Chapter 2

Engine performance parameters

2.1 The definition of thrust

One might be surprised to learn that there is no direct way to determine the thrust generated by a propulsion system. The reason for this is that the flow over and through an installed engine on an aircraft or an engine attached to a test stand is responsible for the total force on the engine and its nacelle. On any part of the propulsion surface the combination of pressure and viscous stress forces produced by the flow may contribute to the thrust or to the drag and there is no practical way to extricate one force component from the other. Even the most sophisticated test facility can measure the thrust produced by an engine only up to an accuracy of about 0.5%. Wind and weather conditions during the test, inaccuracies in measurement, poorly known flow characteristics in the entrance flow and exhaust and a variety of minor effects limit the ability of a test engineer to precisely measure or predict the thrust of an engine. Thus as a practical matter we must be satisfied with a thrust formula that is purely a definition. Such a definition is only useful to the extent that it reflects the actual thrust force produced by an engine up to some reasonable level of accuracy. In the following, we will use mass and momentum conservation over an Eulerian control volume surrounding a ramjet to motivate a definition of thrust. The control volume is indicated as the dashed line shown in Figure 2.1.

The control volume is in the shape of a cylinder centered about the ramjet. Note that the control volume is simply connected. That is, by suitable distortions without tearing, it is developable into a sphere. The surface of the control volume runs along the entire wetted surface of the ramjet and encloses the inside of the engine. The upstream surface is far enough upstream so that flow variables there correspond to free-stream values. The downstream surface of the control volume coincides with the nozzle exit. The reason for positioning the downstream surface this way is that we need a definition of thrust that
Figure 2.1: Ramjet control volume for developing a definition of thrust

is expressed in terms of flow variables that can be determined relatively easily in terms of the thermodynamic and geometrical properties of the engine internal gas flow. Note that the velocity profile in the wake cannot be used to determine thrust since the profile is momentum-less. An integral over the wake profile is proportional to the sum of thrust plus drag and since the engine is not accelerating this sum is zero. We will assume that within the engine all flow variables are area averaged (averaged in the $y$-$z$ plane) and that the flow is steady. Fuel from an onboard tank is injected through the control volume surface. The mass flows through the engine are

$$
\dot{m}_a = \text{air mass flow rate} \\
\dot{m}_f = \text{fuel mass flow rate.}
$$

The fuel mixes and reacts with the incoming air flow releasing heat and the heat is assumed to be uniformly distributed over the engine cross section downstream of the region of combustion. The integrated form of the conservation equations for steady flow with no body forces on an Eulerian control volume is
\[ \int_A \rho \bar{U} \cdot \bar{n} dA = 0 \]

\[ \int_A \left( \rho \bar{U} \bar{U} + \bar{P} \bar{T} - \bar{P} \right) \cdot \bar{n} dA = 0 \]

\[ \int_A \left( \rho h_t \bar{U} - \bar{P} \cdot \bar{U} + \bar{Q} \right) \cdot \bar{n} dA = 0. \]

where \( h_t \) is the stagnation enthalpy of the gas flow.

\[ h_t = e + P v + k \]

**Mass Balance**

The continuity equation integrated over the control volume leads to

\[ \int_A \rho \bar{U} \cdot \bar{n} dA = \int \rho_2 U_2 dA + \rho_c U_c A_e \]

\[ - \rho_0 U_0 (A_2 + A_e) - \dot{m}_f + \int \rho_1 V_1 dA = 0. \]

The first integral in (2.4) involving a flux of mass out of the control volume is carried out over the annular area labeled \( A_2 \) in Figure 2.1. It is a complicated integral in that it involves the wake velocity profile which is not accurately known without a direct measurement. In fact the nozzle exit flow is assumed to be an area averaged plug flow and so all the complexity of the wake profile is thrown into this integral. The last integral in (2.4) is carried out over the outer surrounding surface of the control volume and involves a flux of mass leaving the control volume due to the outward displacement of air produced by the blockage effect of the engine. It too is a complicated integral but one we will be able to easily approximate. Note that this part of the control volume is taken to be straight. It does not follow a streamline. Thus the area of the upstream face of the control volume is equal to \( A_2 + A_e \).

