AA 284a
Advanced Rocket Propulsion

Lecture 13
Component Design Issues

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Mass Model

- **Propulsion System**
  - Tanks (estimate)
  - Feed system
    - Turbo pump (assign a value)
    - Pressurization system (estimate)
    - Shut off and throttling valves (assign a value)
    - Other components (assign a value)
  - Combustion chamber (estimate)
  - Nozzle (estimate)
  - Ignition system (assign a value)

- **Rocket structures** (assign percentage)
- **Attitude control system** (assign a value or estimate)
- **Avionics** (assign a value)
- **Other systems** (assign a value)
- **Payload interface** (Percent of payload mass)
- **Mass margin** (Percent of the estimated structural mass)
Tank Design

- Storage of liquid oxidizer and fuel in hybrid and liquid rockets
- Large component of the structural mass fraction especially for pressure fed systems
- Factors that influence the design
  - Liquid mass-overall volume
  - Geometrical constraints – tank shape and configuration
  - Tank weight – tank material selection
  - Pump fed vs. pressure fed – internal pressure (MEOP)
  - Cryogenic vs storable – insulation
  - Corrosiveness of the liquid – tank material selection
  - Chemical stability of the liquid – tank material selection
  - Gravitational environment - diaphragms for zero g
  - Anti-Slosh - Baffles
Tank Design

• Structural design of the tanks

• Loads
  – Internal pressure
  – Acceleration
  – Point loads

• Primary failure modes
  – Yield/Rupture under internal pressure
  – Buckling (especially for thin walled tanks of pump fed systems)

• Structural materials
  – Metals: aluminum, steel, titanium
  – Composite: Carbon/Epoxy

• For cryogenic oxidizers such as LOX, composite technology is still in the R&D Phase
Tank Design

- Failure envelope for metals
- Yield stress: Significant plastic deformation starts
- Ultimate stress: Material breaks
- Failure criteria based on yield for isotropic ductile materials (i.e. metals)
  - Tresca (maximum shear stress)
    \[
    \tau_{\text{max}} = \max \left[ \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_1 - \sigma_3}{2}, \frac{\sigma_2 - \sigma_3}{2} \right] < \frac{\sigma_y}{2}
    \]
  - Von Mises (maximum strain energy)
    \[
    \sigma_{\text{max}} = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 + \left(\frac{\sigma_2 - \sigma_3}{2}\right)^2} < \sigma_y
    \]
- Tresca is more conservative compared to von Mises
Tank Design

- Important material properties
  - Strength/density
  - Ductility
  - Cost
  - Ease of manufacturing (welding, machining, forming)
  - Low temperature characteristics
  - Liquid compatibility

- Note that welding with no post heat treatment reduces yield strength
- Aluminum 2219 is widely used in cryogenic tank fabrication

<table>
<thead>
<tr>
<th>Structural Material</th>
<th>Tensile Yield Strength, ksi</th>
<th>Specific Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum 2219</td>
<td>60.0 (31.0 welded)</td>
<td>2.7</td>
</tr>
<tr>
<td>Graphite/Epoxy</td>
<td>130.0</td>
<td>1.55</td>
</tr>
<tr>
<td>Steel 4130</td>
<td>125.0</td>
<td>7.83</td>
</tr>
<tr>
<td>Aluminum Lithium</td>
<td>~80</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Tank Design

- Tanks used in propulsion applications are thin walled shells. Use shell theory for structural design.
- For preliminary design, the bending moments can be ignored and the shell equations can be reduced to membrane equations.

