Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Armenia
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  ◦ Lower mortality rate, $\delta = 0.8\%$
  ◦ Higher mortality rate, $\delta = 1.2\%$
  ◦ Infections last longer, $\gamma = 0.15$
  ◦ Cases resolve more quickly, $\theta = 0.2$
  ◦ Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

– Raw data
– Smoothed = 7 day centered moving average
– No “excess deaths” correction (change as of Aug 6 run)
Armenia: Daily Deaths per Million People

Armenia
Armenia: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ $(\beta_t/\gamma)$</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Armenia: Estimates of $R_0(t)$

Armenia

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$

![Graph showing the estimates of $R_0(t)$ for Armenia from May to October 2020. The graph shows fluctuations in $R_0(t)$ over time, with peak values above 2 and trough values approaching 0. The chart indicates that the basic reproduction number $R_0(t)$ for Armenia fluctuates significantly throughout the period, reflecting changes in the transmission dynamics of the disease.](image)
Armenia: Percent Currently Infectious

Peak I/N = 0.19%  Final I/N = 0.04%  δ = 0.010  θ = 0.10  γ = 0.20
Armenia: Growth Rate of Daily Deaths over Past Week (percent)

\[
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it

- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     $$\log R_0(t) = a_0 - \alpha (\text{Daily Deaths})$$

     $\Rightarrow \alpha \approx .05$

     $R_0$ declines by 5 percent for each new daily death, 
     or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in 
  future, so no $\alpha$ adjustment $\Rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Armenia (7 days): Daily Deaths per Million People ($\alpha = .05$)

Armenia

$R_0=1.6/0.9/0.9$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 3/ 3/ 3

DATA THROUGH 11-SEP-2020
Armenia (7 days): Cumulative Deaths per Million (Future, \( \alpha = .05 \))

Armenia

\[ R_0 = 1.6/0.9/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 3/3/3 \]

DATA THROUGH 11-SEP-2020
Armenia (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Armenia

$R_0 = 1.6/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 3/3/3$
Robustness to Mortality Rate, $\delta$
Armenia: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Armenia

$R_0=1.6/0.9/0.9$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect $= 3/3/3$

DATA THROUGH 11-SEP-2020
Armenia: Daily Deaths per Million People ($\delta = .01/.008/.012$)

Armenia

$R_0=1.6/0.9/0.9 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=3/3/3$

DATA THROUGH 11-SEP-2020

Daily deaths per million people

Armenia: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Armenia

$R_0=1.6/0.9/0.9 \hspace{1cm} \delta = 0.010 \hspace{1cm} \alpha=0.05 \hspace{1cm} \theta=0.1 \hspace{1cm} \%\text{Infect}= 3/3/3$

DATA THROUGH 11-SEP-2020
Reopening and Herd Immunity

- **Black**: assumes $R_0(today)$ remains in place forever
- **Red**: assumes $R_0(suppress) = 1/s(today)$
- **Green**: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
- **Purple**: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

**NOTE**: Lines often cover each other up
Armenia: Re-Opening ($\alpha = 0.05$)

Armenia

$R_0(t)=0.9, \ R_0(\text{suppress})=1.0, \ R_0(25/50)=1.2/1.4, \ \delta = 0.010, \ \alpha=0.05$

(Light bars = New York City, for comparison)
Armenia: Re-Opening ($\alpha = 0$)

Armenia

$R_0(t)=0.9$, $R_0\text{(suppress)}=1.0$, $R_0(25/50)=1.2/1.4$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Armenia (7 days): Daily Deaths per Million People ($\alpha = 0$)

Armenia

$R_0 = 1.6/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect = 3/3/3

DATA THROUGH 11-SEP-2020
Armenia (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Armenia

$R_0=1.6/0.9/0.9 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=3/3/3$

DATA THROUGH 11-SEP-2020
Armenia (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Armenia

$R_0=1.6/0.9/0.9$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  $\%$Infect= 3/3/3

New York City

Italy

Cumulative deaths per million people

Armenia: Daily Deaths per Million People ($\delta = 0.8\%$)

Armenia

$R_0 = 1.6/0.9/0.9 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 4/4/4$
Armenia: Cumulative Deaths per Million (δ = 0.8%)
Armenia: Daily Deaths per Million People ($\delta = 1.2\%$)

Armenia

$R_0 = 1.6/0.9/0.9 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 3/3/3$
Armenia: Cumulative Deaths per Million ($\delta = 1.2\%$)

Armenia

$R_0 = 1.6/0.9/0.9$, $\delta = 0.012$, $\theta = 0.1$, $\gamma = 0.2$, $\%\text{Infect} = 3/3/3$
Armenia: Daily Deaths per Million People ($\gamma = 0.2/0.15$)

Armenia

$R_0 = 1.6/0.9/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 3/3/3$

DATA THROUGH 11-SEP-2020
Armenia: Cumulative Deaths per Million $\gamma = 0.2/0.15$)

Armenia

$R_0=1.6/0.9/0.9 \ \delta = 0.010 \ \alpha=0.05 \ \theta=0.1 \ \%\text{Infect}=3/3/3$

DATA THROUGH 11-SEP-2020
Armenia: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Armenia

$R_0 = 1.6/0.9/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 3/3/3$

DATA THROUGH 11-SEP-2020
Armenia: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

$R_0 = 1.6/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 3/3/3

DATA THROUGH 11-SEP-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Armenia: Daily Deaths, Actual and Smoothed

Armenia: Daily deaths, \( d \)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Armenia: Change in Smoothed Daily Deaths

Armenia: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Armenia: Change in (Change in Smoothed Daily Deaths)

Armenia: Delta (Delta d)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]