**Momentum Balance**

Now integrate the \( x \)-momentum equation over the control volume.
\[
\int_A \left( \rho \bar{U} \bar{U} + P \overline{\vec{\tau}} - \overline{\vec{\tau}} \right) \cdot \bar{n} dA = 0
\]

(2.5)

\[
\int_{A_2} \left( \rho_2 U_2^2 + P_2 \right) dA + \left( \rho_e U_e^2 A_e + P_e A_e \right) - \left( \rho_0 U_0^2 + P_0 \right) \left( A_2 + A_e \right) + \\
\int_{A_1} \rho_1 U_1 V_1 dA + \int_{A_w} \left( P \overline{\vec{\tau}} - \overline{\vec{\tau}} \right) \cdot \bar{n} dA = 0
\]

Note that the \(x\)-momentum of the injected fuel mass has been neglected. The first integral involves a complicated distribution of pressure and momentum over the area \(A_2\) and there is little we can do with it. The last integral involves the pressure and stress forces acting over the entire wetted surface of the engine and although the kernel of this integral may be an incredibly complicated function, the integral itself must be zero since the engine is not accelerating or decelerating (the free stream speed is not a function of time).

\[
\int_{A_w} \left( P \overline{\vec{\tau}} - \overline{\vec{\tau}} \right) \cdot \bar{n} dA = \text{Thrust} - \text{Drag} = 0
\]

(2.6)

The second to last integral in (2.5) can be approximated as follows.

\[
\int_{A_1} \rho_1 U_1 V_1 dA \simeq \int_{A_1} \rho_1 U_0 V_1 dA
\]

(2.7)

The argument for this approximation is that at the outside surface of the control volume the \(x\)-component of the fluid velocity is very close to the free stream value. This is a good approximation as long as the control volume surface is reasonably far away from the engine. This approximation allows us to use the mass balance to get rid of this integral. Multiply (2.4) by \(U_0\), and subtract from (2.5). The result is

\[
\rho_e U_e (U_e - U_0) A_e + (P_e - P_0) A_e + \dot{m}_f U_0 + \int_{A_2} \left( \rho_2 U_2 (U_2 - U_0) + (P_2 - P_0) \right) dA = 0. \quad (2.8)
\]

This is as far as we can go with our analysis and at this point we have to make an arbitrary choice. We will define the drag of the engine as
CHAPTER 2. ENGINE PERFORMANCE PARAMETERS

\[ \text{Drag} = \int_{A_2} \left( \rho_2 U_2 (U_0 - U_2) + (P_0 - P_2) \right) dA \]  \hspace{1cm} (2.9) 

and the thrust as

\[ \text{Thrust} = \rho_e U_e (U_e - U_0) A_e + (P_e - P_0) A_e + \dot{m}_f U_0. \]  \hspace{1cm} (2.10)

This is a purely practical choice where the thrust is defined in terms of flow variables that can be determined from a thermo-gas-dynamic analysis of the area-averaged engine internal flow. All the complexity of the flow over the engine has been thrown into the drag integral (2.9) which of course could very well have contributions that could be negative. This would be the case, for example, if some part of the pressure profile had \( P_2 - P_0 > 0 \). The exit mass flow is the sum of the air mass flow plus the fuel mass flow.

\[ \rho_e U_e A_e = \dot{m}_a + \dot{m}_f \]  \hspace{1cm} (2.11)

Using (2.11) the thrust definition (2.10) can be written in the form

\[ T = \dot{m}_a (U_e - U_0) + (P_e - P_0) A_e + \dot{m}_f U_e. \]  \hspace{1cm} (2.12)

In this form the thrust definition can be interpreted as the momentum change of the air mass flow across the engine plus the momentum change of the fuel mass flow. The pressure term reflects the acceleration of the exit flow that occurs as the jet exhaust eventually matches the free stream pressure in the far wake. Keep in mind that the fuel is carried on board the aircraft, and in the frame of reference attached to the engine, the fuel has zero velocity before it is injected and mixed with the air.

