d \pi_c \pi_b \pi_t = 2.34 \tag{4.71}$$

and the relevant temperature ratios are:

$$\frac{T_{t2}}{T_{t0}} = \tau_d = 1; \quad \frac{T_{t3}}{T_{t0}} = \tau_d \tau_c = 2.19$$
$$\frac{T_{t4}}{T_{t0}} = \tau_d \tau_c \tau_b = \frac{1944}{605} = 3.21; \quad \frac{T_{t5}}{T_{t0}} = \tau_d \tau_c \tau_b \tau_t = 2.02 \tag{4.72}$$

Now we can sketch the stagnation pressure and temperature ratios through the engine.


1
1.5
2
3
4
5
e
M0=3
shock
A4t
Pt/Pt0
1
0.76
11.8
2.34
x
Tt/Tt0
1
2.19
3.21
2.02
x

Suppose we add a diverging section to the nozzle. What happens to the thrust?

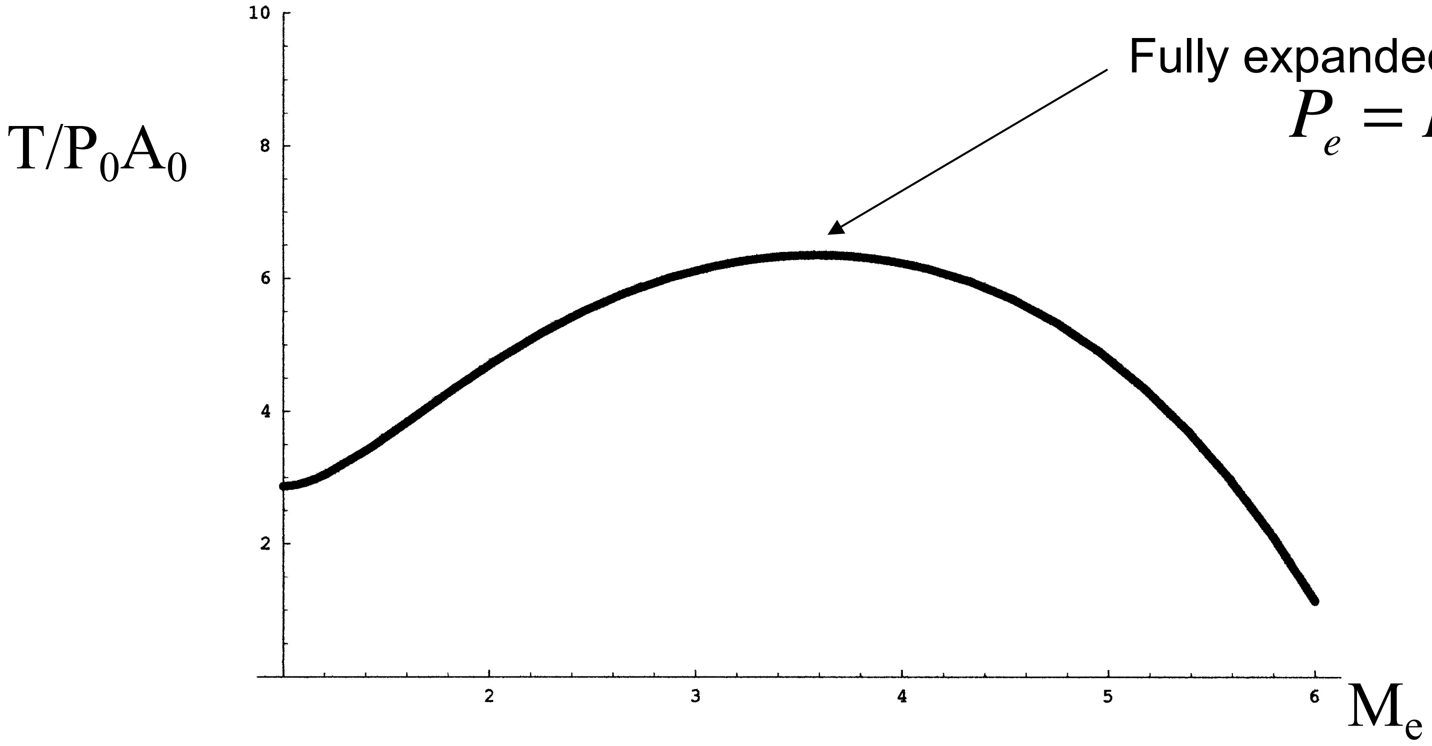
$$\frac{T}{P_0 A_0} = \gamma M_0^2 \left(\frac{M_e}{M_0} \sqrt{\frac{T_e}{T_0}} - 1 \right) + \frac{A_e}{A_0} \left(\frac{P_e}{P_0} - 1 \right) .$$

Rearrange to express the thrust in terms of the nozzle exit Mach number.

$$\frac{T}{P_0 A_0} = \gamma M_0^2 \left(\frac{M_e}{M_0} \sqrt{\frac{T_{te}}{T_0} \left(\frac{T_e}{T_{te}} \right)} - 1 \right) + \frac{A_8}{A_0} \frac{A_e}{A_8} \left(\frac{P_{te}}{P_0} \frac{P_e}{P_{te}} - 1 \right)$$

$$\frac{T}{P_0 A_0}(M_e) = \gamma M_0^2 \left(\frac{M_e}{M_0} \sqrt{\frac{T_{te}}{T_0} \left(\frac{1}{1 + \left(\frac{\gamma - 1}{2} \right) M_e^2} \right)} - 1 \right) + \frac{A_8}{A_0} \left(\frac{1}{f(M_e)} \right) \left(\frac{P_{te}}{P_0} \left(\frac{1}{1 + \left(\frac{\gamma - 1}{2} \right) M_e^2} \right)^{\frac{\gamma}{(\gamma - 1)}} - 1 \right)$$

Plot for $\frac{T_{te}}{T_0} = 5.658$ $\frac{P_{te}}{P_0} = 85.955$ $\frac{A_8}{A_0} = 0.143$



