AA 284a
Advanced Rocket Propulsion

Lecture 14
Stability of Chemical Rockets

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Stability of Chemical Propulsion Systems

- **Instability Definition:**
  - Chamber pressure and/or thrust oscillations
  - Smooth combustion: Peak to peak < 5% of the mean (Sutton)
  - Rough combustion: Pressure oscillations in completely random intervals
  - Unstable combustion: More or less organized - activity at discrete frequencies

- **Importance:**
  - Mechanical and thermal loading on the components - may result in the destruction of the propulsion system
  - Performance variations (i.e. regression rate variations due to DC shift) - May result in mission failure
  - Vibration loads on the structures and payload - May result in mission failure

- **Classification of Instabilities Based on Frequency:**
  - Low Frequency Oscillations: 1-400 Hz, L* (solids), chugging, chuffing
  - Intermediate Frequency Oscillations: 400-1000 Hz, Buzzing
  - High Frequency Oscillations: > 1000 Hz, Screaming, Screeching
Stability of Liquid Propulsion Systems

- **Low Frequency:**
  - Chuffing, chugging: Systems coupling
  - Pogo: Vehicle acceleration propellant mass flow rate coupling. Eliminated by inserting gas accumulators into the propellant feed system

- **Intermediate Frequency:**
  - Buzzing: Feed system combustion chamber coupling
  - Not as destructive as the high frequency

- **High Frequency:**
  - Screaming, screeching
  - Related to the acoustic modes of the chamber
    - Longitudinal
    - Traverse: Radial or tangential
  - Most common and most destructive
  - Increases heat transfer rates up to a factor of 10. Causes metal walls to melt
  - Typically the tangential mode is the most destructive one
Stability of Liquid Propulsion Systems

• **Rating:**
  - Introduce a disturbance and check the time required to return to normal operation or if stays unstable check the amplitude of the oscillations
  - Non-directional bomb, directional explosive pulse, directed flow of inert gas

• **Control of instabilities:**
  - Most stability tests must be done in full scale
  - For chugging instabilities: decouple chamber from the feed system by increasing the injector pressure drop
  - For high frequency instabilities
    • Injector face baffles
    • Acoustic energy absorption cavities (Helmholtz resonators)
    • Combustion chamber liners (Helmholtz resonators)
    • Change injector design
    • Injector end is critical in the production of instabilities
Chamber Gas Dynamic Model

- Transient mass balance in the combustion chamber
  \[
  \frac{d\rho V}{dt} = m_p - m_n
  \]

- For constant volume
  \[
  V \frac{d\rho}{dt} = m_p - \frac{P_c A_{nt}}{c^*}
  \]

- Ideal gas and isothermal process
  \[
  \frac{V}{RT} \frac{dP_c}{dt} = m_p - \frac{P_c A_{nt}}{c^*}
  \]

- Can be rearranged
  \[
  \frac{dP_c}{dt} + \frac{A_{nt}}{V} \frac{RT}{c^*} P_c = \frac{RT}{V} m_p
  \]
  \[
  \frac{dP_c}{dt} + \frac{1}{\tau_c} P_c = \frac{RT}{V} m_p
  \]

- Characteristic chamber filling/emptying time and \( L^* \) are defined as
  \[
  \tau_c \equiv \frac{L^*}{c^* f(\gamma)}
  \]
  \[
  L^* \equiv \frac{V}{A_{nt}}
  \]

- Note that if the propellant gas generation rate is pressure dependent, coupling is possible (Solid rocket \( L^* \) instability)
- Note that this coupling is not possible in a hybrid rocket
- Bulk mode instability – Pressure oscillates uniformly in the chamber
Instabilities increase the regression rate ("DC Shift") and reduce the burn time. This may lead to mission failure.

Motor instability is not as frequent as it is in liquid engines.

Rarely cause a sudden motor failure and disintegration.