The thrust definition (2.12) is very general and applies to much more complex systems. If the selected engine was a turbojet the control volume would look like that shown in Figure 2.2.

The surface of the control volume covers the entire wetted surface of the engine including the struts that hold the rotating components in place as well as the rotating compressor, shaft and turbine. In this case the control volume is of mixed Eulerian-Lagrangian type with part of the control volume surface attached to and moving with the rotating parts. The cut on the engine centerline comes from the wrapping of the control volume about the supports and rotating components. All fluxes cancel on the surface of the cut, which is really a line on the engine axis. The terms arising from the pressure-viscous stress forces on the rotating components are just part of the total surface force integral (2.6) that is still
zero. A mass and momentum balance over the control volume shown in Figure 2.2 would lead to the same result (2.12).

### 2.2 Energy balance

The energy balance across the engine is very simple. The energy equation integrates to

\[
\int_{A} \left( \rho_{f} h_{f} U_{f} - \bar{\rho} \cdot \bar{U} + \bar{Q} \right) \cdot n dA = 0
\]

\[
\int_{A_{2}} \left( \rho_{2} h_{22} U_{2} \right) dA + \rho_{e} h_{e} U_{e} A_{e} - \rho_{0} h_{00} U_{0} (A_{2} + A_{e}) - \bar{m}_{f} h_{f} + \int_{A_{1}} \rho_{1} h_{11} V_{1} dA = 0.
\]

(2.13)

Here the viscous and heat conduction terms across the boundaries of the control volume have been neglected and the flow over the inside and outside surface of the ramjet is assumed to be adiabatic (or at least the temperature of the engine is assumed to be at steady state where any heat conducted into the engine is conducted out elsewhere). This is a very reasonable though not an exact assumption. Some heat is always lost through the engine nacelle but this is a tiny fraction of the enthalpy flow in the exhaust. The viscous power term on the wetted surface is zero due to the no-slip condition. The only contribution over the wetted surface is from the flux of fuel which carries with it its fuel enthalpy \( h_{f} \).
A typical value of fuel enthalpy for JP-4 jet fuel is

\[ h_{f|JP-4} = 4.28 \times 10^7 \text{ J/kg}. \]  

(2.14)

As a comparison, the enthalpy of Air at sea level static conditions is

\[ h_{|Air|288.15K} = C_p T_{SL} = 1005 \times 288.15 = 2.896 \times 10^5 \text{ J/kg}. \]  

(2.15)

The ratio is

\[ \frac{h_{f|JP-4}}{h_{|Air|288.15K}} = 148. \]  

(2.16)

The energy content of a kilogram of hydrocarbon fuel is remarkably large and constitutes one of the important facts of nature that makes extended powered flight possible.

If the flow over the outside of the engine is adiabatic then the stagnation enthalpy of flow over the outside control volume surfaces is equal to the free-stream value and we can write the energy balance as

\[
\int_{A_2} \left( \rho_2 h_{t0} U_2 \right) dA + \rho_e h_{te} U_e A_e - \rho_0 h_{t0} U_0 \left( A_2 + A_e \right) - \dot{m}_f h_f + \int_{A_1} \rho_1 h_{t0} V_1 dA = 0. \]  

(2.17)

Now multiply the continuity equation (2.4) by \( h_{t0} \) and subtract from (2.17). The result is

\[
\rho_e h_{te} U_e A_e - \rho_e h_{t0} U_e A_e - \dot{m}_f \left( h_f - h_{t0} \right) = 0. \]  

(2.18)

Using (2.11) the energy balance across the engine can be written as

\[
\left( \dot{m}_a + \dot{m}_f \right) h_{te} = \dot{m}_a h_{t0} + \dot{m}_f h_f. \]  

(2.19)

The energy balance boils down to a simple algebraic relationship that states that the change in the stagnation enthalpy per second of the gas flow between the exit and entrance of the engine is equal to the added chemical enthalpy per second of the injected fuel flow.
2.3 Capture area

As the operating point of an engine changes, the amount of air passing through the engine may also vary. This is typically the case for an engine operating at subsonic Mach numbers. The capture area of the engine $A_0$ is defined in terms of the air mass flow rate.

$$\dot{m}_a = \rho_0 U_0 A_0$$  \hspace{1cm} (2.20)

The sketches below depict the variation in capture area that can occur as the engine flight Mach number changes from low subsonic to near sonic flight. The geometric entrance area of the engine is $A_1$. Similar changes can occur at a fixed flight Mach number, for example as the engine throttle is changed leading to changes in the demand of the engine for air. More will be said on this topic in a later chapter when we examine how a jet engine operates.

