# Inducing Interpretable Causal Structures in Neural Networks

#### Christopher Potts

Joint work with Atticus Geiger, Zhengxuan (Zen) Wu, Hanson Lu, Josh Rozner, Elisa Kreiss, Karel D'Oosterlinck, Thomas Icard, and Noah Goodman

Stanford Linguistics and the Stanford NLP Group

Mila/McGill, April 8, 2022







## Semantic insights in NLP models

#### Semantic insights in NLP models: 1980s

 Which country bordering the Mediterranean borders a country that is bordered by a country whose population exceeds the population of India?

#### Semantic insights in NLP models: 1980s

 Which country bordering the Mediterranean borders a country that is bordered by a country whose population exceeds the population of India? turkey.

- Which country bordering the Mediterranean borders a country that is bordered by a country whose population exceeds the population of India? turkey.
- How far is London from Paris?

Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusion Co

#### Semantic insights in NLP models: 1980s

```
/* Sentences */
sentence(S) --> declarative(S), terminator(.) .
sentence(S) --> wh_question(S), terminator(?) .
sentence(S) --> yn_question(S), terminator(?) .
sentence(S) --> imperative(S), terminator(!) .

/* Noun Phrase */
np(np(Agmt, Pronoun, []), Agmt, NPCase, def, _, Set, Nil) -->
{is_pp(Set)},
pers_pron(Pronoun, Agmt, Case),
{empty(Nil), role(Case, decl, NPCase)}.

/* Prepositional Phrase */
pp(pp(Prep, Arg), Case, Set, Mask) -->
prep(Prep),
{prep_case(NPCase)},
np(Arg,_NPCase,_, Case, Set, Mask).
```

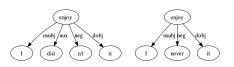
- Which country bordering the Mediterranean borders a country that is bordered by a country whose population exceeds the population of India? turkey.
- How far is London from Paris? I don't understand!

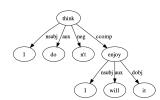
Chat-80; Warren and Pereira 1982

I didn't enjoy it.

I never enjoy it.

I don't think I will enjoy it.

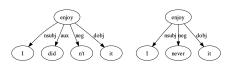


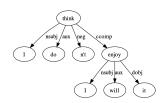


I didn't enjoy it.

I never enjoy it.

I don't think I will enjoy it.





$$neg(x, *) \Rightarrow x_neg$$

$$neg(x, y) \land ccomp(x, z) \Rightarrow x_neg, z_neg$$

a)	Utah	borders	Idaho			
	$\overline{NP}$	$(S \backslash NP)/NP$ $\lambda x. \lambda y. borders(y, x)$	NP			
	utah		idaho ——>			
		$(S \backslash NP)$ $\lambda y.borders(y, idaho)$				
		S borders(utah, idaho)				

b)	What	states	border	Texas
	$\frac{(S/(S\backslash NP))/N}{\lambda f.\lambda g.\lambda x. f(x) \wedge g(x)}$	$\frac{N}{\lambda x.state(x)}$	$(S \backslash NP)/NP$ $\lambda x. \lambda y. borders(y, x)$	NP texas
	$S/(S \backslash NP)$ $\lambda g. \lambda x. state(x)$	$\frac{(S \backslash NP)}{\lambda y.borders(y, texas)}$		

 $\lambda x.state(x) \land borders(x, texas)$ 

rivers

river

run through

the largest

the highest

the longest

a)	Utah	borders	Idaho	b)	What	states	border	Texas	
	$\overline{NP}$	$(S \backslash NP)/NP$	NP		$(S/(S\backslash NP))/N$	N	$(S \backslash NP)/NP$	$\overline{NP}$	
	utah	$\lambda x.\lambda y.borders(y, x)$	) idaho		$\lambda f.\lambda g.\lambda x.f(x) \wedge g(x)$	$\lambda x.state(x)$	$\lambda x. \lambda y. borders(y, x)$	texas	
	$(S \backslash NP) \ \lambda y.borders(y,idaho)$			$S/(S \backslash NF)$			$(S \backslash NP)$		
				$\lambda g.\lambda x.state(x) \wedge g(x)$		$\lambda y.borders(y, texas)$			
	borders(utah, idaho)			$\frac{S}{\lambda x. state(x) \land borders(x, \textit{texas})}$					
		sta	tes	:=	$N: \lambda x.state(x)$				
		ma	jor	:=	$N/N: \lambda f. \lambda x. major(x)$	$f(x) \wedge f(x)$			
		por	pulation	:=	$N : \lambda x.population(x)$	, ,			
		citi	ies	:=	$N: \lambda x.city(x)$				

Figure 6: Ten learned lexical items that had highest associated parameter values from a randomly chosen development run in the Geo880 domain.

 $N: \lambda x.river(x)$ 

 $N: \lambda x.river(x)$ 

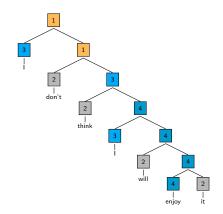
 $(S\backslash NP)/NP: \lambda x. \lambda y. traverse(y, x)$ 

 $NP/N : \lambda f. \arg \max(f, \lambda x. size(x))$ 

 $NP/N: \lambda f. \arg \max(f, \lambda x. elev(x))$ 

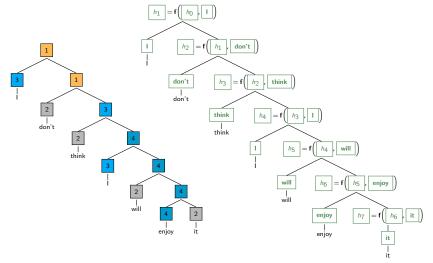
 $NP/N : \lambda f. \arg \max(f, \lambda x. len(x))$ 

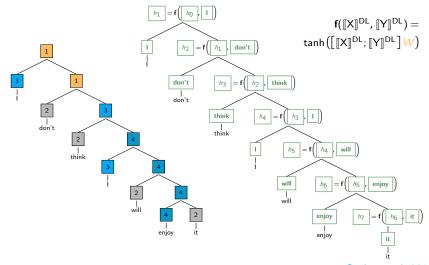
#### Semantic insights in NLP models: 2010s

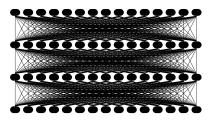


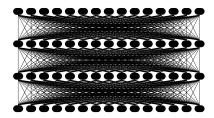
Socher et al. 2013

Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

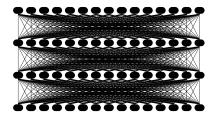






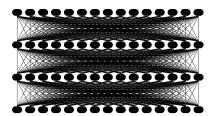


















A low point for connections between linguistics and NLP?

A low point for connections between linguistics and NLP? No!

A low point for connections between linguistics and NLP? No!

A low point for connections between linguistics and NLP? No!

Modern NLP systems based in deep learning (a.k.a. neural networks, connectionism):

Focused on representations

A low point for connections between linguistics and NLP? No!

- Focused on representations
- High-dimensional representations

A low point for connections between linguistics and NLP? No!

- Focused on representations
- High-dimensional representations
- Context-dependent representations

A low point for connections between linguistics and NLP? No!

- Focused on representations
- High-dimensional representations
- Context-dependent representations
- Holistic representations

A low point for connections between linguistics and NLP? No!

- Focused on representations
- High-dimensional representations
- Context-dependent representations
- Holistic representations
- Ambitions to interpret even the most complex language

A low point for connections between linguistics and NLP? No!

Modern NLP systems based in deep learning (a.k.a. neural networks, connectionism):

- Focused on representations
- High-dimensional representations
- Context-dependent representations
- Holistic representations
- Ambitions to interpret even the most complex language

Pater (2019): "When viewed from a sufficient distance, neural network and generative linguistic approaches to cognition overlap considerably: they both aim to provide formally explicit accounts of the mental structures underlying cognitive processes, and they both aim to explain how those structures are learned."

### Overview of today's talk

#### Overview of today's talk

Motivations for bringing semantic insights into NLP models

#### Overview of today's talk

Motivations for bringing semantic insights into NLP models

Characterize representations

Causal inference

Improved models

Motivations for bringing semantic insights into NLP models

Characterize Causal Improved representations inference models

Probing

#### Motivations for bringing semantic insights into NLP models

	Characterize representations	Causal inference	Improved models
Probing	<b>U</b>		
Feature attribution		$\stackrel{\boldsymbol{\square}}{=}$	

#### Motivations for bringing semantic insights into NLP models

	Characterize representations	Causal inference	Improved models
Probing	<u> </u>		<u> </u>
Feature attribution Causal abstraction	<b>₹</b>	<del>**</del>	<b>:</b>

#### Motivations for bringing semantic insights into NLP models

	Characterize representations	Causal inference	Improved models
Probing	<b>U</b>	_	
Feature attribution Causal abstraction			<b>U</b>

Appendix on feature attribution!