$M_{eMaxThrust} = 3.585$ $A_e/A_{8MaxThrust} = 7.347$

$A_e/A_1 = 1.05$

4.11 Inlet operation

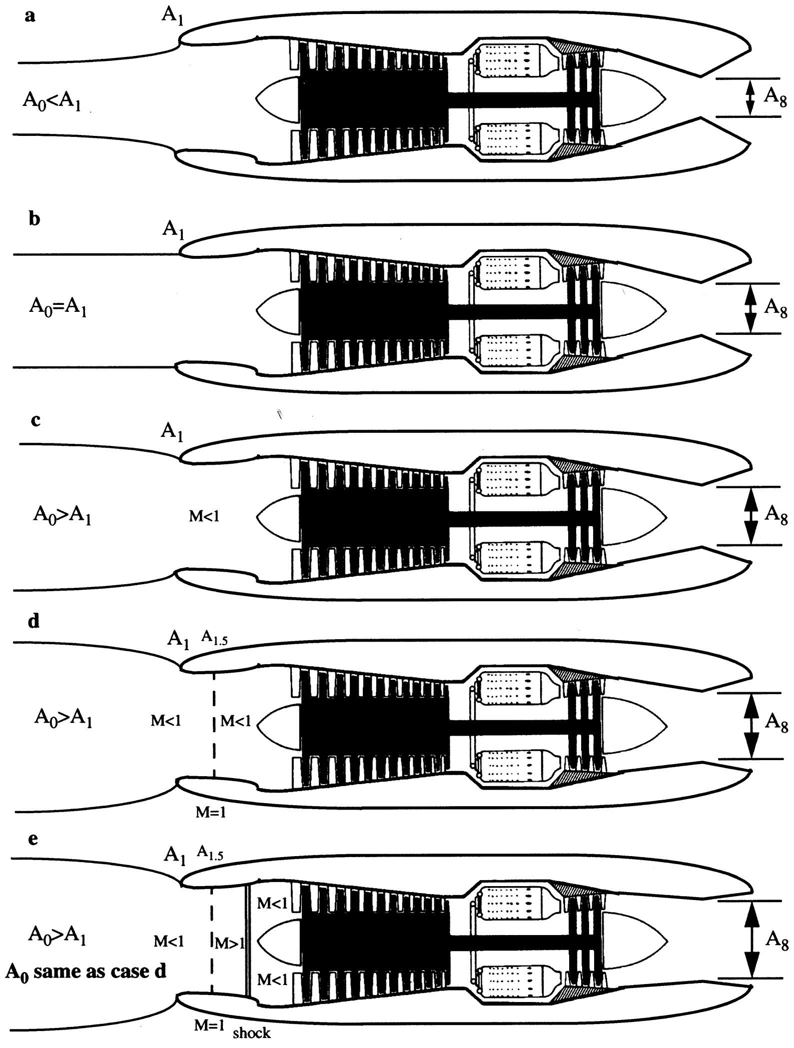


Figure 4.12 Inlet behavior with increasing nozzle throat area in subsonic flow

Condition for inlet choking

$$(P_{t2}A_2/\sqrt{T_{t2}})f(M_2) = (P_{t1.5}A_{1.5}/\sqrt{T_{t1.5}})f(M_{1.5}) \tag{4.104}$$

