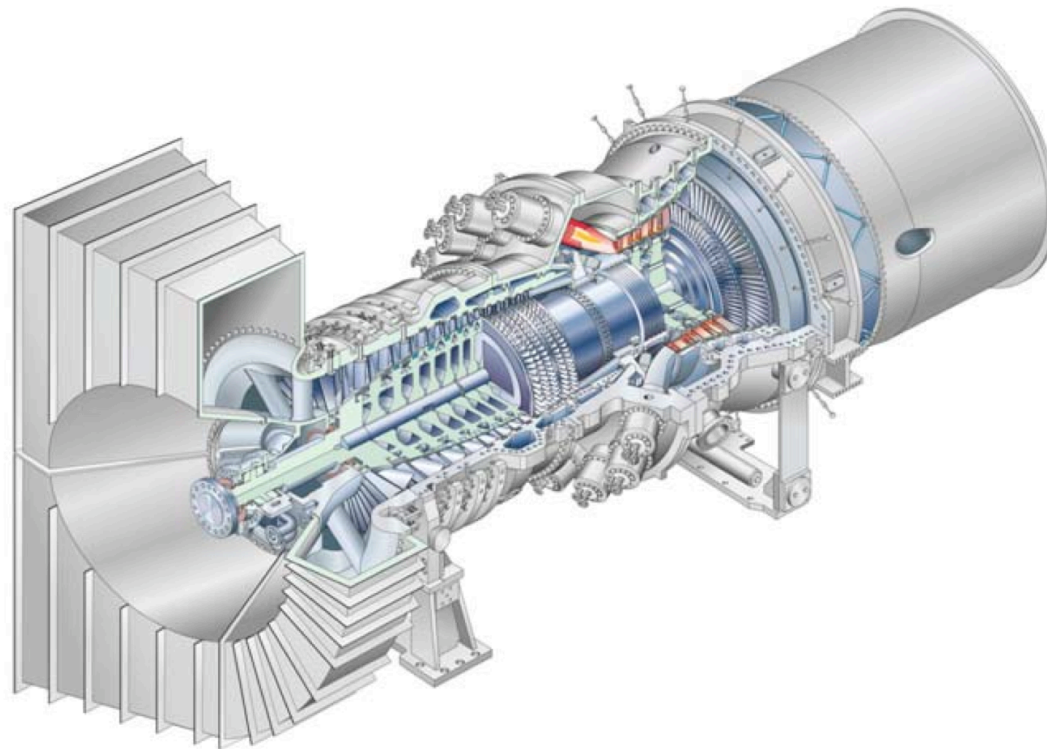
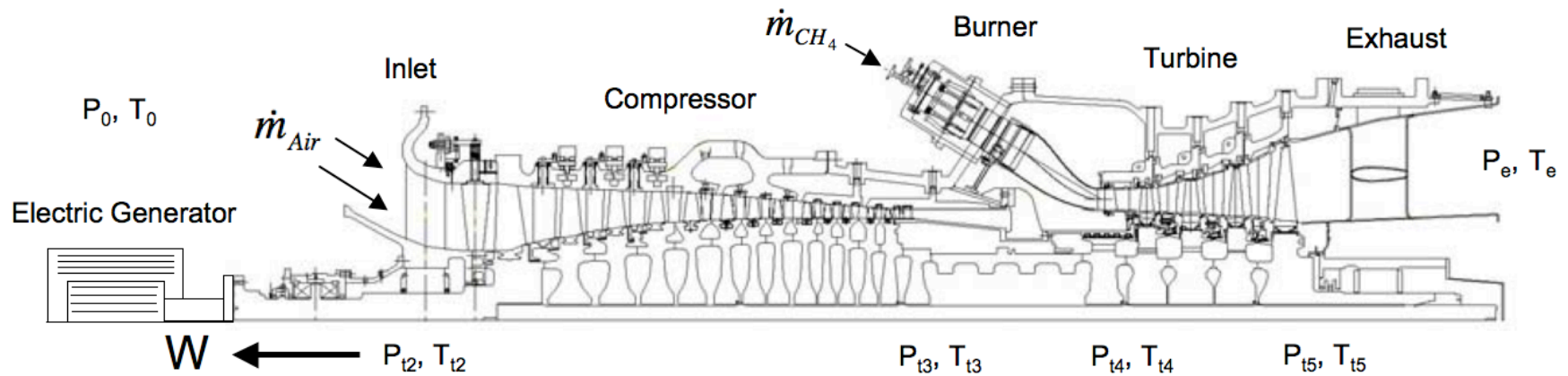