## Motivations

#### Fodor and Pylyshyn (1988):

#### Fodor and Pylyshyn (1988):

"What we mean when we say that linguistic capacities are *systematic* is that the ability to produce/understand some sentences is *intrinsically* connected to the ability to produce/understand certain others."

1. Sandy loves the puppy.

#### Fodor and Pylyshyn (1988):

- 1. Sandy loves the puppy.
- 2. The puppy loves Sandy.

#### Fodor and Pylyshyn (1988):

- 1. Sandy loves the puppy.
- 2. The puppy loves Sandy.
- 3. the turtle ∼ the puppy

#### Fodor and Pylyshyn (1988):

- 1. Sandy loves the puppy.
- 2. The puppy loves Sandy.
- 3. the turtle ∼ the puppy
- 4. The turtle loves the puppy.

#### Fodor and Pylyshyn (1988):

- 1. Sandy loves the puppy.
- 2. The puppy loves Sandy.
- 3. the turtle ∼ the puppy
- 4. The turtle loves the puppy.
- 5. The puppy loves the turtle.

#### Fodor and Pylyshyn (1988):

- 1. Sandy loves the puppy.
- 2. The puppy loves Sandy.
- 3. the turtle ∼ the puppy
- 4. The turtle loves the puppy.
- 5. The puppy loves the turtle.
- 6. The turtle loves Sandy.

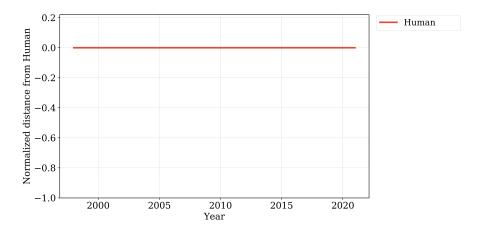
#### Fodor and Pylyshyn (1988):

- 1. Sandy loves the puppy.
- 2. The puppy loves Sandy.
- 3. the turtle ~ the puppy
- 4. The turtle loves the puppy.
- 5. The puppy loves the turtle.
- 6. The turtle loves Sandy.
- 7. ..

#### Fodor and Pylyshyn (1988):

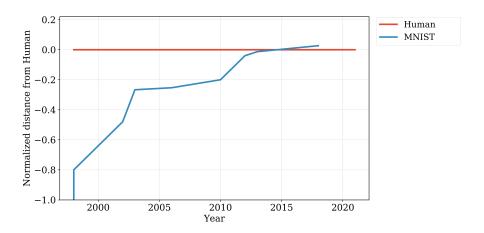
Example		Gold	Prediction
The bakery	sells a mean apple pie.	pos	pos
They	sell a mean apple pie.	pos	pos
She	sells a mean apple pie.	pos	neg
He	sells a mean apple pie.	pos	neg

#### Benchmarks saturate faster than ever

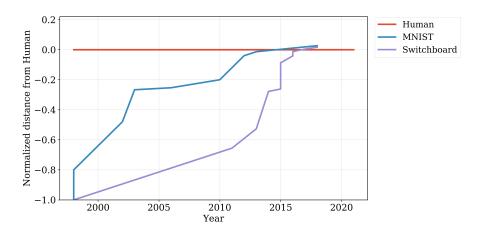


Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

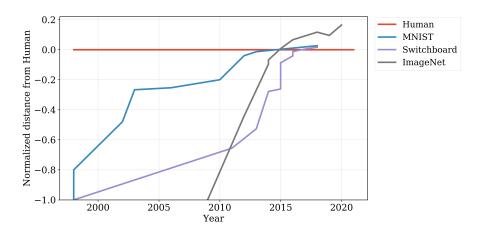
#### Benchmarks saturate faster than ever



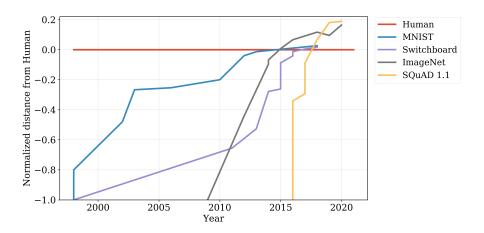
#### Benchmarks saturate faster than ever



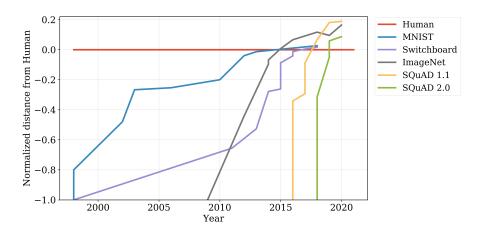
#### Benchmarks saturate faster than ever



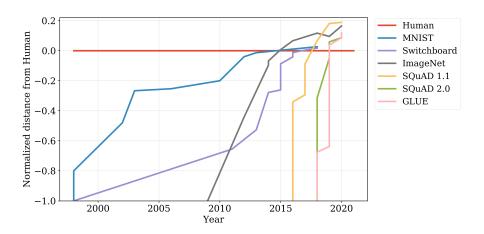
#### Benchmarks saturate faster than ever



#### Benchmarks saturate faster than ever

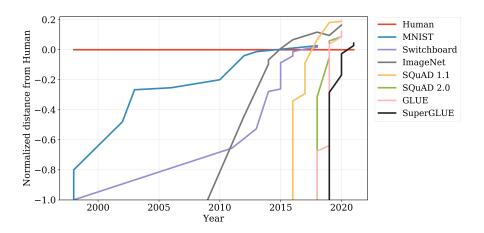


#### Benchmarks saturate faster than ever



Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

#### Benchmarks saturate faster than ever



#### Worrisome behaviors

#### Worrisome behaviors

Me: Can we talk about cats now?

Gwern: Sure, let's talk about cats!

Me: Are cats liquid?

Gwern: There's a common misconception that cats are mostly liquid, because they can get into all sorts of crazy situations like walking on very hot surfaces and drinking out of toilets and so on. This is true. Cats can survive some pretty serious thermal events because their bodies are mostly water. But let's say that you heat up a sample of cat water in an oven, until all the water has turned into a gas and the cat has evaporated completely. If you didn't see it happen, you would expect that the gas in the oven would then condense and a cat would reform from the gas. This is not the case. Cats are not made of gas. In fact, cats cannot be liquefied at any temperature. You can't turn a cat into liquid by heating it up any amount.



#### Worrisome behaviors

Me: Can we talk about cats now?

Gwern: Sure, let's talk about cats!

Me: Are cats liquid?

Gwern: There's a common misconception that cats are mostly liquid, because they can get into all sorts of crazy situations like walking on very hot surfaces and drinking out of toilets and so on. This is true. Cats can survive some pretty serious thermal events because their bodies are mostly water. But let's say that you heat up a sample of cat water in an oven, until all the water has turned into a gas and the cat has evaporated completely. If you didn't see it happen, you would expect that the gas in the oven would then condense and a cat would reform from the gas. This is not the case. Cats are not made of gas. In fact, cats cannot be liquefied at any temperature. You can't turn a cat into liquid by heating it up any amount.



www.nabla.com/



Goal: causal analysis of a model's structure.

- Goal: causal analysis of a model's structure.
- Goal: incorporate linguistic insights to increase systematicity.

- Goal: causal analysis of a model's structure.
- Goal: incorporate linguistic insights to increase systematicity.
- Further questions of
  - fairness
  - bias
  - reliability
  - robustness

are hard to address without guarantees of systematicity.

# Probing

### Recipe for probing

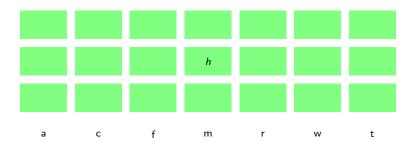
### Recipe for probing

1. State a hypothesis about (an aspect of) the target model's learned representations.

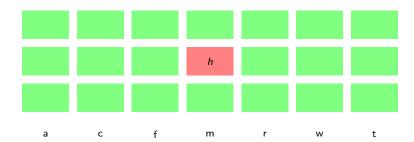
### Recipe for probing

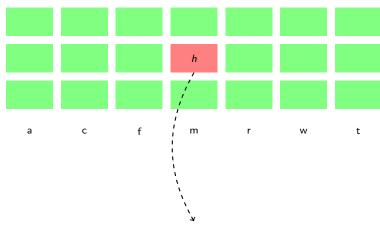
- 1. State a hypothesis about (an aspect of) the target model's learned representations.
- 2. Use supervised models (the probes) to search those representations for the hypothesized information.