**Low Frequency:**
- Bulk mode: Helmholtz mode or $L^*$ mode or chuffing mode
- Frequencies less than 150 Hz
- Due to a coupling between the chamber gas dynamics and thermal lags in the solid (the phase lead character of the thermal lag system is key to instability)

**Intermediate/High Frequency:**
- Acoustic modes: longitudinal or traverse

Many plausible trigger sources (i.e. a propellant chunk flying through the nozzle)

**Amplifying factors**
- Coupling with combustion. Response function: Transfer function between the regression rate and the chamber pressure.
- Vortex shedding
- Flow instabilities
Stability of Solid Propulsion Systems

- Attenuating factors:
  - Viscous damping
  - Particle or droplet damping: due to drag induced by relative velocity. There exists an optimum particle size for a given frequency
  - Nozzle
  - Viscoelastic character of the propellant

- Intrinsic instability of a solid propellant charge: Due to thermal lags and combustion coupling

- Use T-burners to determine the response function of a solid propellant

- Stability Fixes:
  - Change grain geometry
  - Change propellant formulation
    - Al addition helps. Optimal particle size for a given motor size
  - Add mechanical devices to attenuate the unsteady gas motion or alter the natural frequency of the chamber
Hybrids are prone to low frequency instabilities (2-100 Hz)

- High amplitude spiky combustion
- Especially common in liquid oxygen (LOX) based systems
- A number of feasible mechanisms exist
We believe that a LOX motor can be made stable
- Without the use of heaters or TEA injection
- By advanced injector and combustion chamber design

Solutions used in the field
- Lockheed Martin – Michoud and HPDP used hybrid heaters to vaporize $\text{LO}_2$
- AMROC injected TEA (triethylaluminum) to vaporize LOX

Both solutions introduce complexity minimizing the simplicity advantage of hybrids
- Heaters - extra plumbing
- TEA – extra liquid, hazardous material
Stability of Hybrid Propulsion Systems

- Pressure-time history for NASA Ames motor test 4L-05.
- Paraffin-based/GOX

- FFT for test 4L-05
- Three modes are observed:
  - Hybrid low frequency
  - Bulk mode
  - 1-L mode
Stability of Hybrid Propulsion Systems

- Most hybrids show the “Low Frequency” mode which typically dominates the other modes.
- Observed in all hybrid development/research programs to our knowledge.
- The exact frequency is case dependent but ranges in the 2-100 Hz for most practical hybrids.
- Oscillations are in the limit cycle form. Amplitudes are typically in the range of 2-30 % rms of the mean.
- The low frequency mode is typically accompanied by acoustic modes.
- The fore end configuration/volume effects the amplitude. (i.e. axial injection is more stable compared to radial injection).
- The low frequency mode is encountered in both liquid and gaseous oxidizer systems.
- Few theories exist-None of them are based on a mathematical formalism that one commonly encounters in solid/liquid fields.
- TCG coupled theory: Develop transient mathematical models of hybrid subsystems and couple these subsystems to search for instabilities.
<table>
<thead>
<tr>
<th>Physical Phenomenon:</th>
<th>Time Scale:</th>
<th>Explanation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Solid phase kinetic times</td>
<td>$\tau_{sp} &lt; 10^{-3}$ sec</td>
<td>Degradation mechanisms of the polymer</td>
</tr>
<tr>
<td>2) Gas phase kinetic times</td>
<td>$\tau_{gp} &lt; 10^{-3}$ sec</td>
<td>Hydrocarbon combustion mechanisms</td>
</tr>
<tr>
<td>3) Feed system response times</td>
<td>(Varies greatly from system to system)</td>
<td>Response time of the feed system</td>
</tr>
<tr>
<td>4) Evaporation times</td>
<td>$\tau_{evap} = f(U_o, T_1, \Delta P)$</td>
<td>Evaporation process of the liquid oxidizer</td>
</tr>
<tr>
<td>5) Thermal lags in solid</td>
<td>$\tau_{tl} \propto \kappa/r^2 \approx 10^{-1}$ sec</td>
<td>Thermal profile changes in the solid grain</td>
</tr>
<tr>
<td>6) Boundary layer diffusion times</td>
<td>$\tau_{bl} \propto L/ u_e \approx 10^{-1}$ sec (Varies greatly form case to case)</td>
<td>Turbulent boundary layer diffusion processes</td>
</tr>
<tr>
<td>3) Acoustic times (longitudinal)</td>
<td>$\tau_a \propto L/c \approx 10^{-3}$ sec (Varies greatly form case to case)</td>
<td>Propagation of the acoustic waves</td>
</tr>
<tr>
<td>7) Gas dynamic filling times</td>
<td>$\tau_{fill} \propto L^<em>/c^</em> \approx 10^{-1}$ sec (Varies greatly form case to case)</td>
<td>Global mass flow balance</td>
</tr>
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Hybrid Transient Model Schematic