# Gas turbine based power generation

Siemens SGT6-5000F(4) 150 MW power Gas Turbine





$$\eta_{overall} = \frac{\text{Shaft power out (J / sec)}}{\text{fuel mass flow (kg / sec)} \times \text{fuel heating value (J / kg)}} = \frac{W}{\dot{m}_f h_f}$$

$$h_{fCH_4} = 5.543 \times 10^7 \text{ J / kg}$$

$$\text{Net Heat Rate} = \frac{3.6}{\eta_{overall}} \left( \frac{\text{MegaJoules}}{\text{kiloWatt - hour}} \right)$$

## Cycle Analysis

Across the burner.

$$\dot{m}_f h_f = (\dot{m}_a + \dot{m}_f) h_{t4} - \dot{m}_a h_{t3}$$

$$f = \frac{\tau_\lambda - \tau_c}{\tau_f - \tau_\lambda}$$

$$\tau_f = \frac{h_f}{C_p T_0} = \frac{5.543 \times 10^7}{1005 \times 298.15} = 185$$

Power balance.

$$(\dot{m}_a + \dot{m}_f)(h_{t4} - h_{t5}) = \dot{m}_a(h_{t3} - h_{t2}) + W$$

Divide by  $\dot{m}_a C_p T_0$

$$C = (1 + f)\tau_\lambda(1 - \tau_t) - (\tau_c - 1)$$

Work output coefficient.

$$C = \frac{W}{\dot{m}_a C_p T_0}$$

$$\eta_{overall} = \frac{W}{\dot{m}_f h_f} = \frac{\dot{m}_a C_p T_0}{\dot{m}_f h_f} \frac{W}{\dot{m}_a C_p T_0} = \frac{C}{f\tau_f}$$

The system is designed to take as much work as possible out of the turbine. The exit Mach number is very low and the exit pressure is very close to the ambient pressure.

$$P_{t5} = P_{te} \cong P_0$$

$$\frac{P_{te}}{P_0} = \frac{P_{t2}}{P_0} \frac{P_{t3}}{P_{t2}} \frac{P_{t4}}{P_{t3}} \frac{P_{t5}}{P_{t4}} \frac{P_{te}}{P_{t5}} = 1$$

Ideal Cycle.

$$\eta_m = 1 \quad \pi_d = 1 \quad \pi_b = 1 \quad \pi_n = 1 \quad \eta_{pc} = 1 \quad \eta_{pe} = 1$$

$$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}} \quad \pi_t = \tau_t^{\frac{\gamma}{\gamma-1}}$$

$$\tau_c \tau_t = 1$$

$$C = (1 + f) \tau_\lambda (1 - \tau_t) - (\tau_t^{-1} - 1)$$

$$f \ll 1$$

$$\frac{\partial C}{\partial \tau_t} = -\tau_\lambda + \tau_t^{-2} = 0$$

For the ideal cycle the maximum work output coefficient occurs when

$$\tau_t = \frac{1}{\sqrt{\tau_\lambda}}$$

and

$$\tau_c = \sqrt{\tau_\lambda}$$

## Ideal Gas Turbine Power Generator.

$$\tau_{t \text{ max work ideal}} = \frac{1}{\sqrt{\tau_\lambda}}$$

$$\tau_c = \sqrt{\tau_\lambda}$$

$$C_{\text{max work ideal}} = \tau_\lambda - 2\sqrt{\tau_\lambda} + 1$$

$$f = \frac{\tau_\lambda - \sqrt{\tau_\lambda}}{\tau_f - \tau_\lambda}$$

$$\eta_{\text{overall}} = \frac{\tau_\lambda - 2\sqrt{\tau_\lambda} + 1}{\tau_\lambda - \sqrt{\tau_\lambda}}$$