- For axisymmetric geometries the membrane stresses are
  - Meridional stress: \( \sigma_\phi = \frac{Pr_\theta}{2t} \)
  - Circumferential stress (Hoop stress): \( \sigma_\theta = \sigma_\phi \left(2 - \frac{r_\theta}{r_\phi}\right) \)

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Tank Design

• Tanks can be fabricated from combination of the following special geometries:
  - Cylinder: \( r_\theta = r \quad r_\phi \to \infty \)
    \[
    \sigma_\phi = \frac{P r}{2 t} \quad \sigma_\theta = \frac{P r}{t} \]
  - Sphere: \( r_\theta = r_\phi = r \)
    \[
    \sigma_\phi = \sigma_\theta = \frac{P r}{2 t} \]
  - Ellipsoid: (semi-major axis: \( a \), semi-minor axis: \( b \))
    \[
    \sigma_\phi = \frac{P a^2}{2 b t} u \quad \sigma_\theta = \sigma_\phi \left( 2 - \frac{1}{u} \right)^2 \\
u = \sqrt{1 - \varepsilon^2 \left( \frac{r}{a} \right)^2} \quad \varepsilon = \sqrt{1 - \left( \frac{b}{a} \right)^2} \]
Tank Design

- Spherical tank: (with radius \( r \))

  - Stress field: \( \sigma_1 = \sigma_2 = \frac{Pr}{2t} \)

  - From Tresca criterion: \( \tau_{\text{max}} = \frac{\sigma_1}{2} = \frac{\sigma_y}{2} \)

  - Tank wall thickness: \( t_{\text{min}} = k \frac{Pr}{2\sigma_y} \)

  - Tank mass: \( M_t = S_t t_{\text{min}} \rho_t = 4\pi \rho_t r^3 \frac{P}{\sigma_y} \)

  - Liquid mass: \( M_l = V_t \rho_l = \beta \frac{4\pi}{3} r^3 \rho_l \)

  - Define the tank efficiency: \( \eta_t \equiv \frac{M_l}{M_l + M_t} = \frac{1}{1 + \frac{3k}{2} \frac{\rho_t}{\rho_l \beta} \frac{P}{\sigma_y}} \)
Tank Design

- Cylindrical tank:
  - Different designs are possible based on various head geometries
    - Hemisphere:
      - Ideal end closure to minimize stress concentration
      - Expensive to manufacture, Ends are too long
    - Ellipsoidal:
      - Typical $a/b$ is 2
      - Hoop stress is compressive for the outside 20% of the end closure
      - Bending moments are introduced around the ellipsoid cylinder juncture
      - The stress concentration factor and yield criterion is
        \[
        K = \left[ 2 + \left( \frac{a}{b} \right)^2 \right] \frac{2r}{2t} < \sigma_y
        \]
    - Torisphere:
      - Spherical central portion with radius $R$ and a toroidal knuckle of radius $r$
        \[
        K = \left[ 3 + \left( \frac{R}{r} \right)^{1/2} \right] \frac{2t}{2} < \sigma_y
        \]
      - Higher stress concentration, but less expensive to manufacture
    - Flat Plate:
      - No membrane stresses, large stresses due to bending moments
      - Simple fabrication

- Typically 2:1 ellipsoidal design or hemispherical design is adapted for propulsion system tanks and combustion chambers
Tank Design

- Cylindrical tank design with hemispherical ends
- Assume uniform wall thickness
- Tank radius: \( r \), Length of the cylindrical portion: \( L_c \)
- Maximum shear stress:
  - Sphere: \( \tau_{\text{max}} = \frac{P r}{4 t} \)
  - Cylinder: \( \tau_{\text{max}} = \frac{P r}{2 t} \)
- Minimum thickness: \( t_{\text{min}} = k \frac{P}{\sigma_y} r \)
- Tank mass: \( M_t = S_t t_{\text{min}} \rho_t = 2\pi \rho_t kr^2 \left( L_c + 2r \right) \frac{P}{\sigma_y} \)
- Liquid mass: \( M_l = V_t \rho_l = \beta \pi \rho_l r^2 \left( L_c + \frac{4}{3} r \right) \)
- Tank efficiency: \( \eta_t \equiv \frac{M_l}{M_l + M_t} = \frac{1}{1 + 2 \frac{k \rho_t}{\rho_l \beta \sigma_y} \left( \frac{L_c}{r + 2} \right) \left( \frac{L_c}{r + 4/3} \right)} \)
- Tank length: \( L = 2r + L_c \)
Tank Design