![Variation of inlet capture area with engine operating point.](image)

Figure 2.3: Variation of inlet capture area with engine operating point.

2.4 Overall efficiency

The overall efficiency of a propulsion system is defined as
\[ \eta_{ow} = \frac{\text{The power delivered to the vehicle}}{\text{The total energy released per second through combustion}}. \]  \hspace{1cm} (2.21)

That is

\[ \eta_{ow} = \frac{T U_0}{m_f h_f}. \]  \hspace{1cm} (2.22)

It may not be so obvious but the definition of overall efficiency embodies a certain choice of the frame of reference in which the engine is viewed. In particular we have selected a frame in which the thrust generated by the engine \( T \) acts at a speed \( U_0 \). This is a frame in which the surrounding air is at rest and the engine moves to the left at the given speed. This idea is illustrated in Figure 2.4.

Figure 2.4: Frame of reference used to define efficiencies.

Note that in the frame of reference depicted in Figure 2.1 and Figure 2.2 the power generated by the engine thrust is zero.

To the children observing the engine from the ground in Figure 2.4 a parcel of still air is engulfed by the engine moving to the left and exits the engine as a mixture of air and combustion products with a speed to the right equal to \( U_e - U_0 \).
2.5 Breguet aircraft range equation

There are a number of models of aircraft range. The simplest assumes that the aircraft flies at a constant value of lift to drag ratio and constant engine overall efficiency. The range is

\[ R = \int U_0 dt = \int \frac{\dot{m}_f h_f \eta_{ov}}{T} dt. \]  

(2.23)

The fuel mass flow is directly related to the change in aircraft weight, \( \frac{1}{g} \frac{dw}{dt} \), per second.

\[ \dot{m}_f = \frac{1}{g} \frac{dw}{dt} \]  

(2.24)

Since thrust equals drag and aircraft weight equals lift we can write

\[ T = D = \left( \frac{D}{L} \right) L = \left( \frac{D}{L} \right) w. \]  

(2.25)

Now the range integral becomes

\[ R = -\eta_{ov} \frac{h_f}{g} \left( \frac{L}{D} \right) \int_{w_{initial}}^{w_{final}} \frac{dw}{w}. \]  

(2.26)

The result is

\[ R = \eta_{ov} \frac{h_f}{g} \left( \frac{L}{D} \right) Ln \left( \frac{w_{initial}}{w_{final}} \right). \]  

(2.27)

The range formula (2.27) is generally attributed to the great French aircraft pioneer Louis Charles Breguet who in 1919 founded a commercial airline company that would eventually become Air France. This result highlights the key role played by the engine overall efficiency in determining aircraft range. Note that as the aircraft burns fuel it must increase altitude to maintain constant \( L/D \), and the required thrust decreases. The small, time dependent effects due to the upward acceleration are neglected.
2.6 Propulsive efficiency

It is instructive to decompose the overall efficiency into an aerodynamic factor and a thermal factor. To accomplish this, the overall efficiency is written as the product of a propulsive and thermal efficiency.

\[ \eta_{ov} = \eta_{pr} \times \eta_{th} \]  

(2.28)

The propulsive efficiency is

\[ \eta_{pr} = \frac{\text{Power delivered to the vehicle}}{\text{Power delivered to the vehicle} + \Delta \text{kinetic energy of air} \over \text{second} + \Delta \text{kinetic energy of fuel} \over \text{second}} \]  

(2.29)

or

\[ \eta_{pr} = \frac{TU_0}{TU_0 + \left( \frac{\dot{m}_a(U_e-U_0)^2}{2} - \frac{\dot{m}_a(0)^2}{2} \right) + \left( \frac{\dot{m}_f(U_e-U_0)^2}{2} - \frac{\dot{m}_f(0)^2}{2} \right)} \]  

