AA 218 - Homework 4

Spring 2020

Date due: Tuesday May 12

Read Chapter 9

1) Complete EXERCISE 8.3 – See the paper in the resources folder, “Refracted arrival waves in a zone of silence from a finite thickness mixing layer” by Suzuki and Lele in J. Acoust. Soc. Am. 111(2) Feb 2002. Note equation 24 and fig 5.

2) Complete EXERCISE 9.2 - In problem 9.2 you are asked to “experience” the Lie algorithm through a hand calculation. But before you start, try to identify as many groups as possible by inspection. When you reach the determining equations, try to simplify them as much as possible. How many end up actually playing an important role in determining the infinitesimals? For more information see the paper “Self-Similar Diffusion Flame Including Effects of Streamwise Diffusion” in the resources folder.

We are not having any exams this quarter but I would like to give you an opportunity to practice what you have learned about ODEs in a timed environment. On May 12, I will lecture for the first 20 minutes. At the beginning of the lecture I will send you an exam for you to work during the remaining 60 minutes. See how far you can get. There is no need to turn it in and on May 14 we will go through the solution. We will do a similar exercise on PDEs at the end of the quarter.

The midterm I distribute May 12 will have a very similar format to the exam below. You can use this exam to practice for the May 12 exam if you wish. There is no need to turn it in.

AA 218 – MidTerm Exam

Problem 1
Consider the transformation

$$\tilde{x} = x$$
$$\tilde{y} = ye^{sx}$$

1) Show by composition that the transformation is a Lie group.

2) Determine the infinitesimals of the group.

3) Determine the invariant of the group

4) Determine an invariant family.

Problem 2
Consider the first order ODE

$$xy_x - y\ln(y) + xy = 0$$

1) Show that the equation is invariant under the group in Problem 1.

2) Use this group to construct an integrating factor for the equation.

Problem 3
Consider the second order ODE

$$yy_{xx} - y_x^2 + y^2 = 0$$

1) Show that the equation is invariant under the group in Problem 1.

2) Based on inspection would you say the equation is solvable? If so, why?

See if you can work out the general solutions to the first and second order equations given on the exam (no need to turn this in).