### Core method

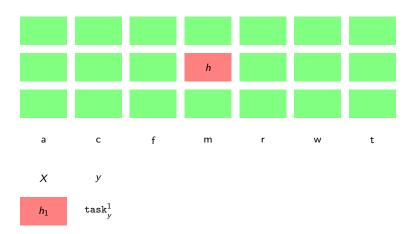


### Core method

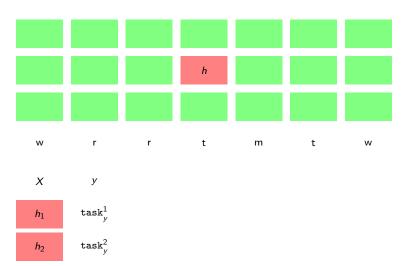


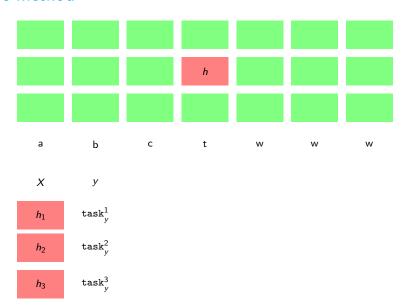


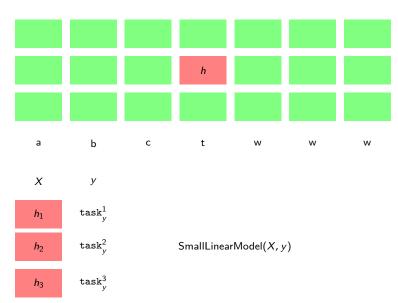
SmallLinearModel(h) = task

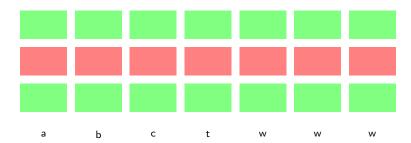


Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusion

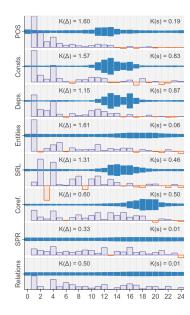








### **Probing BERT**



Probing or learning a new model?

#### Probing or learning a new model?

1. A probe is a supervised model with a particular featurization choice.

#### Probing or learning a new model?

- 1. A probe is a supervised model with a particular featurization choice.
- 2. At least some of the information that we identify is likely to be stored in the probe model.

#### Probing or learning a new model?

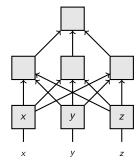
- 1. A probe is a supervised model with a particular featurization choice.
- 2. At least some of the information that we identify is likely to be stored in the probe model.
- 3. Responses:
  - Unsupervised probes (Saphra and Lopez 2019; Clark et al. 2019; Hewitt and Manning 2019)
  - ► Control tasks (Hewitt and Liang 2019)

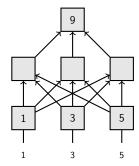
#### Probing or learning a new model?

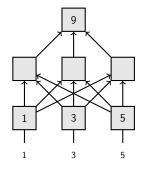
- 1. A probe is a supervised model with a particular featurization choice.
- 2. At least some of the information that we identify is likely to be stored in the probe model.
- 3. Responses:
  - Unsupervised probes (Saphra and Lopez 2019; Clark et al. 2019; Hewitt and Manning 2019)
  - Control tasks (Hewitt and Liang 2019)

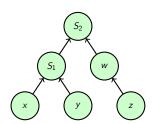
#### No causal inference

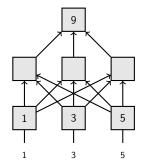
Probes cannot tell us about whether the information that we identify has any *causal* relationship with the target model's behavior (Belinkov and Glass 2019; Geiger et al. 2020, 2021a).

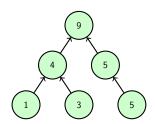


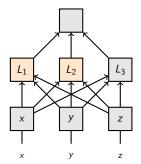




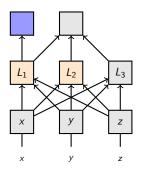




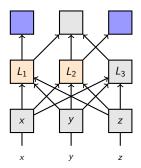




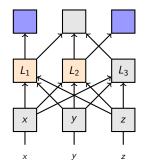
1. Probe  $L_1$ : it computes z

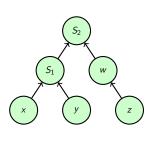


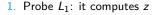
- 1. Probe  $L_1$ : it computes z
- 2. Probe  $L_2$ : it computes x + y



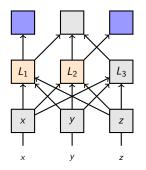
- 1. Probe  $L_1$ : it computes z
- 2. Probe  $L_2$ : it computes x + y
- 3. Aha!

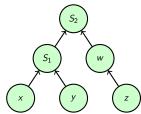






- 2. Probe  $L_2$ : it computes x + y
- 3. Aha!



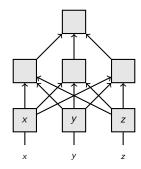


4. But  $L_2$  has no impact on the output!

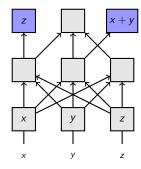
$$W_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$
  $W_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$   $W_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ 

$$\mathbf{w} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad (\mathbf{x}W_1; \mathbf{x}W_2; \mathbf{x}W_3) \mathbf{w}$$

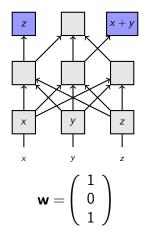
### From probing to multi-task training



### From probing to multi-task training



### From probing to multi-task training



### Summary

•		Characterize representations	Causal inference	Improved models
	Probing	<u></u>		
	Feature attribution		$\stackrel{\boldsymbol{\square}}{\boldsymbol{\square}}$	
	Causal abstraction	$\stackrel{\boldsymbol{\square}}{=}$	$\stackrel{\boldsymbol{\square}}{\boldsymbol{\square}}$	$\stackrel{\boldsymbol{\square}}{\boldsymbol{\square}}$

# Causal abstraction

### Recipe for causal abstraction

### Recipe for causal abstraction

 State a hypothesis about (an aspect of) the target model's causal structure. 

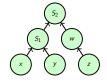
### Recipe for causal abstraction

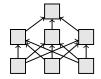
- State a hypothesis about (an aspect of) the target model's causal structure.
- 2. Search for an alignment between the causal model and target model.

emantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusion

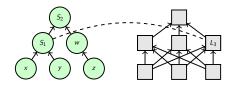
### Recipe for causal abstraction

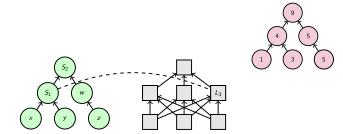
- State a hypothesis about (an aspect of) the target model's causal structure.
- 2. Search for an alignment between the causal model and target model.
- 3. Perform interchange interventions.