(2.30)

If the exhaust is fully expanded so that \( P_e = P_0 \) and the fuel mass flow is much less than the air mass flow \( \dot{m}_f \ll \dot{m}_a \), the propulsive efficiency reduces to

\[ \eta_{pr} = \frac{2U_0}{U_e + U_0} \]  

(2.31)

This is quite a general result and shows the fundamentally aerodynamic nature of the propulsive efficiency. It indicates that for maximum propulsive efficiency we want to generate thrust by moving as much air as possible with as little a change in velocity across the engine as possible. We shall see later that this is the basis for the increased efficiency of a turbofan over a turbojet with the same thrust. This is also the basis for comparison of a wide variety of thrusters. For example, the larger the area of a helicopter rotor the more efficient the lift system tends to be.

2.7 Thermal efficiency

The thermal efficiency is defined as
\[ \eta_{th} = \frac{\text{Power delivered to the vehicle} + \Delta \text{kinetic energy of air} + \Delta \text{kinetic energy of fuel}}{\dot{m}_f h_f} \]  

(2.32)

or

\[ \eta_{th} = \frac{T U_0 + \left( \frac{\dot{m}_a (U_e-U_0)^2}{2} - \frac{\dot{m}_a (0)^2}{2} \right) + \left( \frac{\dot{m}_f (U_e-U_0)^2}{2} - \frac{\dot{m}_f (0)^2}{2} \right)}{\dot{m}_f h_f}. \]  

(2.33)

If the exhaust is fully expanded so that \( P_e = P_0 \) the thermal efficiency reduces to

\[ \eta_{th} = \frac{(\dot{m}_a + \dot{m}_f) \frac{U_e^2}{2} - \dot{m}_a \frac{U_0^2}{2}}{\dot{m}_f h_f}. \]  

(2.34)

The thermal efficiency directly compares the change in gas kinetic energy across the engine to the energy released through combustion.

The thermal efficiency of a thermodynamic cycle compares the work out of the cycle to the heat added to the cycle.

\[ \eta_{th} = \frac{W}{Q_{\text{input during the cycle}}} = \frac{Q_{\text{input during the cycle}} - Q_{\text{rejected during the cycle}}}{Q_{\text{input during the cycle}}} = 1 - \frac{Q_{\text{rejected during the cycle}}}{Q_{\text{input during the cycle}}}. \]  

(2.35)

We can compare (2.34) and (2.35) by rewriting (2.34) as

\[ \eta_{th} = 1 - \left( \frac{\dot{m}_f h_f + \dot{m}_a \frac{U_0^2}{2} - (\dot{m}_a + \dot{m}_f) \frac{U_e^2}{2}}{\dot{m}_f h_f} \right). \]  

(2.36)

This equation for the thermal efficiency can also be expressed in terms of the gas enthalpies. Recall that

\[ h_{te} = h_e + \frac{U_e^2}{2} \]  

(2.37)

\[ h_{t0} = h_0 + \frac{U_0^2}{2}. \]
CHAPTER 2. ENGINE PERFORMANCE PARAMETERS

Replace the velocities in (2.36).

\[ \eta_{th} = 1 - \left( \frac{\dot{m}_f h_f + \dot{m}_a (h_0 - h_0) - (\dot{m}_a + \dot{m}_f) (h_e - h_e)}{\dot{m}_f h_f} \right) \tag{2.38} \]

Use (2.19) to replace \( \dot{m}_f h_f \) in (2.38). The result is

\[ \eta_{th} = 1 - \frac{Q_{\text{rejected during the cycle}}}{Q_{\text{input during the cycle}}} = 1 - \left( \frac{\dot{m}_a + \dot{m}_f) (h_e - h_0) + \dot{m}_f h_0}{\dot{m}_f h_f} \right). \tag{2.39} \]

According to (2.39) the heat rejected during the cycle is

\[ Q_{\text{rejected during the cycle}} = (\dot{m}_a + \dot{m}_f) (h_e - h_0) + \dot{m}_f h_0. \tag{2.40} \]