The mass balance becomes

$$A_2 f(M_2) = A_{1.5} f(M_{1.5}) \tag{4.105}$$

The inlet chokes when $f(M_{1.5}) = 1$. This occurs when

$$f(M_2)\Big|_{inlet\ choking} = A_{1.5}/A_2 \tag{4.106}$$

Supersonic flow

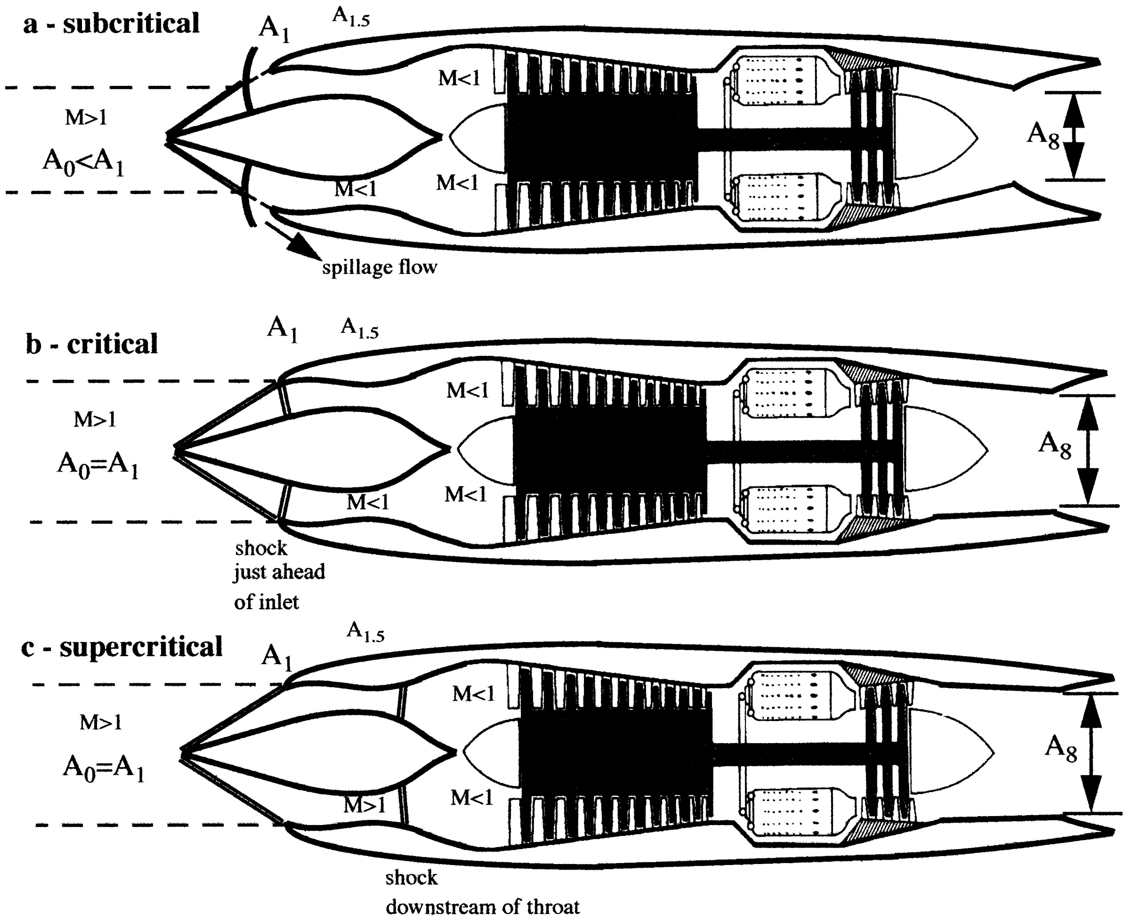


Figure 4.14 Inlet behavior with increasing nozzle throat area in supersonic flow

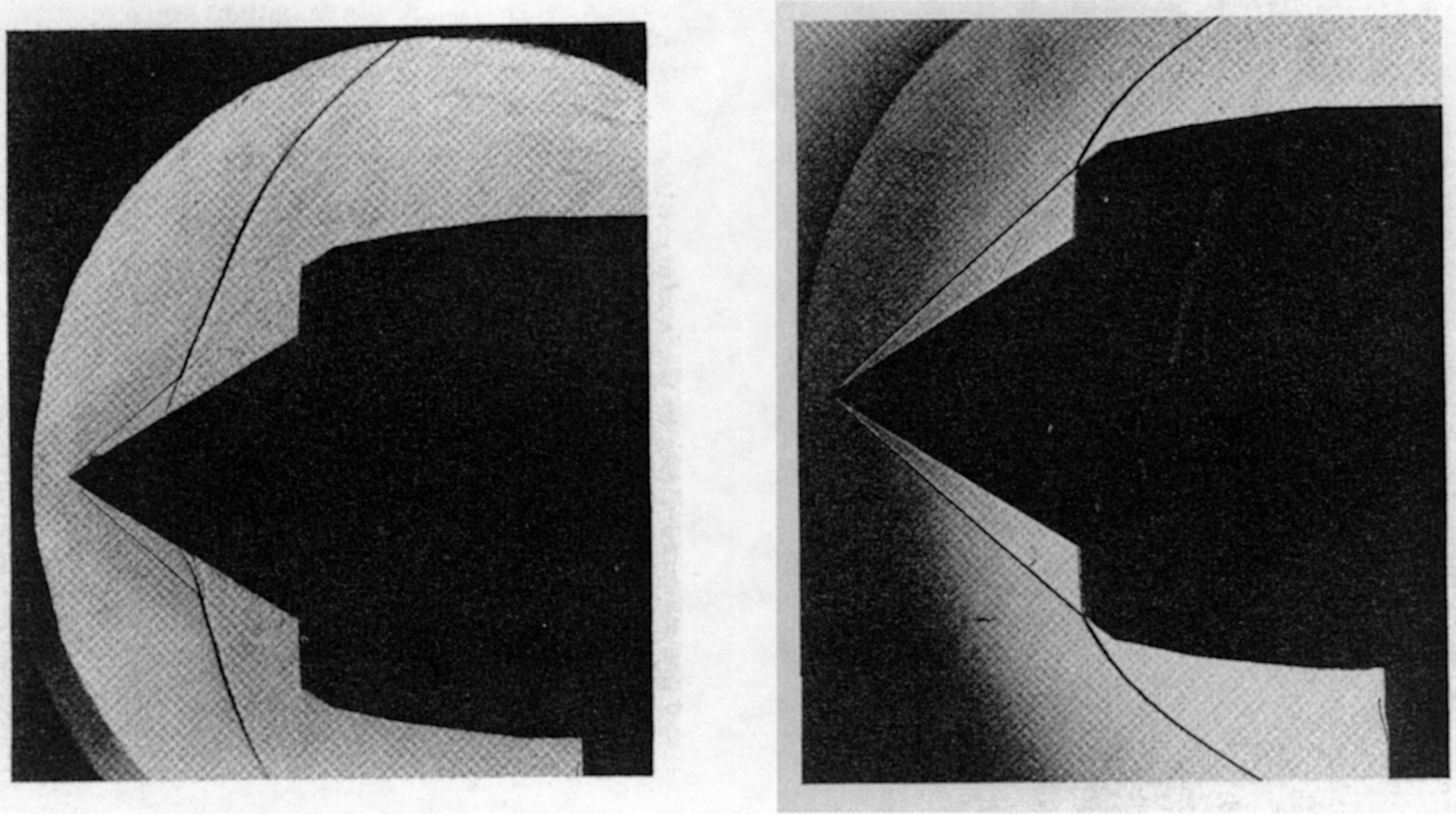


Figure 4.14 Flow over a Mach 3 spike inlet, left photo subcritical behavior, right photo supercritical behavior.

4.8 How does a turbojet work ?

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Recall the engine matching conditions f<<1

$$\tau_t = (A_4^*/A_8)^{\frac{2(\gamma-1)}{\gamma+1}} \qquad (4.54)$$

$$\tau_c - 1 = \frac{\tau_\lambda}{\tau_r}(1 - \tau_t) \qquad (4.55)$$

$$f(M_2) = \frac{\pi_c}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right) \qquad (4.56)$$

$$\left(\frac{1}{\pi_d}\right)\left(\frac{A_0}{A_2}\right)f(M_0) = f(M_2) \qquad (4.57)$$

The compressor operating line

$$\tau_c - 1 = \frac{\tau_\lambda}{\tau_r}(1 - \tau_t)$$

$$f(M_2) = \frac{\pi_c}{\sqrt{\tau_\lambda/\tau_r}}\left(\frac{A_4^*}{A_2}\right)$$

Use (4.55) and (4.56) and

$$\pi_c = (\tau_c)^{\frac{\gamma}{\gamma-1}}, \qquad (4.75)$$

$$\frac{\pi_c}{\left(\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right)^{1/2}} = \left(\frac{1}{1 - \left(\frac{A_4^*}{A_8}\right)^{\frac{2(\gamma-1)}{\gamma+1}}}\right)^{\frac{1}{2}} \frac{A_2}{A_4^*} f(M_2) \qquad (4.76)$$