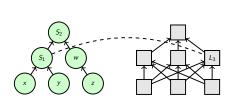


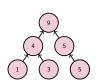


Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

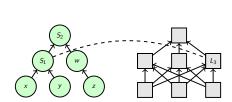


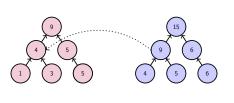


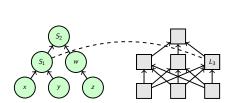


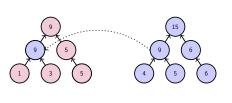


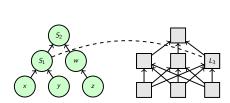


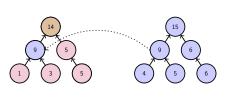


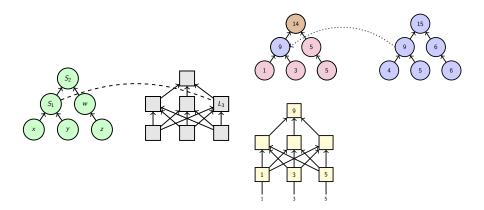


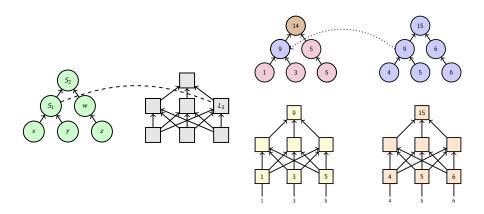


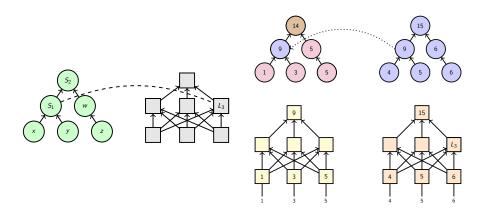


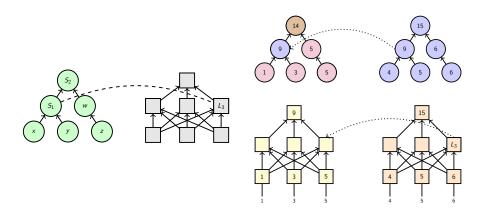


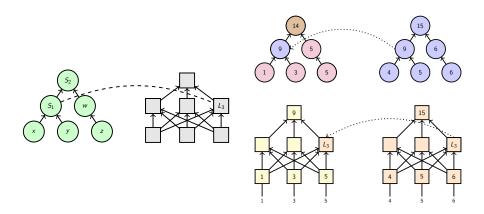


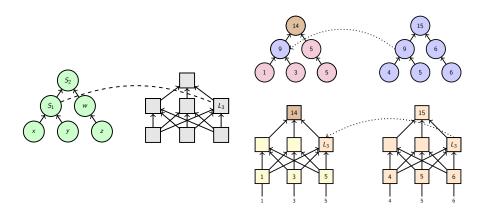


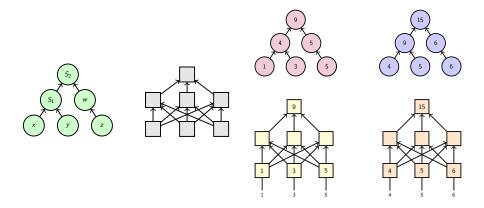




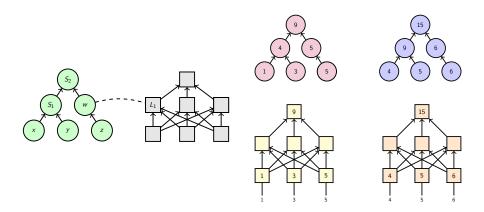


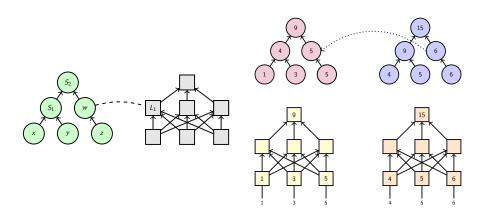


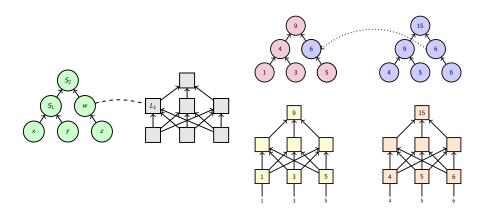


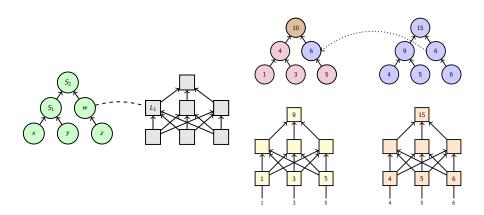


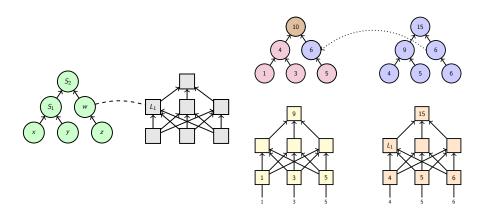
Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

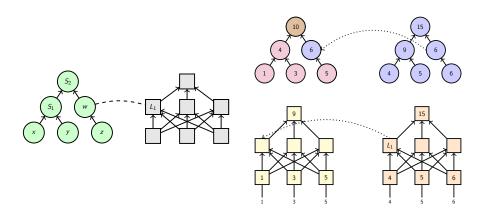


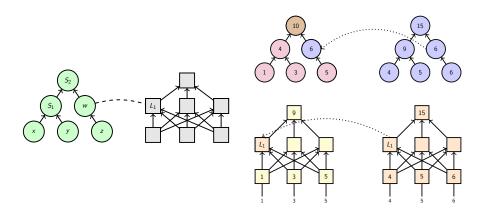


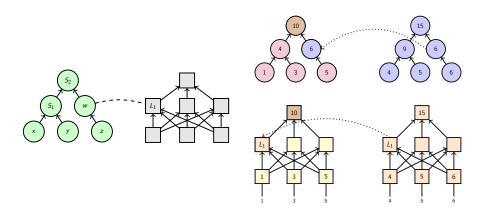


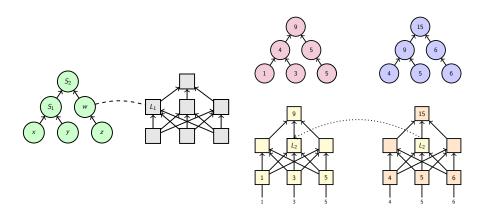












## Connections to the literature

- Constructive abstraction
- Causal mediation analysis
- Role Learning Networks
- CausaLM
- Amnesic Probing

```
(Beckers et al. 2020)
(Vig et al. 2020)
(Soulos et al. 2020)
```

# Summary

	Characterize representations	Causal inference	Improved models
Probing			
Feature attribution Causal abstraction	<b>ॐ</b>		

# Monotonicity NLI (MoNLI)

## Intuitive learning target

If A entails B then not-B entails not-A

## Intuitive learning target

If A entails B then not-B entails not-A

#### Observation

Top-performing NLI models fail to achieve the learning target (Yanaka et al. 2019, 2020; Hossain et al. 2020; Geiger et al. 2020).

## Intuitive learning target

If A entails B then not-B entails not-A

#### Observation

Top-performing NLI models fail to achieve the learning target (Yanaka et al. 2019, 2020; Hossain et al. 2020; Geiger et al. 2020).

#### Tempting conclusion

Top-performing models are incapable of learning negation.

## Intuitive learning target

If A entails B then not-B entails not-A

#### Observation

Top-performing NLI models fail to achieve the learning target (Yanaka et al. 2019, 2020; Hossain et al. 2020; Geiger et al. 2020).

#### Tempting conclusion

Top-performing models are incapable of learning negation.

#### Dataset observation

Negation is severely under-represented in NLI benchmarks.

Positive MoNLI (PMoNLI; 1,476 examples)

## Positive MoNLI (PMoNLI; 1,476 examples)

SNLI hypothesis (A) Food was served.

## Positive MoNLI (PMoNLI; 1,476 examples)

SNLI hypothesis (A)
WordNet

Food was served. pizza □ food

## Positive MoNLI (PMoNLI; 1,476 examples)

SNLI hypothesis (A)

WordNet

New example (B)

Food was served.

pizza ⊏ food

Pizza was served.

## Positive MoNLI (PMoNLI; 1,476 examples)

SNLI hypothesis (A)

WordNet

New example (B)

Positive MoNLI
Positive MoNLI

Food was served.

pizza ⊏ food

Pizza was served.

(A) neutral (B)

(B) entailment (A)

## Positive MoNLI (PMoNLI; 1,476 examples)

SNLI hypothesis (A) Food was served.

WordNet pizza □ food

New example (B) Pizza was served.

Positive MoNLI (A) neutral (B)

Positive MoNLI (B) entailment (A)

## Negative MoNLI (PMoNLI; 1,202 examples)

SNLI hypothesis (A) The children are **not** holding plants.

WordNet flowers □ plants

New example (B) The children are **not** holding flowers.

Negative MoNLI (A) **entailment** (B) Negative MoNLI (B) **neutral** (A)

# MoNLI monotonicity algorithm

## MoNLI monotonicity algorithm

#### Infer(example)

- 1  $lexrel \leftarrow GET-LEXREL(example)$
- 2 **if** CONTAINS-NOT(example)
- 3 return REVERSE(lexrel)
  - 4 **return** lexrel

## MoNLI monotonicity algorithm

#### Infer(example)

- 1  $lexrel \leftarrow GET-LEXREL(example)$
- 2 **if** CONTAINS-NOT(example)
- 3 **return** REVERSE(*lexrel*)
- 4 return lexrel

MoNLI Pizza was served. entailment Food was served. lexrel Pizza entailment Food

## MoNLI monotonicity algorithm

```
INFER(example)
```

- 1  $lexrel \leftarrow GET-LEXREL(example)$
- 2 **if** CONTAINS-NOT(example)
- 3 return REVERSE(lexrel)
- 4 return lexrel

MoNI I Pizza was served. entailment Food was served. entailment lexrel Pizza Food MoNI I Pizza was not served. neutral Food was not served. lexrel Pizza entailment Food REVERSE(lexrel) neutral

## MoNLI as challenge dataset

Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

# MoNLI as challenge dataset

			No MoNLI fine-tuning			
Model	Input pretrain	NLI train data	SNLI	PMoNLI	NMoNLI	
BiLSTM	GloVe	SNLI train	81.6	73.2	37.9	
ESIM	GloVe	SNLI train	87.9	86.6	39.4	
BERT	BERT	SNLI train	90.8	94.4	2.2	

Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

# MoNLI as challenge dataset

			No MoNLI fine-tuning		With NMoNLI fine-tuning		
Model	Input pretrain	NLI train data	SNLI	PMoNLI	NMoNLI	SNLI	NMoNLI
BiLSTM	GloVe	SNLI train	81.6	73.2	37.9	74.6	93.5
ESIM	GloVe	SNLI train	87.9	86.6	39.4	56.9	96.2
BERT	BERT	SNLI train	90.8	94.4	2.2	90.5	90.0

## Probe results for lexrel accuracy



Appendix with full probing results!

Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

#### Probe results for lexrel accuracy



SmallLinearModel(h) = GET-LEXREL(tree, elm)

Appendix with full probing results!

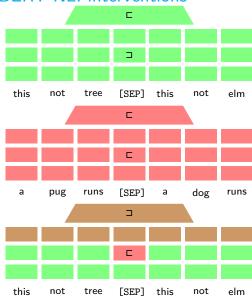
Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

#### Probe results for lexrel accuracy

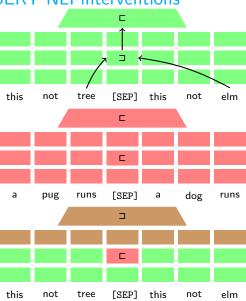


SmallLinearModel(h) = GET-LEXREL(tree, elm)

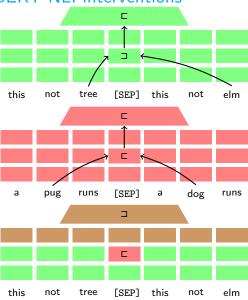
Appendix with full probing results!



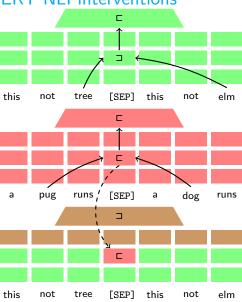
- 1  $lexrel \leftarrow GET-LEXREL(ex)$
- 2 **if** CONTAINS-NOT(ex)
- 3 return REVERSE(lexrel)
- 4 return lexrel



- $lexrel \leftarrow GET-LEXREL(ex)$
- 2 **if** CONTAINS-NOT(*ex*)
- 3 return REVERSE(lexrel)
- 4 return lexrel



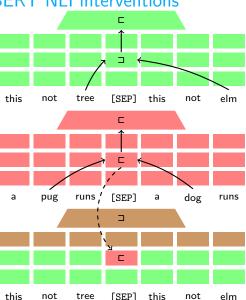
- $lexrel \leftarrow GET-LEXREL(ex)$
- 2 **if** CONTAINS-NOT(ex)
- 3 return REVERSE(lexrel)
- 4 **return** lexrel



- 1  $lexrel \leftarrow GET-LEXREL(ex)$
- 2 **if** CONTAINS-NOT(ex)
- 3 return REVERSE(lexrel)
- 4 return lexrel

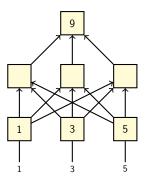
Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training 0000000

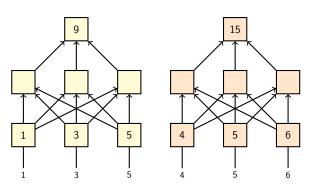
## BERT NLI interventions

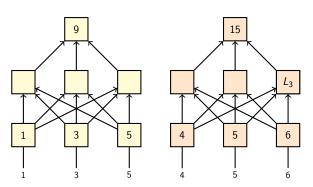


- $lexrel \leftarrow GET-LEXREL(ex)$
- if CONTAINS-NOT(ex)
- return REVERSE(*lexrel*)
- return lexrel

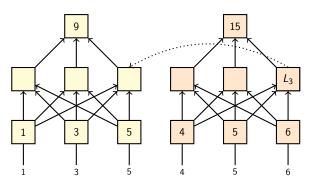
# Interchange intervention training (IIT)



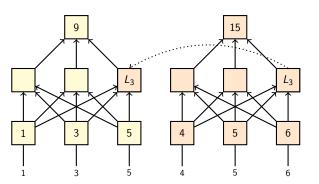


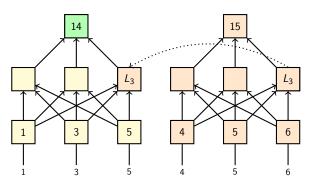


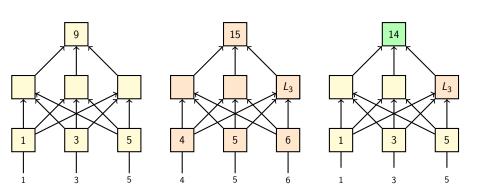
Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio



Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

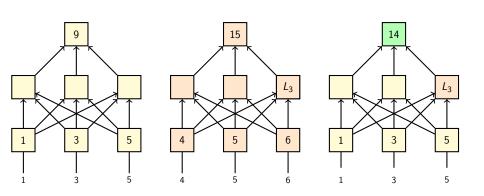






Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

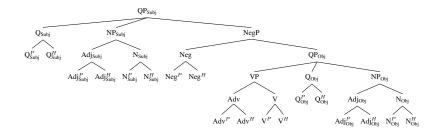
#### IIT: Training models to conform to a hypothesis



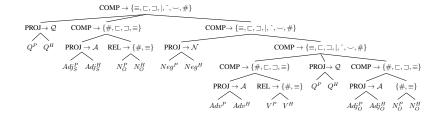
Appendix: IIT induces causal structure!

## MQNLI: Extreme compositional complexity

#### MQNLI: Extreme compositional complexity

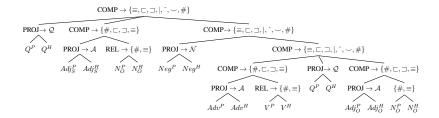


#### MQNLI: Extreme compositional complexity



iemantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusio

#### MQNLI: Extreme compositional complexity



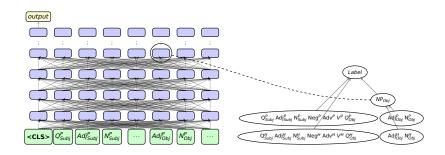
 $\mathcal{E}$  every  $\mathcal{E}$  baker  $\mathcal{E}$   $\mathcal{E}$  eats  $\mathcal{E}$  no  $\mathcal{E}$  bread contradiction  $\mathcal{E}$  no angry baker  $\mathcal{E}$   $\mathcal{E}$  eats  $\mathcal{E}$  no  $\mathcal{E}$  bread

 $\mathcal E$  every silly professor  $\mathcal E$   $\mathcal E$  sells not every  $\mathcal E$  book **neutral**  $\mathcal E$  every silly professor  $\mathcal E$   $\mathcal E$  sells not every  $\mathcal E$  chair

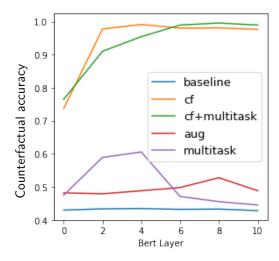
Geiger et al. 2020, 2021a

Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusion

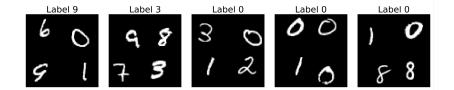
## MQNLI: IIT on the object quantifier model



## MQNLI results



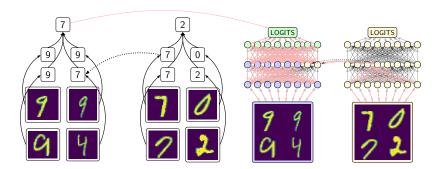
#### MNIST Pointer Value Retrieval



0-3: top right; 4-6: bottom left; 7-9: bottom right

Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusion

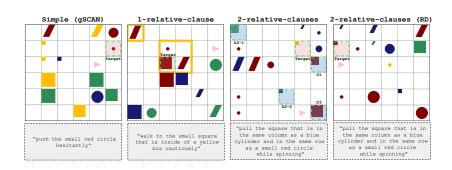
#### MNIST Pointer Value Retrieval



Geiger et al. 2021b

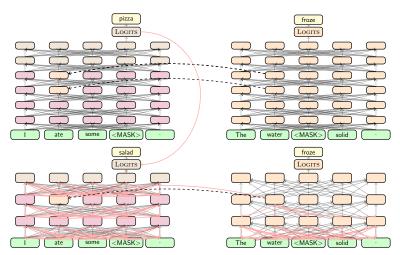
Semantics in NLP Motivations Probing Causal abstraction Monotonicity NLI Interchange intervention training Conclusion

#### ReaSCAN



Geiger et al. 2021b

## Language model distillation



Wu et al. 2021

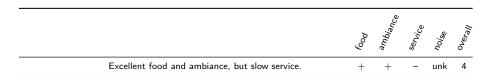
#### CeBaB: A causal benchmark for sentiment

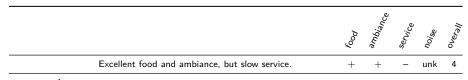


#### CeBaB: A causal benchmark for sentiment

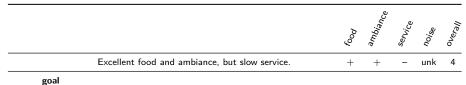


Excellent food and ambiance, but slow service.