- **Chamber Gasdynamics**
  - Pre-Combustion Chamber
  - Post-Combustion Chamber and Nozzle

- **Port-Volume**
  - $P_1$
  - $P_2$

- **Hybrid Combustion**
  - Thermal Lags in the Solid
  - Boundary Layer Combustion
  - $Q_s(x)$

- **Gaseous oxidizer, no feed system dynamics**
Thermal Lag Model

- Input: Wall heat flux schedule
- Output: Regression rate variation in time
- Surface Model:

\[ \dot{r} = A \ e^{-E_a / R_u T_s} \]
Thermal Lag Model Transfer Function

- First perturbation solution around a nominal operating point generates the transfer function

\[ F_T(s) = \frac{R_L(s)}{Q_L(s)} = \frac{2E_{Ea}s}{\left(1+\sqrt{4s+1}\right)\left(s+E_{Ea}\right)-2E_{Ea}+2E_LE_{Ea}s} \]

- Stability character of the thermal lag system:
  - No poles, just a zero at (0, 0)
  - No instabilities can be generated by this system alone
  - No intrinsic instability of an inert fuel (no heterogeneous rxns are permitted)

- The square root terms represents the diffusive character of the system

- Phase lead behavior at low frequencies

- This subsystem is key in the generation of solid rocket intrinsic and L* instabilities
  - In hybrids L* instability is not possible since regression rate is only a weak function of the chamber pressure.
  - In hybrids intrinsic instability is not possible since no heterogeneous rxns are expected.
Initially assume that gas phase is fast compared to solid phase.

Ignore the radiative component of the wall heat flux.

Modify the classical theory (by Marxman in the early 60’s).

Relation between the wall flux, oxidizer mass flux and the fuel regression rate:

\[
\dot{Q}_c(t) = E_L \overline{G}_o^{n/(1-k)} R^{-k/(1-k)}
\]

The regression dependency of the flux comes from a phenomenon called the “Blocking Effect”. \(k\) is the blocking exponent.

\[
\frac{C_H}{C_{Ho}} = B^k
\]

This effect produces a cross coupling mechanism between the gas phase and the solid phase.
Gas Phase Combustion Model: Transient Case

- Boundary layer cannot respond to changes in the oxidizer mass flux and the fuel regression rate.
- Introduce this transient as a time lag:

\[
Q_1(t) = (E_L + 1) \left( \left( \frac{n}{1 - k} \right) G_1(t - \bar{\tau}_{bl1}) - \left( \frac{k}{1 - k} \right) R_1(t - \bar{\tau}_{bl2}) \right)
\]

- From literature for turbulent boundary layers with no combustion and no blowing this delay can be written as

\[
\tau_{bl} = c' \frac{z}{u_e}
\]

- We have estimated the constant to be 0.55.
- Even though this looks like a time flight characteristic time scale, the origin of the formula is based on the radial diffusion period across the boundary layer thickness.
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Thermal-Combustion (TC) Coupled System

• Transfer function for the coupled system (Input: Oxidizer mass flux. Output: Regression Rate)

\[ F_{TC}(s) = \frac{R_{1L}(s)}{I(s)} = \frac{2E_{E_a} \sigma_2 e^{-\tau_{bl1}s}}{(1 + \sqrt{1+4s}) \left(s + E_{E_a}\right) - 2E_{E_a} + 2E_{E_a}s \left(E_L + \sigma_1 e^{-\tau_{bl2}s}\right)} \]

• Stability Character:
  - \( \tau_{bl1} \): No effect on stability
  - If \( \tau_{bl2} = 0 \), no poles
  - If \( \tau_{bl2} > 0 \), a series of poles in the positive real half of the s plane-unstable system
  - We only consider the fundamental mode (pole with the lowest frequency)
\[ \tau_{bl2} = 0 \]

\[ \tau_{bl2} = 38 \text{ m sec} \]
Effect of System Parameters on the TC Coupled Instabilities

- Effect of all the system parameters other than $\tau_{bl2}$ on the oscillation frequency and the amplification rate is negligible for the range of these parameters commonly encountered in hybrid applications.