This expression deserves some discussion. Strictly speaking the engine is not a closed system because of the fuel mass addition across the burner. So the question is; How does the definition of thermal efficiency account for this mass exchange within the concept of the thermodynamic cycle? The answer is that the heat rejected from the exhaust is comprised of two distinct parts. There is the heat rejected by conduction from the nozzle flow to the surrounding atmosphere plus physical removal from the thermally equilibrated nozzle flow of a portion equal to the added fuel mass flow. From this perspective, the fuel mass flow carries its fuel enthalpy into the system by injection in the burner and the exhaust fuel mass flow carries its ambient enthalpy out of the system by mixing with the surroundings. There is no net mass increase or decrease to the system.

Note that there is no assumption that the compression or expansion process operates isentropically, only that the exhaust is fully expanded.

2.8 Specific impulse, specific fuel consumption

An important measure of engine performance is the amount of thrust produced for a given amount of fuel burned. This leads to the definition of specific impulse

\[ I_{sp} = \frac{\text{Thrust force}}{\text{Weight flow of fuel burned}} = \frac{T}{\dot{m}_f g} \tag{2.41} \]

with units of seconds. The specific fuel consumption is essentially the inverse of the specific impulse.
CHAPTER 2. ENGINE PERFORMANCE PARAMETERS

\[ SFC = \frac{\text{Pounds of fuel burned per hour}}{\text{Pounds of thrust}} = \frac{3600}{I_{sp}} \] (2.42)

The specific fuel consumption is a relatively easy to remember number of order one. Some typical values are

\[ SFC|_{JT9D-\text{takeoff}} \approx 0.35 \]
\[ SFC|_{JT9D-\text{cruise}} \approx 0.6 \]
\[ SFC|_{\text{military engine}} \approx 0.9 \text{to} 1.2 \]
\[ SFC|_{\text{military engine with afterburning}} \approx 2. \] (2.43)

The SFC generally goes up as an engine moves from takeoff to cruise, as the energy required to produce a pound of thrust goes up with increased percentage of stagnation pressure losses and with the increased momentum of the incoming air.

2.9 Dimensionless forms

We have already noted the tendency to use both Metric and English units in dealing with propulsion systems. Unfortunately, despite great effort, the US propulsion industry has been unable to move away from the clumsy system of English units. Whereas the rest of the world, including the British, has gone fully metric. This is a real headache and something we will just have to live with, but the problem is vastly reduced by expressing all of our equations in dimensionless form.

*Dimensionless forms of the Thrust.*

\[ \frac{T}{P_0 A_0} = \gamma M_0^2 \left( 1 + f \frac{U_e}{U_0} - 1 \right) + \frac{A_e}{A_0} \left( \frac{P_e}{P_0} - 1 \right) \] (2.44)

\[ \frac{T}{\dot{m}_a a_0} = \left( \frac{1}{\gamma M_0} \right) \frac{T}{P_0 A_0} \]

Normalizing the thrust by \( P_0 A_0 \) produces a number that compares the thrust to a force equal to the ambient pressure multiplied by the capture area. In order to overcome drag it is essential that this be a number considerably larger than one.

*Dimensionless Specific impulse.*
The quantity $f$ is the fuel/air ratio defined as

$$ f = \frac{\dot{m}_f}{\dot{m}_a}. $$

(2.46)

**Overall efficiency.**

$$ \eta_{ov} = \left( \frac{\gamma - 1}{\gamma} \right) \left( \frac{1}{f \tau_f} \right) \left( \frac{T}{P_0 A_0} \right) $$

(2.47)

The ratio of fuel to ambient enthalpy appears in this definition.

$$ \tau_f = \frac{h_f}{C_p T_0} $$

(2.48)

And $T_0$ is the temperature of the ambient air. Note that the fuel/air ratio is relatively small whereas $\tau_f$ is rather large (See (2.16)). Thus $1/(f \tau_f)$ is generally a fraction somewhat less than one.