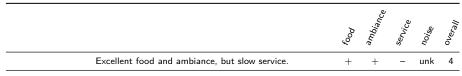




goal



food edit:



goal

food edit: - Terrible food, excellent ambiance, but slow service.

food edit:

		Poog	ambianc	service	<sup>n</sup> o/se	"overa
	Excellent food and ambiance, but slow service.	+	+	_	unk	4
goal						

Terrible food, excellent ambiance, but slow service.

ø

			e e			
		f <sub>00</sub> d	ambian <sub>Ce</sub>	Service	<sup>7</sup> 0/s <sub>e</sub>	Overall
	Excellent food and ambiance, but slow service.	+	+	-	unk	4
goal						
	Terrible food, excellent ambiance, but slow service.  Excellent ambiance, but slow service.	– unk	++	_	unk unk	2

		$t^{600d}$	<sup>a</sup> mbi <sub>ance</sub>	Service	nois <sub>e</sub>	Overall
	Excellent food and ambiance, but slow service.	+	+	_	unk	4
goal						
	Terrible food, excellent ambiance, but slow service. k Excellent ambiance, but slow service.	– unk	++	_	unk unk	
ambiance edit: -	Excellent food, but lousy ambiance and slow service.	+	_	_	unk	3

		f <sub>00</sub> d	$^{amb_{iance}}$	Service	noise	Overall
	Excellent food and ambiance, but slow service.	+	+	-	unk	4
goal						
	Terrible food, excellent ambiance, but slow service. Excellent ambiance, but slow service.	– unk	++	- -	unk unk	
	Excellent food, but lousy ambiance and slow service.  Excellent food, but slow service.	+++	– unk	_	unk unk	3

			600y	$^{am}b_{ianc_{e}}$	Service	noise	Overall
		Excellent food and ambiance, but slow service.	+	+	-	unk	4
goal							
		Terrible food, excellent ambiance, but slow service. Excellent ambiance, but slow service.	– unk	++		unk unk	
		Excellent food, but lousy ambiance and slow service. Excellent food, but slow service.	++	– unk		unk unk	
service edit:	+	Excellent food and ambiance, and premium service.	+	+	+	unk	5

		, 60od	ambiance	Service	noise	"overall
	Excellent food and ambiance, but slow service.	+	+	-	unk	4
goal						
	Terrible food, excellent ambiance, but slow service. Excellent ambiance, but slow service.	– unk	++	_	unk unk	2
	Excellent food, but lousy ambiance and slow service. Excellent food, but slow service.	++	– unk	_	unk unk	3 3
	Excellent food and ambiance, and premium service. Excellent food and ambiance.	++	++	+ unk	unk unk	5 5

			f00d	ambiance	Setvice	nois <sub>e</sub>	overall
		Excellent food and ambiance, but slow service.	+	+	-	unk	4
goal							
		Terrible food, excellent ambiance, but slow service. Excellent ambiance, but slow service.	– unk	++	-	unk unk	2
		Excellent food, but lousy ambiance and slow service. Excellent food, but slow service.	++	– unk	_	unk unk	3
		Excellent food and ambiance, and premium service. Excellent food and ambiance.	+ +	++	+ unk	unk unk	5 5
noise edit:	+	Excellent food, ambiance, and music, but slow service.	+	+	_	+	4

		$t^{6}$	<sup>a</sup> mbi <sub>ance</sub>	Service	noise	overall
	Excellent food and ambiance, but slow service.	+	+	-	unk	4
goal						
	Terrible food, excellent ambiance, but slow service. Excellent ambiance, but slow service.	– unk	++	- -	unk unk	2
	Excellent food, but lousy ambiance and slow service. Excellent food, but slow service.	++	– unk	_	unk unk	3 3
	Excellent food and ambiance, and premium service. Excellent food and ambiance.	++	++	$_{unk}^{+}$	unk unk	5 5
	Excellent food, ambiance, and music, but slow service. Excellent food and ambiance, but slow service, and noisy.	++	++	_	+	4 3

		600y	<sup>a</sup> mb <sub>iance</sub>	service	<sup>n</sup> ois <sub>e</sub>	overall
	Excellent food and ambiance, but slow service.	+	+	-	unk	4
goal						
	Terrible food, excellent ambiance, but slow service. Excellent ambiance, but slow service.	– unk	++	_	unk unk	2
	Excellent food, but lousy ambiance and slow service. Excellent food, but slow service.	++	– unk	_	unk unk	3 3
	Excellent food and ambiance, and premium service. Excellent food and ambiance.	++	++	+ unk	unk unk	5 5
noise edit: noise edit:	Excellent food, ambiance, and music, but slow service. Excellent food and ambiance, but slow service, and noisy.	++	++	_	+	4 3

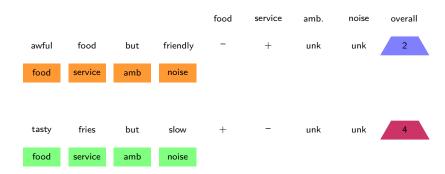
		000	<sup>1</sup> mbiance	ervice	s,ise	
	Finally of Good and ambiguous has also assiste			Š	ξ	o
goal	Excellent food and ambiance, but slow service.	+	+	_	unk	
	Terrible food, excellent ambiance, but slow service. Excellent ambiance, but slow service.	– unk	+++	_	unk unk	2
	Excellent food, but lousy ambiance and slow service. Excellent food, but slow service.	++	– unk	_	unk unk	3
	Excellent food and ambiance, and premium service. Excellent food and ambiance.	++	++	+ unk	unk unk	5 5
noise edit: noise edit:	Excellent food, ambiance, and music, but slow service. Excellent food and ambiance, but slow service, and noisy.	++	++	_	+	4 3

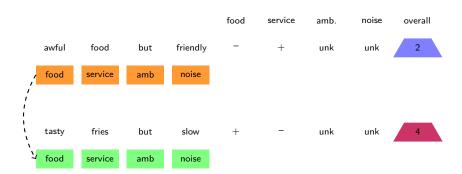
≈15K sentences; 5 validation labels for all examples; 88% have 3/5 majority label.

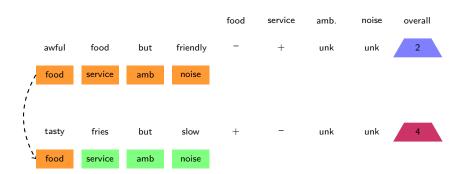
				food	service	amb.	noise	overall
awful	food	but	friendly	-	+	unk	unk	2

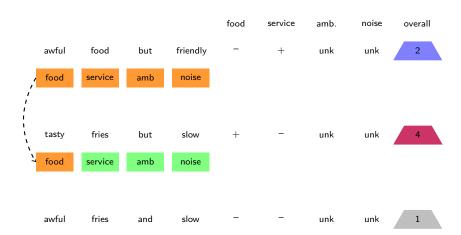


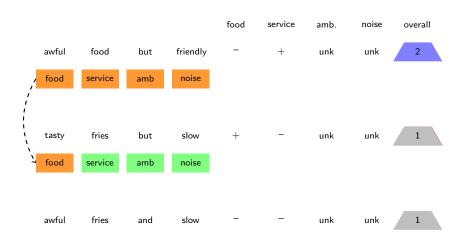












# Conclusion

# Summary

	Characterize representations	Causal inference	Improved models
Probing			
Feature attribution		<u></u>	•
Causal abstraction			

1. Can we more effectively leverage probes to find useful intervention points?

- 1. Can we more effectively leverage probes to find useful intervention points?
- 2. What is the relationship between interchange interventions and integrated gradients?

- 1. Can we more effectively leverage probes to find useful intervention points?
- 2. What is the relationship between interchange interventions and integrated gradients?
- 3. Can we find ways to apply IIT in places where the causal model is approximate and applies to only a subset of examples?

