Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

Jesús Fernández-Villaverde and Chad Jones

Extended results for Arizona
Based on data through September 11, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Arizona: Daily Deaths per Million People

Arizona
Arizona: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Arizona: Estimates of $R_0(t)$

Arizona
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Arizona: Percent Currently Infectious

Peak I/N = 0.60%   Final I/N = 0.12%  \( \delta = 0.010 \)  \( \theta = 0.10 \)  \( \gamma = 0.20 \)
Arizona: Growth Rate of Daily Deaths over Past Week (percent)

\[
\delta = 0.010 \quad \theta=0.10 \quad \gamma=0.20
\]
Notes on Interpreting Results
Guide to Graphs

- **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order!
  
  ROY-G-BIV (old to new, low to high)
  
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  
  - Violet (purple) = one day earlier

- For robustness graphs, same idea
  
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it.

- For future, two approaches:
  1. Alternatively, we fit this equation:

     \[
     \log R_0(t) = a_0 - \alpha(Daily \ Deaths)
     \]

     \Rightarrow \alpha \approx .05

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$. 
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Arizona (7 days): Daily Deaths per Million People ($\alpha = .05$)

$R_0 = 1.2/0.8/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%Infect = 8/8/8$

DATA THROUGH 11-SEP-2020
Arizona (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Arizona

$R_0=1.2/0.8/0.9 \; \delta = 0.010 \; \alpha=0.05 \; \theta=0.1 \; \%Infect=8/8/8$

DATA THROUGH 11-SEP-2020
Arizona (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Arizona

$R_0 = 1.2/0.8/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8$

New York City

Italy
Robustness to Mortality Rate, $\delta$
Arizona: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

DATA THROUGH 11-SEP-2020

Arizona

$R_0 = 1.2/0.8/0.9$ \hspace{1em} $\delta = 0.010$ \hspace{1em} $\alpha = 0.05$ \hspace{1em} $\theta = 0.1$ \hspace{1em} $\%$Infect$= 8/8/8$
Arizona: Daily Deaths per Million People ($\delta = .01/.008/.012$)

Data through 11-Sep-2020

Arizona

$R_0 = 1.2/0.8/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\% \text{Infect} = 8/8/8$
Arizona: Cumulative Deaths per Million ($\delta = .01/0.008/0.012$)

DATA THROUGH 11-SEP-2020

Arizona

$R_0=1.2/0.8/0.9 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 8/8/8$

$\delta = 0.008$
Reopening and Herd Immunity

- **Black**: assumes $R_0(\text{today})$ remains in place forever
- **Red**: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
- **Green**: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
- **Purple**: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

**NOTE**: Lines often cover each other up
Arizona: Re-Opening ($\alpha = .05$)

Arizona

$R_0(t)=0.8, \ R_0(\text{suppress})=1.1, \ R_0(25/50)=1.1/1.4, \ \delta = 0.010, \ \alpha=0.05$

(Light bars = New York City, for comparison)
Arizona: Re-Opening ($\alpha = 0$)

Arizona

$R_0(t)=0.7, \ R_0(\text{suppress})=1.1, \ R_0(25/50)=1.0/1.4, \ \delta = 0.010, \ \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Arizona (7 days): Daily Deaths per Million People ($\alpha = 0$)

Arizona

$R_0=1.2/0.7/0.7$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  $\%$Infect= 8/ 8/ 8

DATA THROUGH 11-SEP-2020
Arizona (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Arizona

$R_0 = 1.2/0.7/0.7 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8$

DATA THROUGH 11-SEP-2020
Arizona (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Arizona

$R_0 = 1.2/0.7/0.7$, $\delta = 0.010$, $\alpha = 0.00$, $\theta = 0.1$, %Infect = 8/8/8
Arizona: Daily Deaths per Million People ($\delta = 0.8\%$)

Arizona

$R_0 = 1.2/0.8/0.9$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\% Infect = 9/10/10$
Arizona: Cumulative Deaths per Million ($\delta = 0.8\%$)

Arizona

$R_0 = 1.2/0.8/0.9$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%\text{Infect} = 9/10/10$
Arizona: Daily Deaths per Million People ($\delta = 1.2\%$)

Arizona

$R_0=1.2/0.7/0.9$  $\delta=0.012$  $\theta=0.1$  $\gamma=0.2$  $%\text{Infect}=6/6/6$
Arizona: Cumulative Deaths per Million ($\delta = 1.2\%$)

Arizona

$R_0 = 1.2/0.7/0.9$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 6/6/6
Arizona: Daily Deaths per Million People ($\gamma = .2/.15$)

Arizona

$R_0 = 1.2/0.8/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%Infect = 8/8/8$

DATA THROUGH 11-SEP-2020
Arizona: Cumulative Deaths per Million $\gamma = .2 / .15$)

Arizona

$R_0 = 1.2 / 0.8 / 0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8 / 8 / 8$

$\gamma \equiv 0.75$

DATA THROUGH 11-SEP-2020
Arizona: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

\[ R_0 = 1.2/0.8/0.9 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8 \]

DATA THROUGH 11-SEP-2020
Arizona: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

DATA THROUGH 11-SEP-2020

Arizona

$R_0 = 1.2/0.8/0.9$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect $= 8/8/8$

$\theta = 0.2$

$\theta = 0.07$

Cumulative deaths per million people

Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Arizona: Daily Deaths, Actual and Smoothed

Arizona: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Arizona: Change in Smoothed Daily Deaths

Arizona: Delta d

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Arizona: Change in (Change in Smoothed Daily Deaths)

Arizona: Delta (Δd)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]