### 2.10 Engine notation

An important part of analyzing the performance of a propulsion system has to do with being able to determine how each component of the engine contributes to the overall thrust and specific impulse. To accomplish this, we will use a standard notation widely used in industry for characterizing the pressure and temperature change across each component. First we need to adopt a standard system for numbering the engine components. Consider the generic engine cross sections shown in Figure 2.5.

The performance of each component is defined in terms of the stagnation pressure and temperature entering and leaving the component. A widely accepted notation is

$$ \tau = \frac{\text{The stagnation temperature leaving the component}}{\text{The stagnation temperature entering the component}} $$

(2.49)

$$ \pi = \frac{\text{The stagnation pressure leaving the component}}{\text{The stagnation pressure entering the component}}. $$
Figure 2.5: Engine numbering and component notation

The various stations are defined as follows.

**Station 0** - This is the reference state of the gas well upstream of the engine entrance. The temperature/pressure parameters are

\[
\tau_r = \frac{T_{r0}}{T_0} = 1 + \left(\frac{\gamma - 1}{2}\right) M_0^2
\]

\[
\pi_r = \frac{P_{r0}}{P_0} = \left(1 + \left(\frac{\gamma - 1}{2}\right) M_0^2\right)^{\frac{\gamma}{\gamma - 1}}.
\]

Note that these definitions are exceptional in that the denominator is the static temperature and pressure of the free stream.

**Station 1** - Entrance to the engine inlet. The purpose of the inlet is to reduce the Mach number of the incoming flow to a low subsonic value with as small a stagnation pressure loss as possible. From the entrance to the end of the inlet there is always an increase in area and so the component is appropriately called a diffuser.

**Station 1.5** - The inlet throat.

**Station 2** - The fan or compressor face. The temperature/pressure parameters across the diffuser are
\[ \tau_d = \frac{T_{t2}}{T_{t1}} \]
\[ \pi_d = \frac{P_{t2}}{P_{t1}} \] (2.51)

In the absence of an upstream shock wave the flow from the reference state is regarded as adiabatic and isentropic so that
\[ T_{t1} = T_{t0} \]
\[ P_{t1} = P_{t0} \] (2.52)

The inlet is usually modeled as an adiabatic flow so the stagnation temperature is approximately constant, however the stagnation pressure decreases due to the presence of viscous boundary layers and possibly shock waves.

**Station 2.5** - All turbofan engines comprise at least two spools. The fan is usually accompanied by a low pressure compressor driven by a low pressure turbine through a shaft along the centerline of the engine. A concentric shaft connects the high pressure turbine and high pressure compressor. Station 2.5 is generally taken at the interface between the low and high pressure compressor. Roll Royce turbofans commonly employ three spools with the high pressure compressor broken into two spools.

**Station 13** - This is a station in the bypass stream corresponding to the fan exit ahead of the entrance to the fan exhaust nozzle. The temperature/pressure parameters across the fan are
\[ \tau_{1c} = \frac{T_{13}}{T_{t2}} \]
\[ \pi_{1c} = \frac{P_{13}}{P_{t2}} \] (2.53)

**Station 18** - The fan nozzle throat.

**Station 1e** - The fan nozzle exit. The temperature/pressure parameters across the fan nozzle are
\[ \tau_{1n} = \frac{T_{1e}}{T_{t13}} \]
\[ \pi_{1n} = \frac{P_{1e}}{P_{t13}}. \]  
(2.54)

**Station 3** - The exit of the high pressure compressor. The temperature/pressure parameters across the compressor are

\[ \tau_c = \frac{T_{t3}}{T_{t2}} \]
\[ \pi_c = \frac{P_{t3}}{P_{t2}}. \]  
(2.55)

Note that the compression includes that due to the fan. From a cycle perspective it is usually not necessary to distinguish the high and low pressure sections of the compressor. The goal of the designer is to produce a compression system that is as near to isentropic as possible.