- 1. Can we more effectively leverage probes to find useful intervention points?
- 2. What is the relationship between interchange interventions and integrated gradients?
- 3. Can we find ways to apply IIT in places where the causal model is approximate and applies to only a subset of examples?
- 4. More generally: where else might causal abstraction analysis and IIT be useful?

# Open questions

- 1. Can we more effectively leverage probes to find useful intervention points?
- 2. What is the relationship between interchange interventions and integrated gradients?
- 3. Can we find ways to apply IIT in places where the causal model is approximate and applies to only a subset of examples?
- 4. More generally: where else might causal abstraction analysis and IIT be useful?

## Thanks!

#### References I

- Sander Beckers, Frederick Eberhardt, and Joseph Y. Halpern. 2020. Approximate causal abstractions. In Proceedings of The 35th Uncertainty in Artificial Intelligence Conference, volume 115 of Proceedings of Machine Learning Research, pages 606–615. PMLR. Yonatan Belinkov and James Glass. 2019. Analysis methods in neural language processing: A survey. Transactions of the Association for Computational Linguistics, 7:49–72.
- Kevin Clark, Urvashi Khandelwal, Omer Levy, and Christopher D. Manning. 2019. What does BERT look at? an analysis of BERT's attention. In Proceedings of the 2019 ACL Workshop BlackboxNLP: Analyzing and Interpreting Neural Networks for NLP, pages 276–286. Florence. Italy. Association for Computational Linguistics.
- Alexis Conneau, German Kruszewski, Guillaume Lample, Loïc Barrault, and Marco Baroni. 2018. What you can cram into a single \$&!#\*
  vector: Probing sentence embeddings for linguistic properties. In Proceedings of the 56th Annual Meeting of the Association for
  Computational Linguistics (Volume 1: Long Papers), pages 2126–2136, Melbourne, Australia. Association for Computational
  Linguistics.
- Yanai Elazar, Shauli Ravfogel, Alon Jacovi, and Yoav Goldberg. 2021. Amnesic probing: Behavioral explanation with amnesic counterfactuals. Transactions of the Association for Computational Linguistics, 9(0):160–175.
- Amir Feder, Nadav Oved, Uri Shalit, and Roi Reichart. 2021. CausaLM: Causal model explanation through counterfactual language models. Computational Linguistics, 47(2):333–386.
- Jerry A. Fodor and Zenon W. Pylyshyn. 1988. Connectionism and cognitive architecture: A critical analysis. Cognition, 28(1):3–71. Atticus Geiger, Hanson Lu, Thomas Icard, and Christopher Potts. 2021a. Causal abstractions of neural networks. In Advances in Neural Information Processing Systems.
- Atticus Geiger, Kyle Richardson, and Christopher Potts. 2020. Neural natural language inference models partially embed theories of lexical entailment and negation. In Proceedings of the Third Blackbox/NLP Workshop on Analyzing and Interpreting Neural Networks for NLP, pages 163–173, Online. Association for Computational Linguistics.
- Atticus Geiger, Zhengxuan Wu, Hanson Lu, Josh Rozner, Elisa Kreiss, Thomas Icard, Noah D. Goodman, and Christopher Potts. 2021b. Inducing causal structure for interpretable neural networks. ArXiv:2112.00826.
- John Hewitt and Percy Liang. 2019. Designing and interpreting probes with control tasks. In Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP), pages 2733–2743, Hong Kong, China. Association for Computational Linguistics.
- John Hewitt and Christopher D. Manning. 2019. A structural probe for finding syntax in word representations. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 4129–4138, Minneapolis, Minnesota. Association for Computational Linguistics.
- Md Mosharaf Hossain, Venelin Kovatchev, Pranoy Dutta, Tiffany Kao, Elizabeth Wei, and Eduardo Blanco. 2020. An analysis of natural language inference benchmarks through the lens of negation. In Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMMLP), pages 9106–9118, Online. Association for Computational Linguistics.

## References II

- Douwe Kiela, Max Bartolo, Yixin Nie, Divyansh Kaushik, Atticus Geiger, Zhengxuan Wu, Bertie Vidgen, Grusha Prasad, Amanpreet Singh, Pratik Ringshia, Zhiyi Ma, Tristan Thrush, Sebastian Riedel, Zeerak Waseem, Pontus Stenetorp, Robin Jia, Mohit Bansal, Christopher Potts, and Adina Williams. 2021. Dynabench: Rethinking benchmarking in NLP. In Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, pages 4110-4124, Online. Association for Computational Linguistics.
- Joe Pater. 2019. Generative linguistics and neural networks at 60: Foundation, friction, and fusion. Language, 95(1):e41-e74.
- Marco Ribeiro, Sameer Singh, and Carlos Guestrin. 2016. "why should I trust you?": Explaining the predictions of any classifier. In Proceedings of the 2016 Conference of the North American Chapter of the Association for Computational Linguistics: Demonstrations, pages 97-101. Association for Computational Linguistics.
- Naomi Saphra and Adam Lopez. 2019. Understanding learning dynamics of language models with SVCCA. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 3257-3267, Minneapolis, Minnesota. Association for Computational Linguistics.
- Avanti Shrikumar, Peyton Greenside, and Anshul Kundaie, 2017. Learning important features through propagating activation differences. In Proceedings of the 34th International Conference on Machine Learning-Volume 70, pages 3145-3153, JMLR, org.
- Karen Simonyan, Andrea Vedaldi, and Andrew Zisserman, 2013. Deep inside convolutional networks: Visualising image classification models and saliency maps. arXiv preprint arXiv:1312.6034.
- Richard Socher, Alex Perelygin, Jean Wu, Jason Chuang, Christopher D. Manning, Andrew Y. Ng, and Christopher Potts. 2013. Recursive deep models for semantic compositionality over a sentiment treebank. In Proceedings of the 2013 Conference on Empirical Methods in Natural Language Processing, pages 1631-1642, Stroudsburg, PA, Association for Computational Linguistics,
- Paul Soulos, R. Thomas McCov, Tal Linzen, and Paul Smolensky, 2020. Discovering the compositional structure of vector representations with role learning networks. In Proceedings of the Third BlackboxNLP Workshop on Analyzing and Interpreting Neural Networks for NLP, pages 238-254, Online. Association for Computational Linguistics.
- Mukund Sundararajan, Ankur Taly, and Oigi Yan, 2017, Axiomatic attribution for deep networks. In Proceedings of the 34th International Conference on Machine Learning-Volume 70, pages 3319-3328, JMLR, org.
- Ian Tenney, Dipanian Das, and Ellie Paylick, 2019, BERT rediscovers the classical NLP pipeline, In Proceedings of the 57th Annual
- Meeting of the Association for Computational Linguistics, pages 4593-4601, Florence, Italy. Association for Computational Linguistics. Jesse Vig. Sebastian Gehrmann, Yonatan Belinkov, Sharon Qian, Daniel Nevo, Yaron Singer, and Stuart Shieber, 2020, Causal mediation analysis for interpreting neural nlp: The case of gender bias.
- David H. D. Warren and Fernando C. N. Pereira. 1982. An efficient easily adaptable system for interpreting natural language queries. American Journal of Computational Linguistics, 8(3-4):110-122.
- Zhengxuan Wu, Atticus Geiger, Josh Rozner, Elisa Kreiss, Hanson Lu, Thomas Icard, Christopher Potts, and Noah D. Goodman. 2021. Causal distillation for language models. ArXiv:2112.02505.

#### References III

- Hitomi Yanaka, Koji Mineshima, Daisuke Bekki, and Kentaro Inui. 2020. Do neural models learn systematicity of monotonicity inference in natural language? In Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics, pages 6105–6117, Online. Association for Computational Linguistics.
- Hitomi Yanaka, Koji Mineshima, Daisuke Bekki, Kentaro Inui, Satoshi Sekine, Lasha Abzianidze, and Johan Bos. 2019. HELP: A dataset for identifying shortcomings of neural models in monotonicity reasoning. In Proceedings of the Eighth Joint Conference on Lexical and Computational Semantics (\*SEM 2019), pages 250–255, Minneapolis, Minnesota. Association for Computational Linguistics.
- Matthew D Zeiler and Rob Fergus. 2014. Visualizing and understanding convolutional networks. In European conference on computer vision, pages 818–833. Springer.
- Luke S. Zettlemoyer and Michael Collins. 2005. Learning to map sentences to logical form: Structured classification with probabilistic categorial grammars. In Proceedings of the Twenty First Conference on Uncertainty in Artificial Intelligence.