![Graph showing the relationship between frequency and boundary layer delay time]

Curve Fit $f = \frac{0.48}{\tau_{bl2}}$

TC Coupled Theory Prediction:

$f = \frac{0.48}{\tau_{bl2}}$
Hybrid Low Frequency Instability Coupling Mechanism

- The cross coupling of three important phenomena generates the TC coupled instabilities:
  - Wall transfer blocking effect, k
  - Heat transfer in the solid
  - Boundary layer dynamics, $\tau_{bl2}$
A gas dynamic component is required to complete the basic transient modeling of hybrid transients.

Also needed to convert the regression rate oscillations into chamber pressure or thrust oscillations.

Model: 2 Volume-Port, Isothermal
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Important Results: Gas Dynamic Model

- A transfer function for the linearized gas dynamic system is derived.
- Numerical simulations on the full nonlinear system are also performed.
- Gas dynamic system by itself is stable.
- The model resolves the filling/emptying and longitudinal acoustic behavior of the chamber.
- When coupled with the combustion subsystem with delay $\tau_{bl1} > 0$, the system preserved its stability character.
Thermal-Combustion-Gas Dynamics (TCG) Coupling

- TCG coupled transfer function

- Shows the TC coupled poles
- Frequency/amplification rate are not altered
- The instabilities are now in terms of chamber pressure oscillations
- Shows most critical transient aspects of a gaseous hybrid with a decoupled feed system:
  - Low frequency oscillations
  - Filling/Emptying
  - Longitudinal acoustic behavior
Hybrid Oscillation Frequency Scaling Law

- Boundary layer delay time in terms of $L^*$: (AMROC)
  \[
  \tau_{bl2} = c' \frac{V_p}{V_m} \left[ \frac{(1 + O/F)}{(1 + 2O/F)} \right] L^* c_{exp}
  \]

- Boundary layer delay time in terms of operational parameters group A: (HPDP, JIRAD, Arizona State)
  \[
  \tau_{bl2} = c' \frac{LP}{(G_o + G_t)RT_{ave}}
  \]

- Boundary layer delay time in terms of operational parameters group B: (Ames/Stanford)
  \[
  \tau_{bl2} = c' \frac{LP}{2 + \frac{1}{O/F}} G_oRT_{ave}
  \]
Comparison to Hybrid Motor Test Data

- 43 motor tests used in this comparison.
- $c'$ value of 2.01 used for all calculations.
- Motor test data covers a wide range of variables:
  - 5 programs
  - Three oxidizers (LOX, GOX, N2O)
  - Wide range of motor dimensions (5” OD to 72” OD)
  - Wide range of operating conditions
  - Several fuel formulations (HTPB, HTPB/Escorez, paraffin-based)
Hybrid Low Frequency Instabilities - Overall Picture

- Linear TC coupled theory predicts indefinite growth of oscillations in time.
- The amplitudes are determined by nonlinear phenomenon and the strength/spectral content of the excitation source (Limit cycle).
- Unlike the amplitude, the frequency of the oscillation is set by the TC coupled theory.
- All hybrids have the TC coupled instability mechanism (root cause).
- Some motors show very low amplitude oscillations because TC coupled mode is not disturbed strongly at the frequency content that it prefers to amplify.
- The possible source of disturbance is the fore-end flow configuration.
- Note that it has been observed that the fore end configuration (i.e. injection, geometry) determines the amplitude of the oscillation but not the frequency.
Hybrid Low Frequency Instabilities-Analogy

- An analog system that works on the same principle is flue organ pipe.
Conclusions- Hybrid Transient Modeling

• A plausible mechanism that generates low frequency chamber pressure oscillations is developed.
• The oscillation frequency for a hybrid system can be predicted by this universal formula:

\[ f = 0.119 \left( 2 + \frac{1}{O/F} \right) \frac{G_o RT_{ave}}{L P} \]

• The amplitudes can not be predicted by this simple model.
• We believe that the stability character of a hybrid motor is determined by the spectral features of the disturbances generated at the fore-end of the motor.