Example, let  $\tau_\lambda = 4$

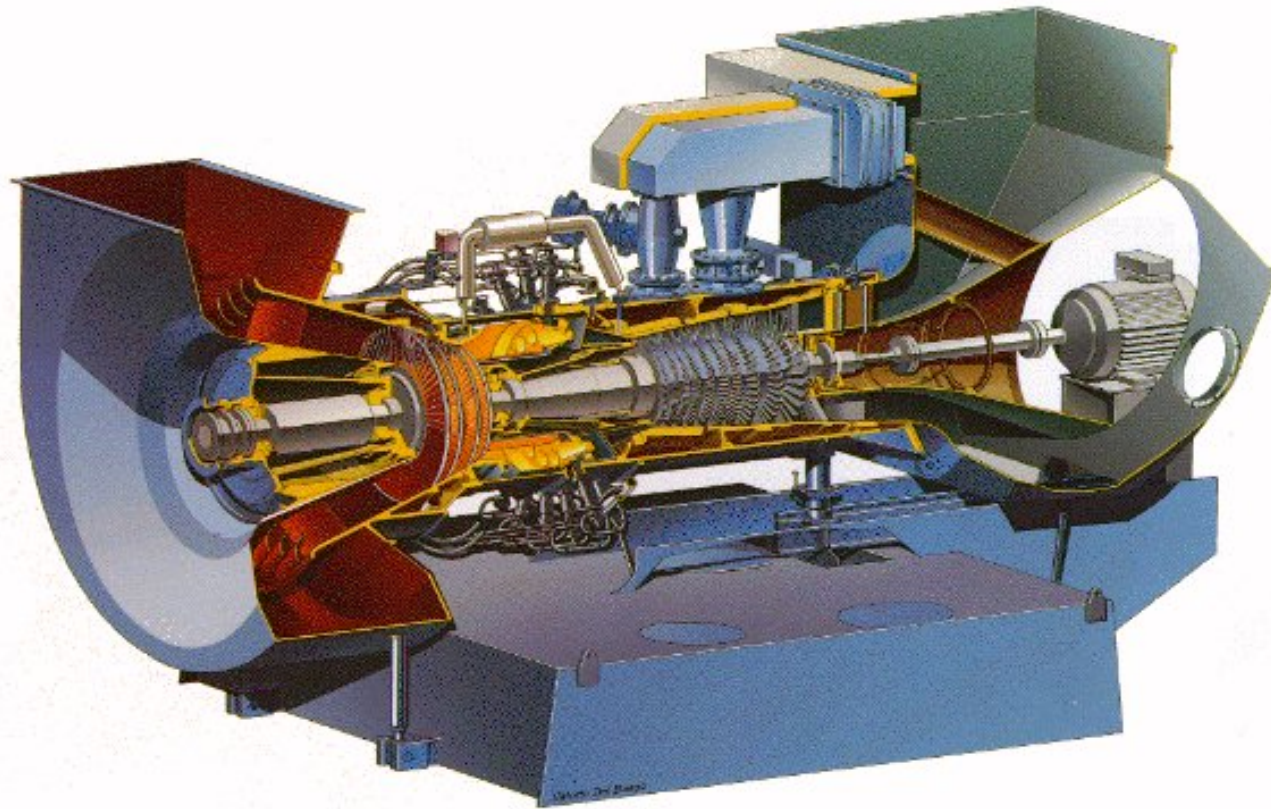
$$\tau_{t \text{ max workideal}} = \frac{1}{2} \quad \tau_c = 2 \quad \pi_c = 11.31 \quad C_{\text{max workideal}} = 1$$