- Toroidal Tank:
  - Assume uniform thickness based on most critical stress
  - Tank inside radius: \( a \), Tank outside radius: \( b \), Length of the cylindrical portion: \( L_c \), Radius of toroidal head: \( r \)
- Maximum shear stress:
  - Inside cylinder:
    \[
    \tau_{\text{max}} = \frac{P r}{2 t} \left[ \frac{r}{2a+1} + \frac{a}{r} \right]
    \]
- Minimum thickness:
  \[
  t_{\text{min}} = k \frac{P}{\sigma_y} \frac{r}{4} \left[ \frac{r}{2a+1} + \frac{a}{r} \right]
  \]
- Tank mass:
  \[
  M_t = S_t t_{\text{min}} \rho_t = 4 \pi k \rho_t \frac{P}{\sigma_y} r (r + a) \left( L + \pi r \right) \left[ \frac{r}{2a+1} + \frac{a}{r} \right]
  \]
- Liquid mass:
  \[
  M_l = V_t \rho_l = \beta 4 \pi \rho_l r (r + a) \left( L_c + \frac{\pi}{4} r \right)
  \]
- Tank efficiency:
  \[
  \eta_t \equiv \frac{M_l}{M_l + M_t} = \frac{1}{1 + k \frac{\rho_t}{\rho_l \beta} \frac{P}{\sigma_y} \left( \frac{L_c}{r + \pi} \right) \left( \frac{r}{2a} + \frac{a}{r} + 1 \right)}
  \]
- Tank length:
  \[
  L = 2 \left( r + L_c \right)
  \]
Tank Design - Effect of $Lc/r$ for Cylindrical Tanks

- Tank Pressure: 700 psi, Beta = 0.97
- Liquid Density: 1140 kg/m$^3$, N = 1
- Tank Material Density: 2700 kg/m$^3$
- Tank Material Yield Strength: 35 ksi
- Safety Factor = 1.2
- Oxidizer Mass: 10,000 kg

- Spherical Tank
- Cylindrical Tank
Tank Design - Effect of Number of Cylindrical Tanks

- Spherical
- Toroidal Tank
- Cylindrical Tank
- Spherical Tank

Tank Pressure = 700 psi, Beta = 0.97
Liquid Density = 1140 kg/m^3
Tank Material Density = 2700 kg/m^3
Tank Material Yield Strength = 35 ksi
Safety Factor = 1.2 Oxidizer Mass = 10,000 kg
For Toroidal Tank: a = 0.4 m, Lc = 1.5 m
Combustion chamber inside the tank
- No common wall for the combustion chamber and tank
- Combustion chamber volume is estimated using
  \[ V_{cc} = \frac{M_f}{\rho_f V_L} \]
- Length can be estimated from the volume and assumed radius
  \[ L_c = \frac{V_{cc}}{\pi a^2} \]
- As the combustion chamber radius increases toroidal tank becomes very inefficient
Other Tank Design Issues

• Include the mass of the other tank components into the mass budget
  – Baffles
  – Diaphragms
  – Mounting supports
  – Insulation (for cryogenic liquids)

• The first three items are difficult to estimate in the preliminary design phase. Account for them by increasing the safety factor.

• Insulation can be estimated based on the total surface area of the tank

• For certain cases the stiffness of the tank may become critical

• For pump fed systems, the tanks are designed for a small but finite pressure (50-75 psi)
  – For pump fed systems check that the calculated wall thickness is more than the minimum acceptable material thickness (minimum gauge)

• Note that the yield stress for metals increases with decreasing temperature. Useful feature for cryogenic liquids
Combustion Chamber Design

- For hybrids and solids fuel/propellant storage volume also serves as the combustion chamber
- Pressure vessel design equations derived for tanks are valid
- Design to Maximum Expected Operating Pressure (MEOP)
- In most cases combustion chambers are cylindrical vessels with 2:1 ellipsoid end caps or hemispherical end caps
- Common combustion chamber case materials are
  - Carbon fiber composite, Kevlar
  - Alloy steel
  - Aluminum
  - Titanium
- Must include the following items in the mass budget for the combustion chamber
  - Fuel sliver mass/web support material
  - Insulation material
  - Injector for hybrids
  - Igniter system
- For liquid systems combustion chambers are small and typically made out of metals
Feed System Components

