Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Arkansas
Based on data through October 9, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Arkansas: Daily Deaths per Million People (Smoothed)
### Brief Summary of Model

- See the **paper** for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Arkansas: Estimates of $R_0(t)$

Arkansas

$\delta = 0.010$  $\theta=0.10$  $\gamma=0.20$
Arkansas: Percent Currently Infectious

Arkansas
Peak I/N = 0.48%  Final I/N = 0.32%  δ = 0.010  θ = 0.10  γ = 0.20
Arkansas: Growth Rate of Daily Deaths over Past Week (percent)

Arkansas
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order! ROY-G-BIV (old to new, low to high)
  - Black = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  - Alternatively, we fit this equation:
    \[
    \log R_0(t) = a_0 - \alpha(Daily\ Deaths)
    \]
    \[
    \Rightarrow \alpha \approx 0.05
    \]
    $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline
- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Arkansas (7 days): Daily Deaths per Million People ($\alpha = .05$)

Arkansas

$R_0 = 1.1/1.1/1.1$ $\delta = 0.010$ $\alpha = 0.05$ $\theta = 0.1$ $\%$ Infect $= 7/8/14$

DATA THROUGH 09-OCT-2020
Arkansas (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

$R_0 = 1.1/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 7/8/14$

DATA THROUGH 09-OCT-2020
Arkansas (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Arkansas

$R_0 = 1.1/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 7/8/14

New York City

Italy
Robustness to Mortality Rate, $\delta$
Arkansas: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Arkansas

$R_0 = 1.1/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 7/8/14$

DATA THROUGH 09-OCT-2020
Arkansas: Daily Deaths per Million People ($\delta = .01/ .008 / .012$)

Arkansas

$R_0 = 1.1 / 1.1 / 1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 7 / 8 / 14$

DATA THROUGH 09-OCT-2020
Arkansas: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

$R_0 = 1/1/1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 7/8/14

DATA THROUGH 09-OCT-2020
Reopening and Herd Immunity

– **Black**: assumes $R_0(\text{today})$ remains in place forever
– **Red**: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
– **Green**: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
– **Purple**: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

**NOTE**: Lines often cover each other up
Arkansas: Re-Opening ($\alpha = 0.05$)

Arkansas

$R_0(t)=1.1$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.3/1.5$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Arkansas: Re-Opening ($\alpha = 0$)

Arkansas

$R_0(t)=1.1$, $R_0(\text{suppress})=1.1$, $R_0(25/50)=1.3/1.5$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Arkansas (7 days): Daily Deaths per Million People ($\alpha = 0$)

Arkansas

$R_0=1.1/1.1/1.1 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%Infect=7/8/11$

DATA THROUGH 09-OCT-2020
Arkansas (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Arkansas

$R_0=1.1/1.1/1.1 \; \delta = 0.010 \; \alpha=0.00 \; \theta=0.1 \; \%\text{Infect}=7/8/11$

DATA THROUGH 09-OCT-2020
Arkansas (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Arkansas

$R_0 = 1.1/1.1/1.1$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  %Infect= 7/8/11
Arkansas: Daily Deaths per Million People \((\delta = 0.8\%)\)

Arkansas

\[ R_0 = 1.1/1.1/1.2 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \% \text{Infect} = 8/11/17 \]
Arkansas: Cumulative Deaths per Million ($\delta = 0.8\%$)

Arkansas

$R_0 = 1.1/1.1/1.2 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 8/11/17$
Arkansas: Daily Deaths per Million People ($\delta = 1.2\%$)

Arkansas

$R_0 = 1.1/1.1/1.1$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 5/7/11
Arkansas: Cumulative Deaths per Million ($\delta = 1.2\%$)

Arkansas

$R_0 = 1.1/1.1/1.1 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 5/7/11$
Arkansas: Daily Deaths per Million People ($\gamma = .2/.15$)

DATA THROUGH 09-OCT-2020

Arkansas

$R_0=1.1/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=7/8/14$
Arkansas: Cumulative Deaths per Million $\gamma = 0.2/0.15$
Arkansas: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

Arkansas

$R_0 = 1.1/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 7/8/14$

DATA THROUGH 09-OCT-2020
Arkansas: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Arkansas

$R_0 = 1.1/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 7/8/14$

DATA THROUGH 09-OCT-2020

$\theta = 0.07$

$\theta = 0.1$

$\theta = 0.2$
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Arkansas: Daily Deaths, Actual and Smoothed

Arkansas: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Arkansas: Change in Smoothed Daily Deaths

Arkansas: Delta $d$

$\delta = 0.010$  $\theta = 0.10$  $\gamma = 0.20$
Arkansas: Change in (Change in Smoothed Daily Deaths)

Arkansas: Delta (Delta \( d \))

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]