$$f = 0.0123 \quad \eta_{\text{overall}} = \frac{1}{2}$$

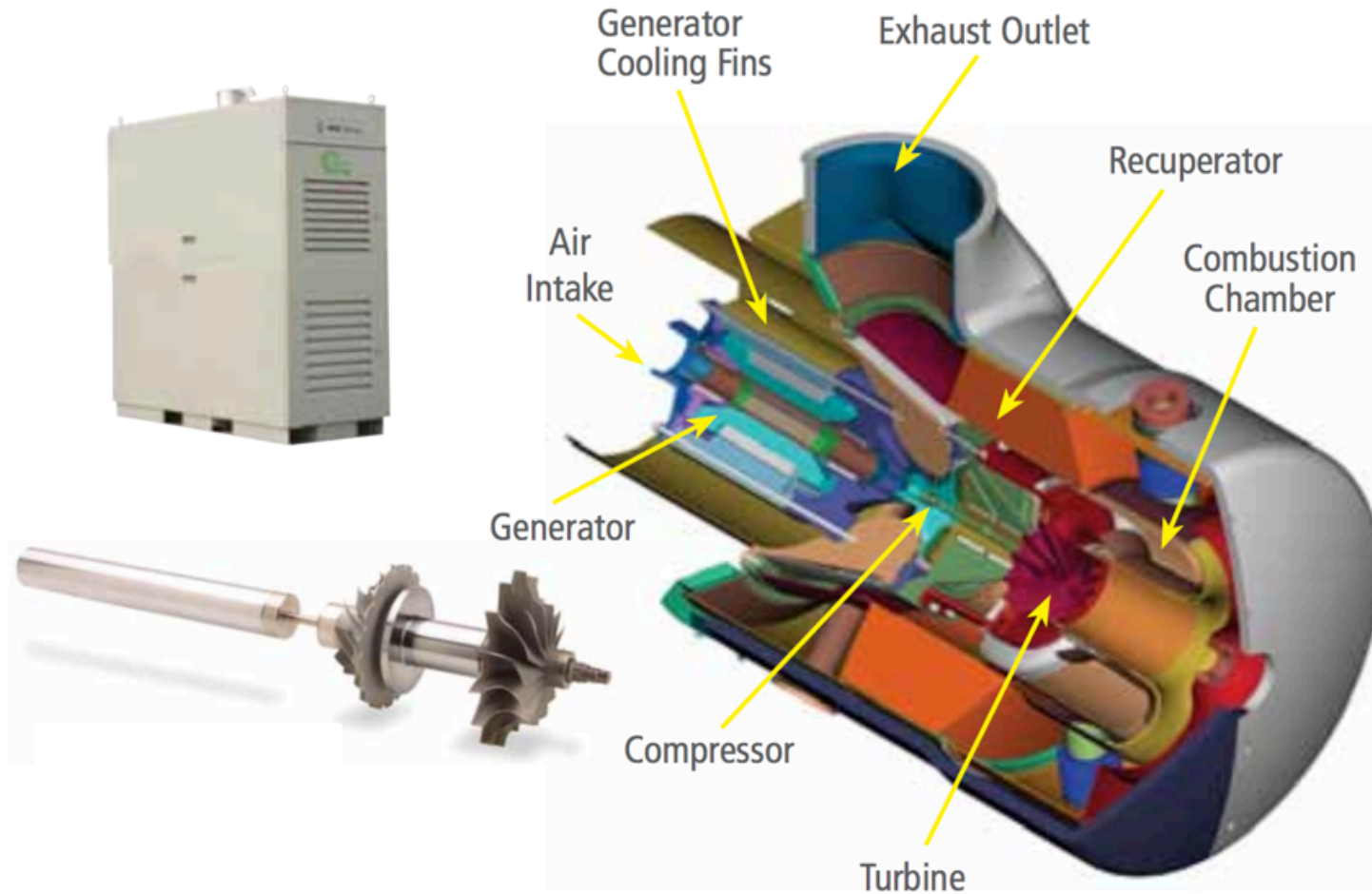


## Gas Turbines for Power Generation

25MW Gas Turbine GT10

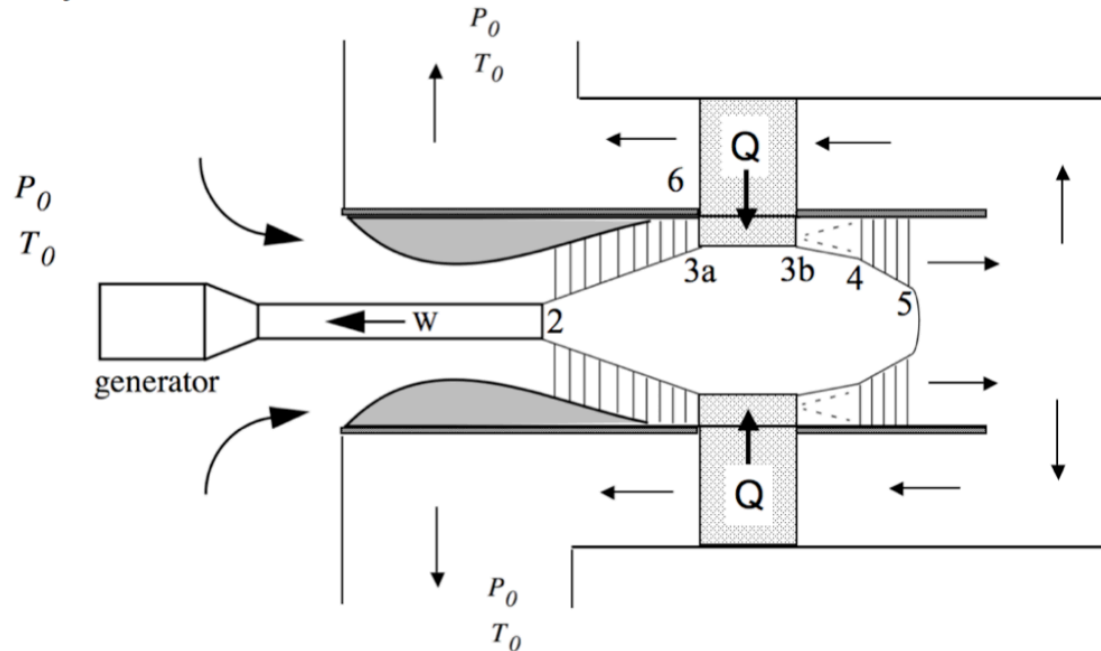


## Capstone Turbine Corp - microturbine



## Example from a previous midterm

**Problem 1** – A power gas turbine is shown below. The inlet (no shock), burner, turbine and exhaust operate ideally.



A heat exchanger is used to transfer heat from the turbine exhaust flow to the air entering the burner. Let

$$\tau_x = T_{t3b} / T_{t3a} \quad \pi_x = P_{t3b} / P_{t3a} \quad \tau_{1x} = T_{t6} / T_{t5} \quad \pi_{1x} = P_{t6} / P_{t5}$$

Assume the heat exchanger operates ideally  $\pi_x = \pi_{1x} = 1$ . The system is designed so that the stagnation pressure of the exhaust flow equals the ambient pressure,  $P_{t6} = P_0$  and the stagnation temperature of the exhaust flow equals the compressor outlet temperature,  $T_{t6} = T_{t3a}$ . Assume  $f \ll 1$  where appropriate.

1) **(4 points)** Write down the enthalpy balance that governs the heat transferred by the heat exchanger between the exhaust flow and the compressor exit flow. Use this balance to show that

$$\tau_{1x} = \frac{1}{\tau_x} \quad \tau_t \tau_\lambda (1 - \tau_{1x}) = \tau_c (\tau_x - 1)$$

The appropriate thermal balance is

$$(\dot{m}_a + \dot{m}_f)(h_{t5} - h_{t6}) = \dot{m}_a (h_{t3b} - h_{t3a})$$

$$(T_{t5} - T_{t6}) = (T_{t3b} - T_{t3a})$$

$$T_{t6} = T_{t3a} \text{ therefore } T_{t5} = T_{t3b}$$

$$\frac{T_{t6}}{T_{t5}} = \frac{T_{t3a}}{T_{t3b}} \text{ or } \tau_{1x} = \frac{1}{\tau_x}$$

$$\tau_t \tau_\lambda (1 - \tau_{1x}) = \tau_c (\tau_x - 1)$$

$$\tau_t \tau_\lambda \left( 1 - \frac{1}{\tau_x} \right) = \tau_c (\tau_x - 1)$$