## Feature attribution

- 1. captum.ai
- 2. Integrated gradients: Intuition
- 3. Integrated Gradients: Central properties
- 4. Integrated Gradients: Computation
- 5. Reliable insights about causal structure

# captum.ai

- 1. Integrated gradients
- 2. Gradients
- 3. Saliency Maps
- 4. DeepLift
- 5. Deconvolution
- 6. LIME
- Feature ablation
- 8. Feature permutation
- 9. ...

```
(Sundararajan et al. 2017)
```

(Simonyan et al. 2013)

(Shrikumar et al. 2017)

(Zeiler and Fergus 2014)

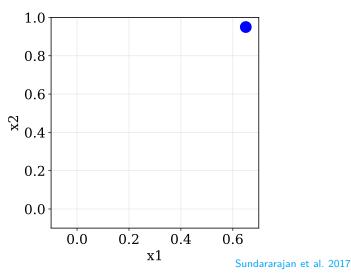
(Ribeiro et al. 2016)

https://captum.ai

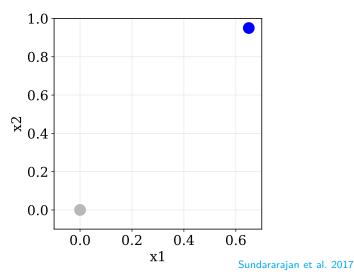
# Plug for integrated gradients!

- It's common for people to use gradients as estimates of feature importance in deep learning models, but these aren't reliable signals.
- Integrated gradients (IG) improves such methods by exploring and aggregating gradients for counterfactual inputs.
- IG can be shown to measure causal effects (Geiger et al. 2021a).
- Easy to use with captum.ai or AllenNLP!

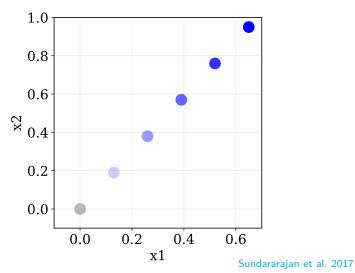
## Integrated gradients: Intuition



## Integrated gradients: Intuition



## Integrated gradients: Intuition



### Integrated gradients: Central properties

### Sensitivity

If two inputs x and x' differ only at dimension i and lead to different predictions, then feature  $f_i$  has non-zero attribution.

$$M([1, 0, 1]) = positive$$
  
 $M([1, 1, 1]) = negative$ 

MoNLI causal abstraction analysis details

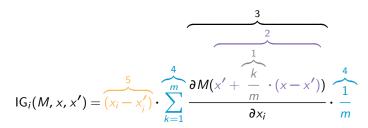
#### Completeness

For input x and baseline x', the sum of attributions for x is equal to M(x) - M(x').

#### Implementation invariance

If two models M and M' have identical input/output behavior, then the attributions for M and M' are identical.

### Integrated Gradients: Computation

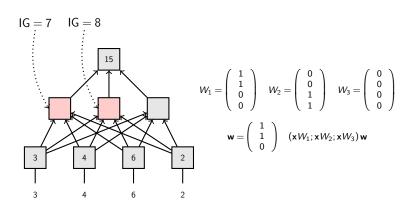


- 1. Generate  $\alpha = [1, \ldots, m]$
- 2. Interpolate inputs between baseline x' and actual input x
- 3. Compute gradients for each interpolated input
- 4. Integral approximation through averaging
- 5. Scaling to remain in the space region as the original

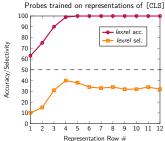
MoNLI causal abstraction analysis details

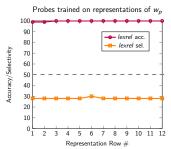
MoNLI causal abstraction analysis details

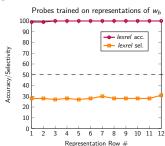
### Reliable insights about causal structure



## Probe results for lexrel accuracy







## MoNLI causal abstraction analysis details

- 1. A systematic generalization task
- 2. Methods and findings
- 3. Largest exchangeable cluster
- 4. Which algorithm is BERT implementing then?

## A systematic generalization task

NMoNLI Train		NMoNLI Test		
person instrument food machine woman music tree boat fruit produce fish plant jewelry anything hat man horse gun adult shirt shoe cake individual clothe weapon creature	198 100 94 60 58 52 46 42 40 40 38 36 34 20 20 20 16 12 10 8 6 6 4 4 4 4	dog building ball car mammal animal	88 64 28 12 4 4	models know these lexical relations (live MoNLI accuracy) and will be compe mbine this knowledge with what they le t negation during Negative MoNLI f g.

# Methods and findings

- 1. Find a useful intervention point.
- 2. Interchange interventions for every pair of examples at that site.
- 3. Find clusters of examples in which BERT mimics the causal dynamics of INFER.
- 4. The largest subsets we found 98, 63, 47, and 37.
  - a. For a random graph, the expected number of subsets larger than 20 is effectively 0.
  - b. If the site perfectly captured INFER, we would get a single huge cluster.

```
INFER(example)
   lexrel \leftarrow GET-LEXREL(example)
   if CONTAINS-NOT(example)
3
        return REVERSE(lexrel)
   return lexrel
```

INFER<sub>lexrel(i) 
$$\rightarrow$$
 lexrel(j)(i) =
$$\begin{cases} INFER(i) & lexrel(i) = lexrel(j) \\ REVERSE(INFER(i)) & lexrel(i) \neq lexrel(j) \end{cases}$$</sub>

INFER<sub>lexrel(i) 
$$\rightarrow$$
 lexrel(j)(i) = BERT<sub>L(i)  $\rightarrow$  L(j)(i)</sub></sub>

MoNLI causal abstraction analysis details

# Largest exchangeable cluster

		(dogs,huski						
(house,location) (den,location)			(dog,husky) (dog,chihuahua)			(hood,thing)		
(ghetto,location)	(backyard,location)	(park,location) (residence,location)	(do	g,retriever)	(dog,maltese)	(nut,thing)	(capsule,thing)	
(jungle,location)			(dog,te	(dog,terrier) (dog,pomerani		(pouch,thing)	(structure,thing)	
,	(meadow,location)			(beetle,insect)		(root,thing)	(nugget,thing)	
(laboratory,location)	(playground, location)		(grassho	(grasshopper,insect) (bee,insect)			(tube,thing)	
(slum,location)	(station, location)	(farm,location)	(wasp,insect)	(fly,insect	(cricket,insect)			
(lab,location)	(campsite,location)	(lailii,location)	(butterfly,insect)		(bumblebee,insect)	(box,o	bject)	
(town,location) (lawn,location)		n)	, ,	,		(object,sweater)	(hat,object)	
		(flea,insect)	(roach,insect osquito,insect	, , , , ,	(object,jacket)	(toy,object)		
(saxophone,instrument) (flute,instrument) (bass,instrument) (piano,instrument)		t)	(person,vegetarian) (person,lunatic)				( )	
		**)					(cane,object)	
(violin,instrument) (tuba,instrument)		11)	(person,republican) (person,trooper) ent) (person,business) (person,navigator)		(person trooper)	(water,rainwater) (water,saltwater)		
		ment)						
		(person,navigator)						
(harmonica,instrument) (person			varu)		(person,goalkeeper)			
			(person,farmer)		()	(sculptor,artist)		
(liquid,whiskey) (person,s			ophomore) (person,housekeeper)					
(liquid,margarita) (liquid,tequila)			(person,cleaner) (person,physicist) (person,cop)			(berry,blueberry)		
(liquid,alcohol)			(person,cambodian) (person,detective)					
			(person,cambodian) (person,detective)				cypress)	
(woman,granny)			(person,genius) (person,sergeant) (person,californian)			(tree,magnolia	) (trees,elms)	
	(woman,widow)				(F,Cumormun)	(tree,maple)		
		(person.do	octor) /_					

## Which algorithm is BERT implementing then?

```
INFER(example)
                                               INFER(example)
    lexrel ← GET-LEXREL(example)
                                                    if INCLUSTER(C_1, example)
    if CONTAINS-NOT(example)
                                                         lexrel_1 \leftarrow GET-LEXREL(example)
3
         return REVERSE(lexrel)
                                                         if CONTAINS-NOT(example)
    return lexrel
                                                              return REVERSE(lexrel<sub>1</sub>)
                                                         return lexrel1
                                                    if INCLUSTER(C_2, example)
                                                         lexrel_2 \leftarrow GET-LEXREL(example)
                                                8
                                                         if CONTAINS-NOT(example)
                                                              return REVERSE(lexrel<sub>2</sub>)
                                               10
                                                         return lexrelo
                                               11
                                                    if INCLUSTER(C_3, example)
                                                         lexrel_3 \leftarrow GET-LEXREL(example)
                                               12
                                                         if CONTAINS-NOT (example)
                                               13
                                               14
                                                              return REVERSE(lexrel3)
                                               15
                                                         return lexrel3
```

16