The compressor operating line is approximately a straight line

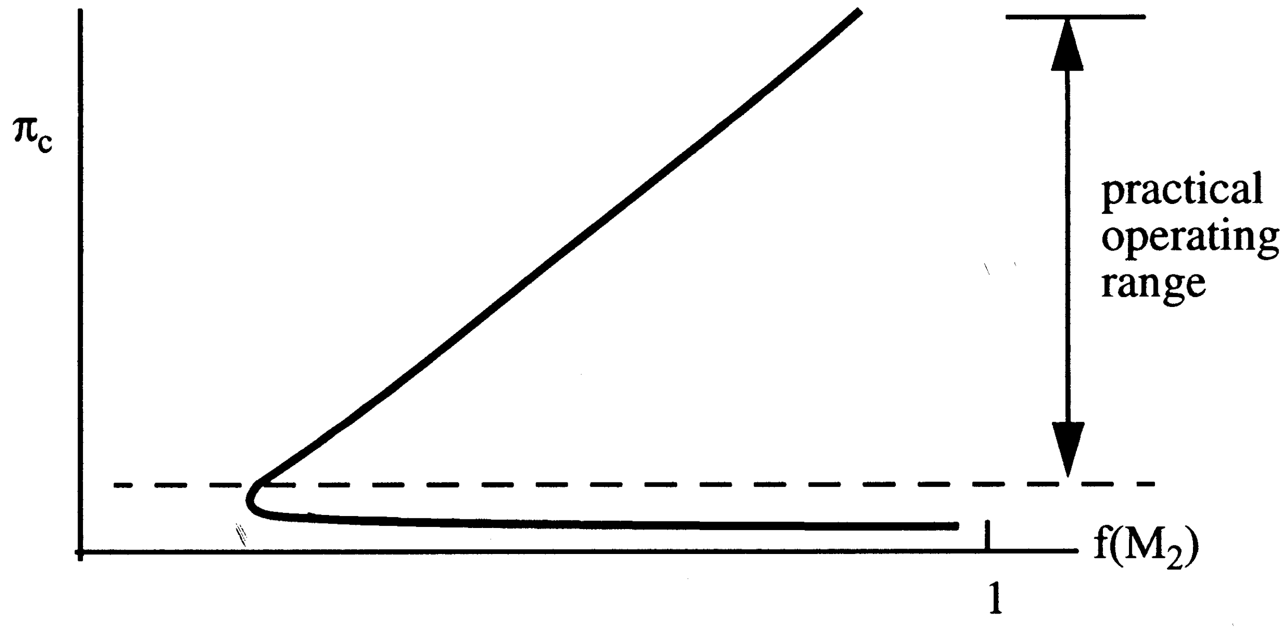

Figure 4.6 Schematic of the compressor operating line Equation (4.59)

4.10 Turbojet engine control

The two main inputs to the control of the engine are:

1) The throttle, which we can regard as controlling T_{t4}, or equivalently, at a fixed altitude, τ_λ and,

2) the nozzle throat area A_8^*.

The logic of engine operation is as follows.

Case 1- Vary A_8^* keeping τ_λ constant. Equation (4.54) determines τ_t which is used in (4.55) to determine τ_c. This determines π_c through (4.58) and $f(M_2)$ through (4.56). Given $f(M_2)$, the combination $(1/\pi_d)(A_0/A_2)$ is now known. This quantity completely defines the inlet operation. The change of $f(M_2)$ and π_c is achieved by an increase in compressor speed according to the behavior indicated on the compressor map. The compressor operating point moves along a constant τ_λ/τ_r characteristic.

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<u>Case 2</u> - Vary τ_λ keeping A_8^* the same. The logic in this case is very similar to case 1 except that the compressor-turbine work matching condition, has $\tau_t = constant$. This determines π_c through (4.58) and $f(M_2)$ through the compressor operating line (4.59). Given $f(M_2)$ then $(1/\pi_d)(A_0/A_2)$ is known and the inlet operation is defined. As in case 1 the change of $f(M_2)$ and π_c is achieved by an increase in compressor speed according to the compressor map. The compressor operating point moves along the operating line (4.59) which crosses the constant τ_λ/τ_r characteristics as shown in Figure 4.11.

4.13 The effect of afterburning

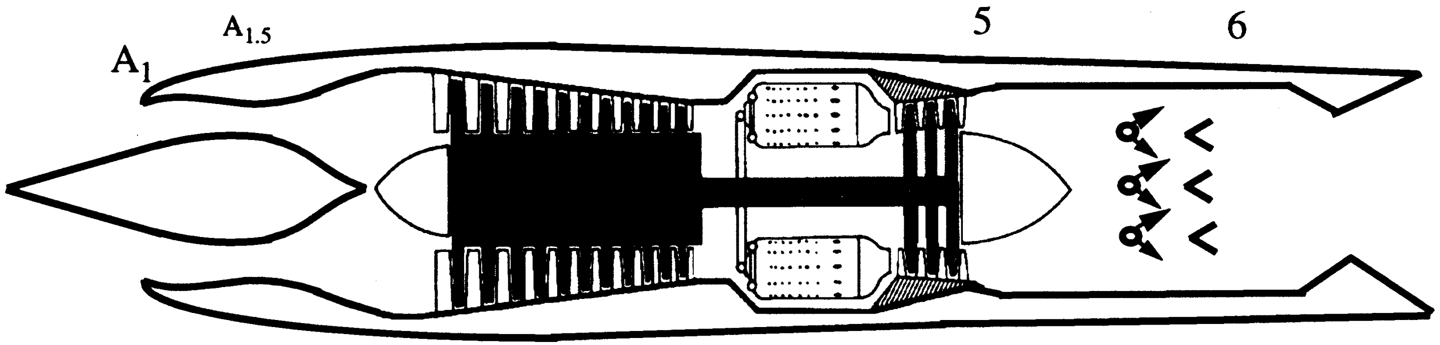

Figure 4.18 Turbojet with afterburner

Assume no stagnation pressure loss across the afterburner

$$\pi_a = P_{t6}/P_{t5} \cong 1 \qquad (4.123)$$

Velocity ratio

$$\frac{U_e}{U_0} = \frac{M_e}{M_0}\sqrt{\frac{T_e}{T_0}} = \frac{M_e}{M_0}\sqrt{\frac{1}{T_0}}\left(\frac{T_{te}}{1+\left(\frac{\gamma-1}{2}\right)M_e^2}\right)^{1/2} \tag{4.124}$$

Exit stagnation temperature

$$T_{te} = T_{t5}\left(\frac{T_{te}}{T_{t5}}\right) = T_{t5}\tau_a \tag{4.125}$$

Note that the exit velocity increases in proportion to the square root of the increase in stagnation temperature ratio across the burner.

4.14 Nozzle operation

With the afterburner on, the turbine temperature ratio is

$$\tau_t = \left(\sqrt{\tau_a} A_4^* / A_8\right)^{\frac{2(\gamma - 1)}{\gamma + 1}} . \qquad (4.126)$$

When the afterburner is turned on the nozzle area must be increased to keep the temperature across the turbine the same. Otherwise a reduction in corrected mass flow, f(M), through the engine may occur.

4.12 The non-ideal turbojet cycle

Inlet - shocks, boundary layer losses, non-adiabatic flow

Nozzle - incomplete expansion, shocks, boundary layer losses, non-adiabatic flow

Burner - stagnation pressure loss due to heat addition, burner drag, non-adiabatic flow, incomplete combustion