**Station 4** - The exit of the burner. The temperature/pressure parameters across the burner are

\[ \tau_b = \frac{T_{t4}}{T_{t3}} \]
\[ \pi_b = \frac{P_{t4}}{P_{t3}}. \]  
(2.56)

The temperature at the exit of the burner is regarded as the highest temperature in the Brayton cycle although generally higher temperatures do occur at the upstream end of the burner where combustion takes place. The burner is designed to allow an influx of cooler compressor air to mix with the combustion gases bringing the temperature down to a level that the high pressure turbine structure can tolerate. Modern engines use sophisticated cooling methods to enable operation at values of \( T_{t4} \) that approach 3700\( R \) (2050K), well above the melting temperature of the turbine materials.

**Station 4.5** - This station is at the interface of the high and low pressure turbines.

**Station 5** - The exit of the turbine. The temperature/pressure parameters across the turbine are
CHAPTER 2. ENGINE PERFORMANCE PARAMETERS

\[ \tau_t = \frac{T_{t5}}{T_{t4}} \]

\[ \pi_t = \frac{P_{t5}}{P_{t4}}. \]

(2.57)

As with the compressor the goal of the designer is to produce a turbine system that operates as isentropically as possible.

**Station 6** - The exit of the afterburner if there is one. The temperature/pressure parameters across the afterburner are

\[ \tau_a = \frac{T_{t6}}{T_{t5}} \]

\[ \pi_a = \frac{P_{t6}}{P_{t5}}. \]

(2.58)

The Mach number entering the afterburner is fairly low and so the stagnation pressure ratio of the afterburner is fairly close to, and always less than, one.

**Station 7** - The entrance to the nozzle.

**Station 8** - The nozzle throat. Over the vast range of operating conditions of modern engines the nozzle throat is choked or very nearly so.

**Station e** - The nozzle exit. The temperature/pressure component parameters across the nozzle are

\[ \tau_n = \frac{T_{te}}{T_{t7}} \]

\[ \pi_n = \frac{P_{te}}{P_{t7}}. \]

(2.59)

In the absence of the afterburner, the nozzle parameters are generally referenced to the turbine exit condition so that

\[ \tau_n = \frac{T_{te}}{T_{t5}} \]

\[ \pi_n = \frac{P_{te}}{P_{t5}}. \]

(2.60)
In general the goal of the designer is to minimize heat loss and stagnation pressure loss through the inlet, burner and nozzle.

There are two more very important parameters that need to be defined. The first is one we encountered before when we compared the fuel enthalpy to the ambient air enthalpy.

\[
\tau_f = \frac{h_f}{C_p T_0}
\] (2.61)

The second parameter is, in a sense, the most important quantity needed to characterize the performance of an engine.

\[
\tau_\lambda = \frac{T_{t4}}{T_0}
\] (2.62)

In general every performance measure of the engine gets better as \(\tau_\lambda\) is increased and a tremendous investment has been made over the years to devise turbine cooling and ceramic coating schemes that permit ever higher turbine inlet temperatures, \(T_{t4}\).

### 2.11 Problems

**Problem 1** - Suppose 10% of the heat generated in a ramjet combustor is lost through conduction to the surroundings. How would this change the energy balance (2.19)? How would it affect the thrust?

**Problem 2** - Write down the appropriate form of the thrust definition (2.12) for a turbofan engine with two independent streams. Suppose 5% of the air from the high pressure compressor is to be used to power aircraft systems. What would be the appropriate thrust formula?

**Problem 3** - Consider the flow through a turbojet. The energy balance across the burner is

\[
(\dot{m}_a + \dot{m}_f) h_{t4} = \dot{m}_a h_{t3} + \dot{m}_f h_f.
\] (2.63)

The enthalpy rise across the compressor is equal to the enthalpy decrease across the turbine. Show that the energy balance (2.63) can also be written

\[
(\dot{m}_a + \dot{m}_f) h_{t5} = \dot{m}_a h_{t2} + \dot{m}_f h_f.
\] (2.64)
The inlet and nozzle are usually assumed to operate adiabatically. Show that (2.64) can be expressed as

\[(\dot{m}_a + \dot{m}_f) h_{te} = \dot{m}_a h_{t0} + \dot{m}_f h_f\]  \hspace{1cm} (2.65)

which is the same as the overall enthalpy balance for a ramjet (2.19).

**Problem 4** - Work out the dimensionless forms in Section 2.9.