* Feed system components
  - Oxidizer (and fuel) pumps or pressurization system
  - Main shut off valves for oxidizer (and fuel)
  - Other components (i.e. pipes etc)

* Turbo pump weight and cost are difficult to estimate
  - Typically pump fed systems are lighter but more complex and expensive
  - Pump weight and cost increases with increasing chamber pressure and liquid mass flow rate
  - Another system is needed to derive the turbine (H2O2 or solid/hybrid gas generators)
  - Assume a reasonable weight value for the preliminary design. Base the guess on the existing pump systems with similar operational characteristics

* The weight of the pressure fed systems can easily be estimated
The mass of pressurant gas in the oxidizer tank at burn out

\[
M_g = \Delta V_{ox} \frac{P_f}{R_p T_{oxf}} = \frac{\beta M_{ox}}{R_p T_{oxf}} \frac{P_f}{\rho_{ox} R_p T_{oxf}}
\]

The mass change in the pressurization tank

\[
\Delta M_{pt} = V_p \left( \frac{P_{pi}}{R_p T_{pi}} - \frac{P_f + \Delta P}{R_p T_{pf}} \right)
\]
Pressurization System

- From mass balance \( M_g = \Delta M_{pt} \)
  \[
  M_g = \frac{\beta M_{ox} P_f}{\rho_{ox} T_{oxf}}
  \]
  \[
  V_p = \frac{P_{pi}}{P_{pf}} - \frac{P_f + \Delta P}{T_{pi}}
  \]

- Tank weight and geometry can be calculated from the volume requirement

- Pressurization gas mass in the tank (assume ideal gas)
  \[
  M_{pg} = V_p \frac{P_{pi}}{R_p T_{pi}} = \frac{\beta M_{ox} P_f}{\rho_{ox} T_{oxf}} \frac{P_{pi}}{R_p T_{pi}} - \frac{P_f + \Delta P}{T_{pi}}
  \]

- In order to minimize the total pressurant gas mass use light gases (i.e. He)

- Total mass of the pressurization system
  \[
  M = M_{pg} + M_{ptank} + M_{pother}
  \]
Pressurization System

- Mass of regulators valves and other small components must be included in $M_{other}$.
- The initial and final pressurization tank temperatures are related according to the polytropic relation

$$\frac{T_{pi}}{T_{pf}} = \left( \frac{P_{pi}}{P_f + \Delta P} \right)^n$$

- For cold gas pressurization systems, exponent $n$ is in the range of 0.1-0.28 for most cases ($n=0$ corresponds to isothermal process).
- The oxidizer temperature in the tank at burn out can be calculated as

$$\frac{T_{oxf}}{T_{pi}} \equiv a_2$$

Where

<table>
<thead>
<tr>
<th>Ppi/Ppf</th>
<th>10</th>
<th>7</th>
<th>4</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2$</td>
<td>0.75</td>
<td>0.80</td>
<td>0.87</td>
<td>0.90</td>
</tr>
</tbody>
</table>

- Note that the pressurization system mass has the following general variation with the pressurization system pressure

$$M = \frac{A P_{pi}}{P_{pi} - C} + M_{other}$$
Pressurization System

Pressurization Gas: Cold He
Press. Tank: 1 Spherical Composite
Safety Factor=1.4
Feed System Pressure: 700psi
Oxidizer Density=1140 kg/m^3
Mox=10,000 kg, Beta=0.97
Tgas=300 K, Tcryo=120 K, n=0.11
Regular Mass=10 kg, DeltaP=20 psi
Pressurization System

Pressurization Gas: Cold He
Press. Tank: 1 Spherical Composite
Safety Factor=1.4
Feed System Pressure: 700psi
Oxidizer Density=1140 kg/m$^3$
Mox=10,000 kg, Beta=0.97
Tgas=300 K, Tcryo=120 K, n=0.11
Regular Mass=10 kg, DeltaP=20 psi
Large pressures are desirable to minimize the system size.

Practical considerations such as tank availability determine the design pressure.