A rule of thumb

$$\pi_b = 1 - constant \times \gamma M_3^2 \qquad (4.107)$$

Combustor efficiency

$$\eta_b = \frac{(1+f)h_{t4} - h_{t3}}{f h_f} \qquad (4.108)$$

Compressor - turbine - shaft efficiency

$$\eta_m = \frac{h_{t3} - h_{t2}}{(1+f)(h_{t4} - h_{t5})} \qquad (4.109)$$

Compressor and turbine efficiencies

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Compressor efficiency

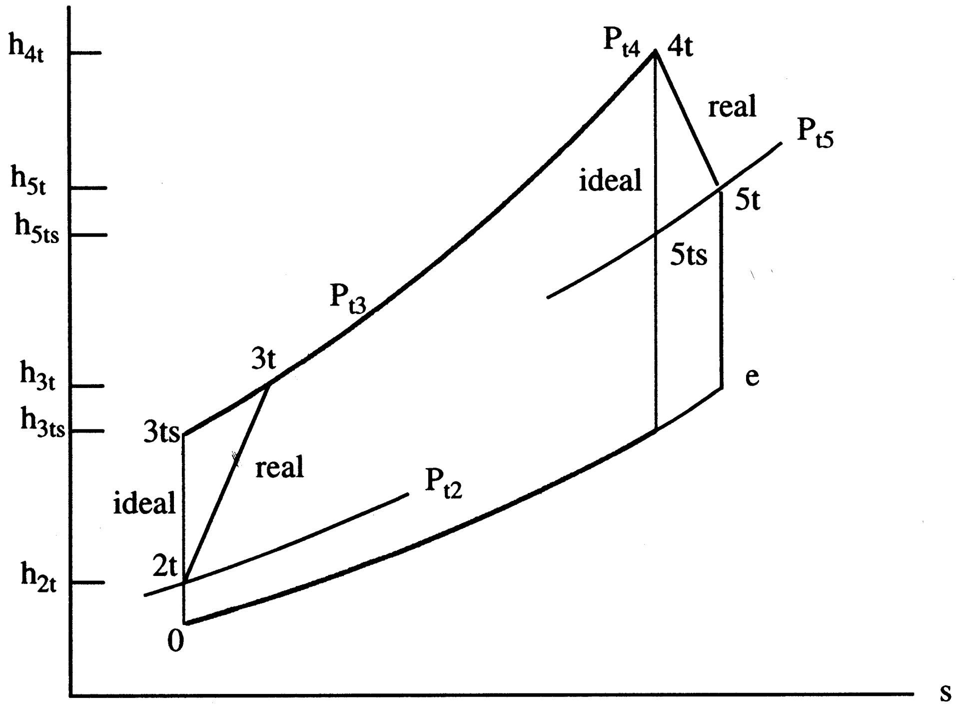

Figure 4.15 h-s path of a turbojet with non-ideal compressor and turbine.

$$\eta_c = \frac{\text{The work needed to reach } P_{t3}/P_{t2} \text{ in an isentropic compression process}}{\text{The work needed to reach } P_{t3}/P_{t2} \text{ in the real compression process}} \tag{4.110}$$

$$\eta_c = \frac{h_{t3s} - h_{t2}}{h_{t3} - h_{t2}}$$

Turbine efficiency

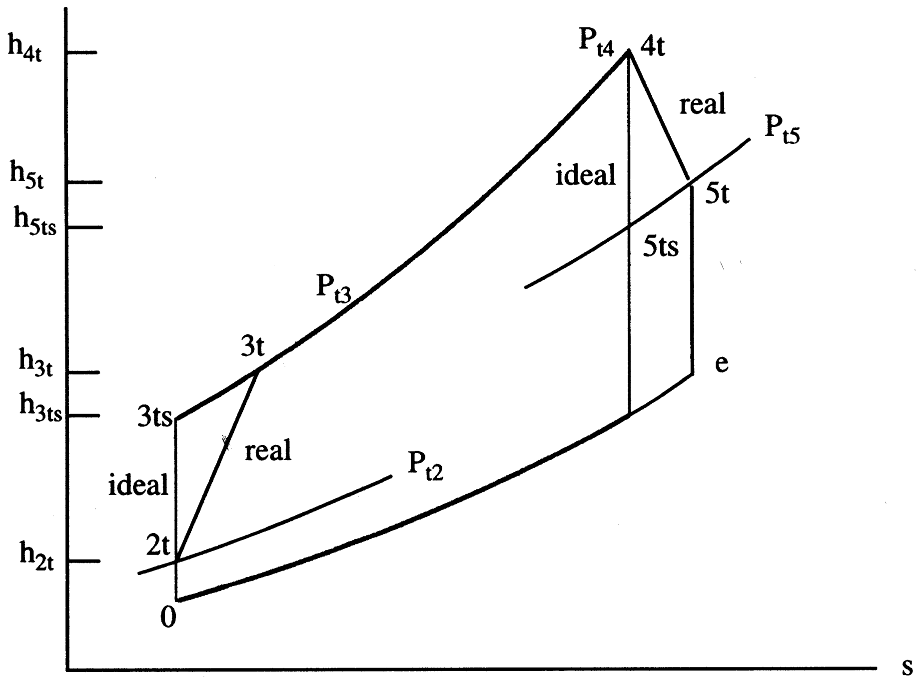

Figure 4.15 h-s path of a turbojet with non-ideal compressor and turbine.

$$\eta_e = \frac{\text{The work output in reaching } P_{t5}/P_{t4} \text{ in the real expansion process process}}{\text{The work output in reaching } P_{t5}/P_{t4} \text{ in an isentropic expansion process}} . \quad (4.111)$$

$$\eta_e = \frac{h_{t5} - h_{t4}}{h_{t5s} - h_{t4}}$$

In terms of temperature

$$\eta_c = \frac{T_{t3s} - T_{t2}}{T_{t3} - T_{t2}} \qquad \eta_e = \frac{T_{t5} - T_{t4}}{T_{t5s} - T_{t4}} \qquad (4.112)$$

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Polytropic efficiency of compression

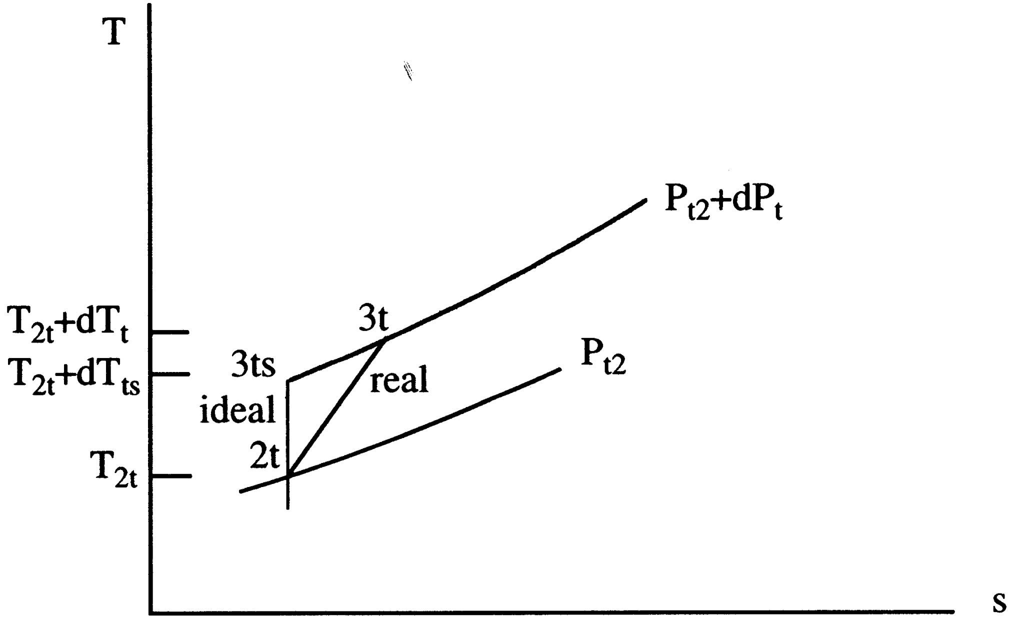

Figure 4.16 Infinitesimal compression process

Define

$$\eta_{pc} = \frac{dT_{ts}}{dT_t} \tag{4.113}$$

For an isentropic process

$$\frac{dT_{ts}}{T_t} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{dP_t}{P_t}. \qquad (4.114)$$

