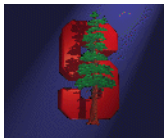# AA 284a
Advanced Rocket Propulsion

Lecture 7b
Gravitational Loss

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Gravity Loss Estimation

Objective:

- Derive a formula for the gravitational loss and gain intuition for this important loss mechanism
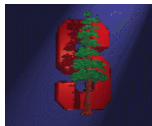
Trajectory Equation:

$$M \frac{dV}{dt} = T - M \ g_s$$

$$g_s = g \ \sin(\gamma)$$

$$dV = -I_{sp} g_o \frac{dM}{M} - g_s dt$$

Real Delta V (velocity increment in gravitational field)

$$\Delta V_r = V_f - V_i = I_{sp} g_o \ln\left(\frac{M_i}{M}\right) - \int_o^t g_s dt$$

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Specific energy:

$$e = \frac{1}{2}V^2 + \int_o^s g_s ds$$

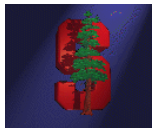Here s is the distance along the trajectory.

$$de = VdV + g_s ds$$

Trajectory Eqn.:

$$VdV = -I_{sp}g_o V \frac{dM}{M} - g_s Vdt$$

Eliminate *dt*

$$\frac{ds}{dt} = V \qquad\qquad VdV = -I_{sp}g_o V \frac{dM}{M} - g_s ds$$

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Combine:

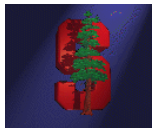$$de = -I_{sp}g_o V \frac{dM}{M}$$

Ideal and real velocity increase

$$de_{id} = -I_{sp}g_o V_{id} \frac{dM}{M} \qquad de_r = -I_{sp}g_o V_r \frac{dM}{M}$$

Gravity loss:

$$dG \equiv de_{id} - de_r = -I_{sp}g_o \left(V_{id} - V_r\right)\frac{dM}{M}$$

$$dG = -I_{sp}g_o \frac{dM}{M} \int_0^t g_s dt$$

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Gravity Loss Estimation

Use of chain rule:
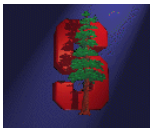
$$dG = -I_{sp}g_o \, d\left[\ln\left(\frac{M}{M_i}\right) \int_0^t g_s \, dt \right] + I_{sp}g_o \ln\left(\frac{M}{M_i}\right) g_s \, dt$$

Integrate over burn time:

$$G = -I_{sp}g_o \ln\left(\frac{M_b}{M_i}\right) \int_0^{t_b} g_s \, dt + I_{sp}g_o \int_0^{t_b} \ln\left(\frac{M}{M_i}\right) g_s \, dt$$

$$G = I_{sp}g_o \int_o^{t_b} \left[\ln\left(\frac{M}{M_i}\right) + \ln\left(\frac{M_i}{M_b}\right) \right] g_s \, dt$$

$$G = I_{sp}g_o \int_o^{t_b} \ln\left(\frac{M}{M_b}\right) g_s \, dt$$

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Reintroduce time: $\dot{m} = -dM/dt$

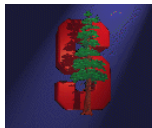$$G = -I_{sp}g_o \int_{M_i}^{M_b} \frac{g_s}{\dot{m}} \ln\left(\frac{M}{M_b}\right) dM$$

For constant g_s/m

$$G = I_{sp}g_o g_s \frac{M_b}{\dot{m}} \left\{ 1 + \frac{M_i}{M_b}\left[\ln\left(\frac{M_i}{M_b}\right) - 1 \right] \right\}$$

Constant acceleration

$$a = \frac{T}{M} = \frac{\dot{m} I_{sp}g_o}{M} \qquad G = \frac{1}{2}\frac{g_s}{a}\left[I_{sp}g_o \ln\left(\frac{M_i}{M_b}\right) \right]^2$$

$$\frac{G}{1/2 V_e^2} = \frac{g_s}{a} \ln^2\left(\frac{M_i}{M_b}\right)$$

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Gravity Loss Estimation

Conclusions:

- The gravity loss decreases with increasing mass flow rate or acceleration (decreasing burn time)

- In the coast phase the gravity loss is zero.

- Gravity loss decreases with decreasing g_s (or with decreasing average trajectory angle)

- The coast phase reduces the overall gravity loss by decreasing the combined burn time for all stages and also by reducing the average trajectory angle for the upper stages. Note that during the coast phase there is no gravity loss but the trajectory angle continues to drop (gravity turn).

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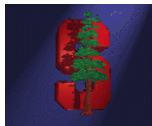
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Gravity Loss Estimation

- The formula for the specific energy increase given in slide 4 is instrumental in showing the relative importance of Isp for upper stages. This equation can be rearranged to express the increase in specific energy for unit mass of propellant expelled from the rocket, namely the sensitivity coefficient .

$$\sigma = \left| \frac{de}{dM} \right| = \frac{I_{sp} g_o V}{M}$$

- Note that for an efficient rocket system the sensitivity coefficient should be maximized. Since the upper stages of a launch vehicle are lighter and move faster, a unit increase in the Isp value results in a larger increase in the sensitivity parameter for the upper stages.

- The loss described in this write up is not limited to the gravitational forces. As can be deduced from gravity loss formula, any force that limits the velocity built up of the rocket (i.e. any force that acts over a finite period against the direction of the thrust force) will result in a loss of useful energy. Some examples of such forces are

  - Drag force
  - Static firing of a rocket. The force exerted by the thrust stand results in a delta V loss equal to the delivered delta V by the rocket.

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