For typical systems pressure is 4-10 ksi.

Heating the pressurant gas reduces the mass and volume requirements.
Nozzle Design – Rao’s Method

- Ideal nozzle has zero 3D flow losses
- Ideal nozzle length, fit to Rao’s curve for $\gamma = 1.23$

\[ \frac{L_{ni}}{D_{nt}} = 2.231 \left( AR - 1.8055 \right)^{0.556} \]

- Use the following correction on $C_F$ for the non-ideal nozzle

\[ \eta_{n3D} = \frac{C_F}{C_{Fi}} \]

<table>
<thead>
<tr>
<th>Ln/Lni</th>
<th>1</th>
<th>0.9</th>
<th>0.7</th>
<th>0.6</th>
<th>0.5</th>
<th>0.45</th>
<th>0.40</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF/CFi</td>
<td>1</td>
<td>1</td>
<td>0.9975</td>
<td>0.9950</td>
<td>0.990</td>
<td>0.985</td>
<td>0.970</td>
</tr>
</tbody>
</table>

- Average cone angle for the non-ideal nozzle

\[ \tan(\theta_c) = \frac{\sqrt{AR} - 1}{2} \frac{L_{ni}}{L_n} \cdot \frac{1}{2.231(AR - 1.8055)^{0.556}} \]
Nozzle Design

• Estimate the throat area from the $c^*$ equation

$$\frac{\pi D_{nt}^2}{4} = A_t = \frac{\dot{m}_p c_{theo}^*}{C_d P_c}$$

• Select a nozzle area ratio. Estimate the nozzle exit area from

$$D_{ne} = \sqrt{AR} \ D_{nt}$$

• If the nozzle exit diameter is matched to the chamber or tank diameter, increasing chamber pressure allows for higher area ratio (better Isp)

• Estimate the ideal nozzle length from Rao’s expression

• Select a 3D nozzle efficiency. Estimate the nozzle length for the selected 3D nozzle loss.

$$L_n = L_{ni} f(\eta_{n3D})$$

• Estimate the total nozzle loss (kinetic losses + 2 phase flow losses + 3D flow losses)

$$\eta_n = \eta_{n3D} \eta_{nk} \eta_{n2P}$$

• Estimate the nozzle mass

• Iterate on area ratio and nozzle 3D loss selection for optimum condition

• A good value for the 3D nozzle efficiency is 0.985
The length and 3D flow efficiency for a parabolic nozzle can be written as:

\[
\frac{L_n}{D_{nt}} = \sqrt{\frac{AR}{4}} - 1 \left[ \frac{1}{\tan(\theta_n)} + \frac{1}{\tan(\theta_e)} \right] \quad \eta_{n3D} \approx \frac{1 + \cos(\theta_e)}{2}
\]

\[
\frac{2}{\tan(\theta_c)} = \left[ \frac{1}{\tan(\theta_n)} + \frac{1}{\tan(\theta_e)} \right]
\]
• Simple, effective, generally light
• All solids/hybrids and some liquids
• Ablative inner shell (Thickness based on ablation rate x burn time)
• Structural outer shell (Thickness based on internal pressure + other loads)
Nozzle Erosion

- Ablative nozzle surface slowly recesses. The effective heat of gasification protects the structure of the nozzle from heat.
- The heat transfer is typically diffusion limited (as in a hybrid rocket system).
- The nozzle regression rate can be written in terms of the local flux

\[
\dot{r}_n = a_n \left( \frac{O}{F} \right) G_n^m
\]

- Note that

\[
G_n = \frac{\dot{m}_P}{A_n}
\]

- Using the c* equation one can write

\[
G_n = \frac{P_c C_d}{c_{\text{theo}} \eta_c} \frac{A_{nt}}{A_n}
\]

- Combine to yield

\[
\dot{r}_n = a_n \left( \frac{O}{F} \right) \left( \frac{C_D}{c_{\text{theo}} \eta_c} \right)^m \left( \frac{A_{nt}}{A_n} \right)^m P_c^m = B \left( \frac{O}{F} \right) \left( \frac{A_{nt}}{A_n} \right)^m P_c^m
\]
Nozzle Erosion