For a real process

$$\frac{dT_t}{T_t} = \left(\frac{\gamma - 1}{\gamma\eta_{pc}}\right)\frac{dP_t}{P_t} \qquad (4.115)$$

Assume the polytropic efficiency is constant over the range of a finite compression. Across the compressor

$$\frac{P_{t3}}{P_{t2}} = \left(\frac{T_{t3}}{T_{t2}}\right)^{\frac{\gamma\eta_{pc}}{\gamma - 1}} \qquad (4.116)$$

Compressor efficiency

$$\eta_c = \frac{\dfrac{T_{t3s}}{T_{t2}} - 1}{\dfrac{T_{t3}}{T_{t2}} - 1} = \frac{\left(\dfrac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\dfrac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\gamma\eta_{pc}}} - 1} \qquad (4.117)$$

Polytropic efficiency of expansion

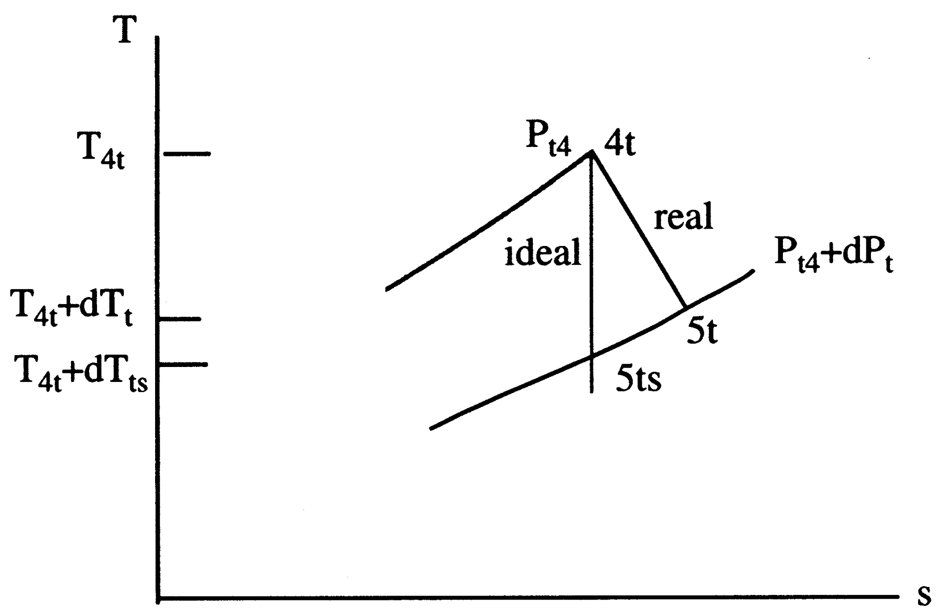

Figure 4.17 Infinitesimal compression process

Define

$$\eta_{pe} = \frac{dT_t}{dT_{ts}} \qquad (4.118)$$

For an isentropic process

$$\frac{dT_{ts}}{T_t} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{dP_t}{P_t}. \qquad (4.119)$$

For a real process

$$\frac{dT_t}{T_t} = \left(\frac{\eta_{pe}(\gamma - 1)}{\gamma}\right)\frac{dP_t}{P_t} \qquad (4.120)$$

Assume the polytropic efficiency is constant over the range of a finite expansion. Across the turbine

$$\frac{P_{t5}}{P_{t4}} = \left(\frac{T_{t5}}{T_{t4}}\right)^{\frac{\gamma}{(\gamma - 1)\eta_{pe}}} \qquad (4.121)$$

Turbine efficiency

$$\eta_e = \frac{\dfrac{T_{t5}}{T_{t4}} - 1}{\dfrac{T_{t5s}}{T_{t4}} - 1} = \frac{\left(\dfrac{P_{t5}}{P_{t4}}\right)^{\frac{(\gamma-1)\eta_{pe}}{\gamma}} - 1}{\left(\dfrac{P_{t5}}{P_{t4}}\right)^{\frac{\gamma-1}{\gamma}} - 1} \qquad (4.122)$$

Combined non-ideal engine analysis

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AA283 Homework 4 2020 - 2021

Cantwell Winter 2020-21

Due February 9, 2021

Reading: Read the remainder of Chapter 4

Problem

The figure below shows several views of the Northrup F-5/T-38 fighter/trainer aircraft used by a variety of nations since it was introduced to service in 1962.

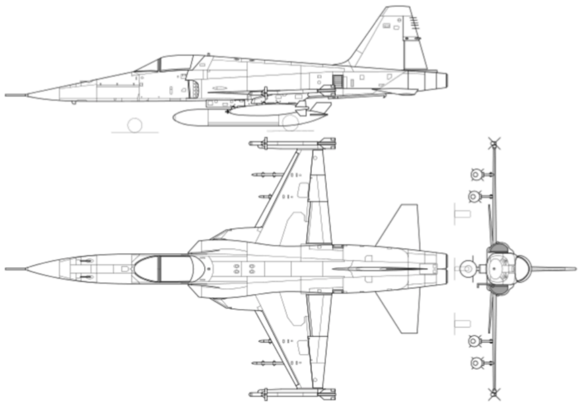


Figure 1: F-5 in several views

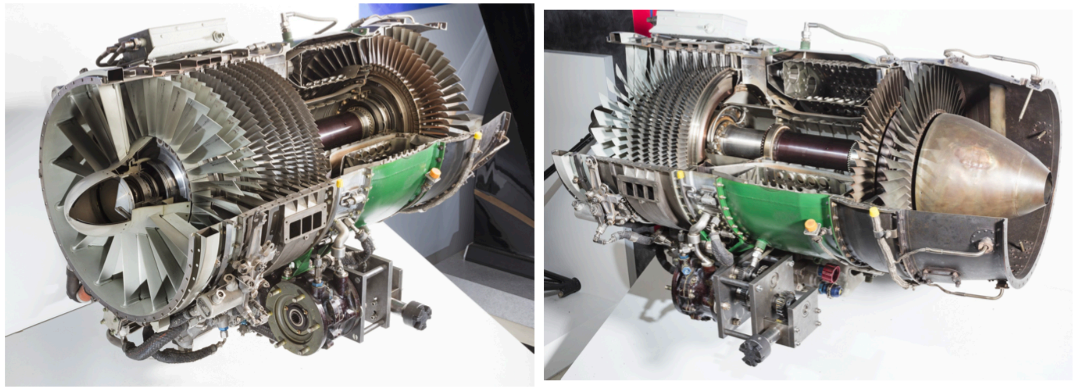
Although the aircraft can fly supersonically, the cruise speed for maximum range of the clean (no stores) aircraft is $M_0 = 0.8$.