2) **(4 points)** Write down the enthalpy balance that governs the heat released across the burner. Use this balance to show that

$$f = \frac{\tau_\lambda - \tau_x \tau_c}{\tau_f}$$

$$\dot{m}_f h_f = (\dot{m}_a + \dot{m}_f) h_{t4} - \dot{m}_a h_{t3b}$$

$$f \tau_f = \tau_\lambda - \frac{T_{t3b}}{T_{t3a}} \frac{T_{t3a}}{T_0} = \tau_\lambda - \tau_x \tau_c$$

3) **(4 points)** Write down the enthalpy balance that governs the power transferred between the turbine, compressor and generator shaft. Use this balance to show that the work output coefficient

$$C = W / \dot{m}_a C_p T_0 \text{ is}$$

$$C = \tau_\lambda (1 - \tau_t) - (\tau_c - 1)$$

$$(\dot{m}_a + \dot{m}_f)(h_{t4} - h_{t5}) = \dot{m}_a (h_{t3a} - h_{t2}) + W$$

$$C = \tau_\lambda (1 - \tau_t) - (\tau_c - 1)$$

4) **(3 points)** The compressor does not operate ideally,  $\pi_c = \tau_c^{\frac{\gamma}{\eta_{pc}}}$ . Show that  $\tau_c = 1 / \tau_t^{1/\eta_{pc}}$ .

$$\text{Let } \pi_c = \tau_c^{\frac{\gamma}{\eta_{pc}}} \text{ and } \pi_t = \tau_t^{\frac{\gamma}{\eta_{pc}}}$$

$$\text{Note } P_{t6} = \pi_c \pi_t P_0 = \tau_c^{\frac{\gamma}{\eta_{pc}}} \tau_t^{\frac{\gamma}{\eta_{pc}}} P_0 = \left( \tau_c^{\eta_{pc}} \tau_t \right)^{\frac{\gamma}{\eta_{pc}}} P_0 \text{ and since } P_{t6} = P_0 \text{ thus } \tau_t = 1 / \tau_c^{\eta_{pc}} \text{ and}$$

$$\tau_c = 1 / \tau_t^{1/\eta_{pc}}$$

5) (3 points) Show that the turbine temperature ratio that maximizes the work output coefficient is

$$\tau_{t \max} = \frac{1}{\left(\eta_{pc} \tau_{\lambda}\right)^{\frac{\eta_{pc}}{1+\eta_{pc}}}}$$

Now

$$C = \tau_{\lambda}(1 - \tau_t) - \left( \frac{1}{\tau_t^{1/\eta_{pc}}} - 1 \right)$$

$$\frac{\partial C}{\partial \tau_t} = -\tau_{\lambda} + \left( \frac{1}{\eta_{pc}} \right) \frac{1}{\tau_t^{\frac{1}{\eta_{pc}}+1}} = 0$$

Now

$$\tau_{t \max} = \frac{1}{\left(\eta_{pc} \tau_{\lambda}\right)^{\frac{\eta_{pc}}{1+\eta_{pc}}}}$$

$$\tau_{c \max} = \left(\eta_{pc} \tau_{\lambda}\right)^{\frac{1}{1+\eta_{pc}}}$$

and

$$C = \tau_{\lambda} \left( 1 - \frac{1}{\left(\eta_{pc} \tau_{\lambda}\right)^{\frac{\eta_{pc}}{1+\eta_{pc}}}} \right) - \left( \left(\eta_{pc} \tau_{\lambda}\right)^{\frac{1}{1+\eta_{pc}}} - 1 \right)$$

6) **(3 points)** Use the result in (1) to show that at this condition  $\tau_{x\max} = 1/\eta_{pc}$ .