- Nozzle erosion dynamics

\[ \frac{dD_n}{dt} = 2\dot{r}_n = 2B_n (O / F) \ P_c^m \]

- Or

\[ D_n^{2m} dD_n = \frac{2^{2m+1}}{\pi^m} a_n m_p^m dt \]

- This ODE can be integrated to find the change in the nozzle area ratio at any point in the nozzle at any instant

- For a linearly throttled rocket, the relation between the initial and final area ratios is (exit plane erosion is ignored)

\[
\frac{AR_i}{AR_f} = \left[ 1 + \frac{2m+1}{m+1} \ 2^{0.5} \ a_n \left( \frac{C_d}{c_{theo} \eta_c} \right)^{m+0.5} \ \frac{P_{ci}^{m+0.5} \ t_b^{1.5}}{M_p^{0.5}} \ (1 + TR)^{0.5} \ (1 - TR^{m+1}) \right]^{2/(2m+1)}
\]

- The throttling ratio is defined as

\[ TR = \dot{m}_{pf} / \dot{m}_{pi} \]
Nozzle Erosion

- Define a reference pressure for which the nozzle erosion rate is known (for a selected average O/F for the motor). Solve for the unknown $a_n$

$$ (\dot{r}_n)_{ref} = B_n \ P_{cref}^m = a_n \left( \frac{C_d}{c_{theo} \eta} \right)^m \ P_{cref}^m $$

- For most systems it is reasonable to assume that $m=1$
- The erosion rate increases with the increasing mass fraction of the oxidizing agents in the nozzle exhaust.
  - CO, HO, H2O, 2 x O2, O
- This value is high in hybrid and liquid systems resulting in high erosion rates
- In hybrids and liquids the erosion rate is a strong function of the O/F of the motor. For high O/F the erosion rates can be quite high.
- Aluminum addition typically reduces the erosion rate for hybrids since Al2O3 formation decreases the mass fraction of oxidizing agents
- In solid rockets the nozzle throat erosion rate for various nozzle throat insert materials are
  - ATJ Graphite: 0.004-0.006 in/sec
  - 3D Carbon-Carbon: 0.0005-0.001 in/sec
- 2D carbon/carbon or 3D carbon/carbon nozzle inserts are not suitable for liquid or hybrid rockets due to the oxidizer attack to the surface
Nozzle Erosion Data – ATJ Graphite

- Propellants: GOX/Paraffin
- Nozzle material: ATJ graphite
- Burn time: 8 sec nominal
- Chamber pressure: 800 psi nominal
- Nozzle throat diameter: 2” nominal
Nozzle Design Issues

- Nozzle erosion effects the performance adversely due to
  - Reduction of the nozzle area ratio in time, Isp loss
  - Increase in nozzle weight, structural mass fraction increase
- Note that the erosion (or the effect of nozzle erosion) can be minimized by
  - Keeping the chamber pressure low (reduce the nozzle mass flux)
  - Running at low O/F
  - Formulating the propellants to minimize the mass fraction of the oxidizing agents (can use the results of the Isp code)
  - Selecting a suitable nozzle material
  - Introducing a cool film on the surface of the nozzle
- Nozzle weight can be estimated from the nozzle erosion rate equation by estimating the required thickness of the ablative material. Use a safety factor (i.e. 1.5). The weight of the structural shell can be calculated using the hoop stress induced by the pressure inside the nozzle.
- For hybrids silica phenolic is commonly used as the ablative nozzle material over the entire nozzle surface. Silica phenolic is resistant to oxidizer attack.
- For small inexpensive hybrid systems ATJ graphite is also commonly used. Note that ATJ is a brittle material. One must minimize the stress concentration areas.
- Use the following reference erosion rate for preliminary design purposes (LOX/HC hybrids running at O/F less than 2.5)
  - Erosion rate: 0.007 in/sec at 500 psi \((m=1)\)
• LOX/paraffin Hybrid
• Ablative shell: silica phenolic
• Structural shell: glass phenolic
• Increase in area ratio improves Isp but increases the structural mass fraction