1) Determine the altitude for minimum drag at the beginning and end of cruise. Assume 15% of fuel mass is needed to reach the cruise condition and 5% of fuel mass is used for descent and landing. Relevant data is as follows.

Empty mass = 4347 kg
Gross weight (clean) = 7142 kg
Max takeoff weight = 11,192 kg
Fuel volume = 2.56 m^3
Fuel density = 789.5 kg/m^3
Wingspan = 8.13 m
Wing area = 17.3 m^2
Frontal area = 0.32 m^2
C_{dp} = 0.012
Span efficiency = 0.6

2) Determine the aircraft drag at the beginning of cruise.

3) The engine used to drive the aircraft is the General Electric J-85 shown in Figures 2a and 2b which has been in service for over 60 years.



(a) Front perspective

(b) Rear perspective

Figure 2: Cutaway views of the J85 engine.

Use the following data to show that this engine can provide the thrust and range needed to drive the aircraft at the cruise flight condition.

Thermal parameters
$h_f = 4.28 \times 10^7 J/kg$
$T_{t4} = 1173.15K$

Engine geometry
Compressor entrance area, $A_2 = 0.1591m^2$
$A_2/A_4{}^* = 8.0$
$A_8/A_4{}^* = 2.275$
Choose A_e/A_8 *so that* $P_e = P_0$ *at the given flight condition.*

Nonideal loss parameters
$\eta_{shaft} = 0.995$
$\eta_{burner} = 0.98$
$\pi_b = 0.96$
$\pi_d = 0.97$
$\pi_n = 0.98$

Compressor and turbine efficiencies
$\eta_{pe} = 0.90$
$\eta_{pc} = 0.86$

Determine the following engine parameters at the given operating point.

1) Turbine parameters, τ_t and π_t
2) Compressor parameters, τ_c and π_c
3) The fuel/air ratio, f
4) Compressor entrance flow, $f(M_2)$ and M_2
5) Freestream capture area to compressor entrance area ratio, A_0/A_2
6) Nozzle area ratio, A_e/A_8. Assume isentropic flow between stations 8 and e.
7) Nozzle exit Mach number, M_e
8) Nozzle pressure ratio, P_e/P_0
9) Nozzle temperature ratio, T_e/T_0
10) Nozzle velocity ratio, U_e/U_0
11) Dimensionless engine thrust based on the compressor entrance area, $T/(P_0 A_2)$
12) Dimensionless engine thrust based on the capture area, $T/(P_0 A_0)$
13) Dimensionless specific impulse, $I_{sp}g/a_0$
14) Overall efficiency, η_{ov}
15) Propulsive efficiency, η_{pr}
16) Thermal efficiency, η_{th}

Determine the engine performance at the given operating point.

17) Thrust, T in Newtons. Compare to the aircraft drag.
19) Air mass flow rate, $\dot{m}_a$ in kg/sec
20) Fuel mass flow rate, $\dot{m}_f$ in kg/sec
21) Aircraft flight time and range during cruise in km
22) Aircraft L/D at the beginning and end of cruise

4) Determine the same engine parameters at sea level and $M_0 = 0$. Use the same turbine inlet temperature, T_{t4}. Let $A_2/A_4{}^* = 8.0$. Change the nozzle throat area so that $A_8/A_4{}^* = 2.5$ and change the nozzle area ratio A_e/A_8 so as to maintain $P_e = P_0$ at sea level and $M_0 = 0$.

Non-ideal engine matching relations and performance

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Turbine

$$\tau_t = \frac{1}{(\pi_n (A_8/A_4{}^*))^{2\eta_{pe}(\gamma-1)/(2\gamma-\eta_{pe}(\gamma-1))}}$$

$$\pi_t = (\tau_t)^{\gamma/\eta_{pe}(\gamma-1)}$$

Compressor

$$\tau_c = \frac{1}{\tau_r}\frac{\eta_{comb}\tau_f\eta_m\tau_\lambda(1-\tau_t)+\tau_r(\eta_{comb}\tau_f-\tau_\lambda)}{\eta_{comb}\tau_f-\tau_\lambda+\eta_m\tau_\lambda(1-\tau_t)}$$

$$\pi_c = (\tau_c)^{\eta_{pc}\gamma/(\gamma-1)}$$

Fuel air ratio

$$f = \frac{\tau_\lambda - \tau_r\tau_c}{(\eta_{comb}\tau_f - \tau_\lambda)}$$

f(M_2)

$$f(M_2) = \frac{1}{(1+f)}\pi_b\pi_c\frac{A_4{}^*}{A_2}\left(\frac{\tau_r}{\tau_\lambda}\right)^{1/2}$$

Inlet/diffuser

$$\frac{A_0}{A_2} = \pi_d\frac{f(M_2)}{f(M_0)}$$

Air mass flow

$$\dot{m}_a = \gamma\left(\frac{2\tau_r}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\pi_d f(M_2)\times\frac{P_0A_2}{a_0}$$

Thrust

$$\frac{T}{P_0A_2} = \gamma\left(\frac{2\tau_r}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\pi_d f(M_2)\left((1+f)M_e\left(\frac{T_e}{T_0}\right)^{1/2}-M_0\right)+\frac{A_e}{A_2}\left(\frac{P_e}{P_0}-1\right)$$

$$\frac{A_e}{A_2} = \frac{A_e}{A_8}\times\frac{A_8}{A_4{}^*}\times\frac{A_4{}^*}{A_2}$$