$$\tau_t \tau_\lambda \left( 1 - \frac{1}{\tau_x} \right) = \tau_c (\tau_x - 1)$$

$$\frac{\tau_\lambda^{\frac{1}{1+\eta_{pc}}}}{(\eta_{pc})^{\frac{\eta_{pc}}{1+\eta_{pc}}}} \left( 1 - \frac{1}{\tau_x} \right) = (\eta_{pc} \tau_\lambda)^{\frac{1}{1+\eta_{pc}}} (\tau_x - 1)$$

$$\left( 1 - \frac{1}{\tau_x} \right) = \eta_{pc} (\tau_x - 1)$$

$$\tau_x^2 - \left( 1 + \frac{1}{\eta_{pc}} \right) \tau_x + \frac{1}{\eta_{pc}} = 0 \quad \text{Roots are } \tau_x = 1 \text{ and } \tau_x = \frac{1}{\eta_{pc}}$$



7) (4 points) Let  $\tau_\lambda = 4$  and  $\eta_{pc} = 0.75$ . Determine the overall efficiency  $\eta_{overall\ max} = W / \dot{m}_f h_f$ . How does this compare with the overall efficiency of the case  $\eta_{pc} = 1.0$ ? How does this compare with the overall efficiency of the case with no heat exchanger?

$$\eta_{overall\ max} = \frac{\tau_\lambda \left( 1 - \frac{1}{(\eta_{pc} \tau_\lambda)^{\frac{\eta_{pc}}{1+\eta_{pc}}}} \right) - \left( (\eta_{pc} \tau_\lambda)^{\frac{1}{1+\eta_{pc}}} - 1 \right)}{\tau_\lambda - \tau_x (\eta_{pc} \tau_\lambda)^{\frac{1}{1+\eta_{pc}}}}$$

In the ideal case with  $\tau_x = 1$

$$\eta_{overall\ max\ ideal} = \frac{\tau_\lambda \left( 1 - \frac{1}{(\tau_\lambda)^{\frac{1}{2}}} \right) - \left( (\tau_\lambda)^{\frac{1}{2}} - 1 \right)}{\tau_\lambda - (\tau_\lambda)^{\frac{1}{2}}} = \frac{4 \left( 1 - \frac{1}{2} \right) - (2 - 1)}{4 - 2} = \frac{1}{2}$$

In the real case with  $\tau_x = 1$

$$\eta_{overall\ max\ real} = \frac{\tau_\lambda \left( 1 - \frac{1}{(\eta_{pc} \tau_\lambda)^{\frac{\eta_{pc}}{1+\eta_{pc}}}} \right) - \left( (\eta_{pc} \tau_\lambda)^{\frac{1}{1+\eta_{pc}}} - 1 \right)}{\tau_\lambda - \tau_x (\eta_{pc} \tau_\lambda)^{\frac{1}{1+\eta_{pc}}}} = \frac{4 \left( 1 - \frac{1}{(3)^{\frac{3}{7}}} \right) - \left( (3)^{\frac{4}{7}} - 1 \right)}{4 - (3)^{\frac{4}{7}}} = 0.29561$$

In the real case the efficiency is only about 30% compared to 50% for the ideal case



In the real case with  $\tau_x = \frac{1}{\eta_{pc}} = \frac{4}{3}$

$$\begin{aligned}
 \eta_{overall\ max} &= \frac{\tau_\lambda \left( 1 - \frac{1}{\left( \eta_{pc} \tau_\lambda \right)^{\frac{\eta_{pc}}{1+\eta_{pc}}}} \right) - \left( \left( \eta_{pc} \tau_\lambda \right)^{\frac{1}{1+\eta_{pc}}} - 1 \right)}{\tau_\lambda - \left( \frac{1}{\eta_{pc}} \right) \left( \eta_{pc} \tau_\lambda \right)^{\frac{1}{1+\eta_{pc}}}} = \\
 &= \frac{4 \left( 1 - \frac{1}{\left( 3 \right)^{\frac{3}{7}}} \right) - \left( \left( 3 \right)^{\frac{4}{7}} - 1 \right)}{4 - \left( \frac{4}{3} \right) \left( 3 \right)^{\frac{4}{7}}} = 0.418508
 \end{aligned}$$

The recuperator improves the efficiency of the real engine by 12%