Nozzle area ratio/Mach number

$$\frac{P_{te}}{P_0} = \pi_r\pi_d\pi_c\pi_b\pi_t\pi_n$$

$$M_e = \left(\frac{2}{\gamma-1}\right)^{1/2}\left(\left(\frac{P_{te}}{P_0}\right)^{\frac{(\gamma-1)}{\gamma}}-1\right)^{1/2}$$

$$\frac{A_e}{A_8} = \frac{1}{f(M_e)}$$

Efficiencies and I_{sp}

$$\frac{T_e}{T_0} = \frac{\tau_\lambda\tau_t}{1+\frac{\gamma-1}{2}M_e{}^2}$$

$$\eta_{ov} = \frac{1}{f\tau_f}\frac{\gamma-1}{\gamma}\frac{T}{P_0A_0}$$

$$\eta_{th} = \frac{\gamma-1}{2}\left(\frac{(1+f)M_e{}^2\left(\frac{T_e}{T_0}\right)-M_0{}^2}{f\tau_f}\right)$$

$$\frac{I_{sp}g}{a_0} = \frac{1}{f}\frac{1}{\gamma M_0}\frac{T}{P_0A_0}$$

Problem 1

A test facility designed to measure the mass flow and pressure characteristics of a jet engine compressor is shown in Figure 1. An electric motor is used to power the compressor. The facility draws air in from the surroundings which is at a pressure of one atmosphere and a temperature of 300 K. The air passes through the inlet throat at station 1, is compressed from 2 to 3 and then exhausted through a simple convergent nozzle at station e. Assume the compressor (2-3) operates ideally. Relevant area ratios of the rig are $A_1/A_e = 8$ and $A_1/A_2 = 1/2$. Suppose the power to the compressor is slowly increased from zero.

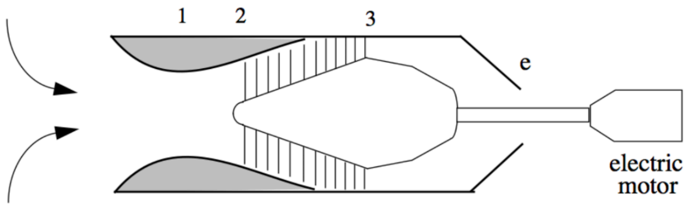

Figure 1: Compressor test facility.

1) Determine the compressor pressure ratio P_{t3}/P_{t2} at which the nozzle chokes.
2) Determine the compressor pressure ratio P_{t3}/P_{t2} at which the inlet throat chokes.
3) Plot the overall pressure ratio P_{te}/P_0 versus the temperature ratio T_{te}/T_0 over the full range from less than sonic flow at station e to beyond the point where a normal shock forms in the inlet.
4) It has been proposed to put a compressor facility like this in one of the basement labs in Durand to support propulsion research. It would operate up to a maximum air mass flow rate of $10 kg/sec$. How much power would be required to operate the facility? Stanford pays about $0.20 per kilowatt-hour for energy. What would be the hourly cost for energy to run the facility?

Problem 2

A turbojet engine is at rest, set for takeoff. The inlet, compressor, burner, turbine and nozzle operate ideally.

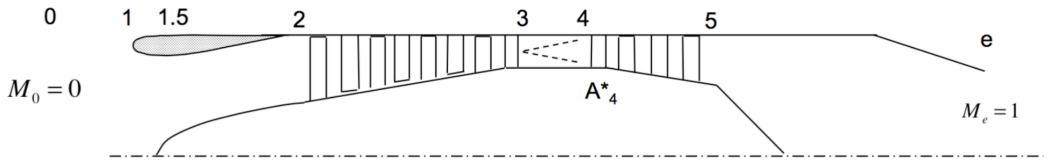

Figure 2: Turbojet ready for take-off.

The nozzle is of simple convergent type with $M_e = 1.0$. Assume $f \ll 1$. The free stream temperature is $300K$ and the turbine inlet temperature is $1500K$. Relevant area ratios are $A_e/A_4{}^* = 2$ and $A_4{}^*/A_2 = 1/8$.
1) Determine τ_t and π_t.
2) Determine τ_c and π_c.
3) Determine $f(M_2)$.
4) Suppose the pilot reduces the throttle to the point where the engine is idling and the exit nozzle is on the verge of un-choking. The engine continues to operate ideally. What value of T_{t4} would produce this condition? Note that the nozzle being on the verge of unchoking means that $M_e = 1.0$ *and* $P_e = P_0$.

Problem 3

Figure 3 shows a turbojet engine flying supersonically. Figure 4 shows typical stagnation pressure and stagnation temperature ratios at various points inside the engine (the figures are not drawn to scale).

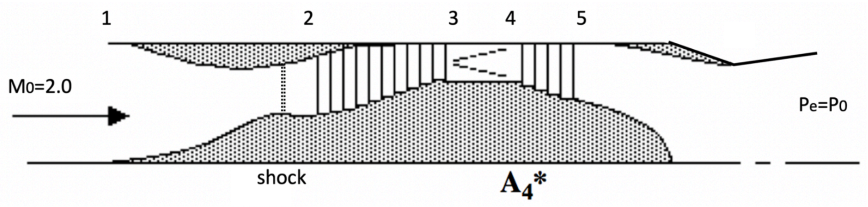

Figure 3: Turbojet flying at Mach 2.0.

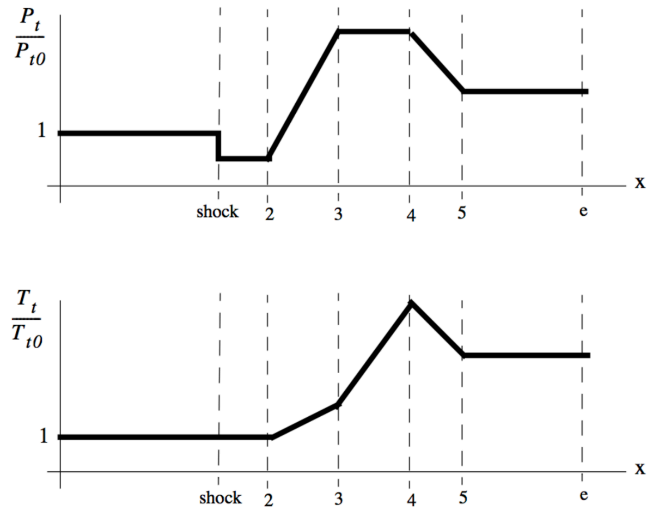

Figure 4: Stagnation pressure and stagnation temperature through a turbojet engine with inlet shock.

The turbine inlet and nozzle throat are choked, and the compressor, burner and turbine operate ideally. At the condition shown $P_e = P_0$. Supersonic flow is established in the inlet and a normal shock is positioned downstream of the inlet throat. Neglect wall friction and assume $f \ll 1$.

Suppose τ_λ is increased while the flight Mach number and engine areas including the nozzle throat and exit area are all constant.

1) Show whether P_{t3}/P_{t0} increases, decreases or remains the same.
2) At each of the stations indicated above explain how the stagnation pressure and stagnation temperature change in response to the increase in τ_λ.
3) Does P_e/P_0 increase of decrease?