Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Arizona
Based on data through October 9, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)

• Brief summary of the model

• Baseline results \( \delta = 1.0\%, \gamma = 0.2, \theta = 0.1 \)

• Simulation of re-opening – possibilities for raising \( R_0 \)

• Results with alternative parameter values:
  o Lower mortality rate, \( \delta = 0.8\% \)
  o Higher mortality rate, \( \delta = 1.2\% \)
  o Infections last longer, \( \gamma = 0.15 \)
  o Cases resolve more quickly, \( \theta = 0.2 \)
  o Cases resolve more slowly, \( \theta = 0.07 \)

• Data underlying estimates of \( R_0(t) \)
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Arizona: Daily Deaths per Million People

Arizona

Daily deaths per million people
Arizona: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

- Inferred from daily deaths, and
- the change in daily deaths, and
- the change in (the change in daily deaths)
(see end of slide deck for this data)
Arizona: Estimates of $R_0(t)$

Arizona

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Arizona: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the *original paper*.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

\[
\log R_0(t) = a_0 - \alpha (\text{Daily Deaths})
\]

\[\Rightarrow \alpha \approx 0.05\]

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Arizona (7 days): Daily Deaths per Million People ($\alpha = .05$)

Arizona

$R_0 = 1.2/0.4/0.4$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 8/8/8

DATA THROUGH 09-OCT-2020
Arizona (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

Arizona

$R_0 = 1.2/0.4/0.4 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8$

DATA THROUGH 09-OCT-2020
Arizona (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Arizona

$R_0=1.2/0.4/0.4 \; \delta = 0.010 \; \alpha=0.05 \; \theta=0.1 \; \%Infect=8/8/8$
Robustness to Mortality Rate, $\delta$
Arizona: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

DATA THROUGH 09-OCT-2020

Arizona

$R_0 = 1.2/0.4/0.4$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 8/8/8

Cumulative deaths per million people
Arizona: Daily Deaths per Million People ($\delta = .01 / .008 / .012$)

Arizona

$R_0 = 1.2 / 0.4 / 0.4 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8 / 8 / 8$

DATA THROUGH 09-OCT-2020
Arizona: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

Arizona

$R_0 = 1.2/0.4/0.4$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 8/8/8$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people

Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = \text{“normal”}$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = \text{“normal”}$

NOTE: Lines often cover each other up
Arizona: Re-Opening ($\alpha = .05$)

$$R_0(t) = 0.4, \quad R_0(\text{suppress}) = 1.1, \quad R_0(25/50) = 0.8/1.2, \quad \delta = 0.010, \quad \alpha = 0.05$$

(Light bars = New York City, for comparison)
Arizona: Re-Opening ($\alpha = 0$)

Arizona

$R_0(t)=0.4$, $R_0\text{(suppress)}=1.1$, $R_0(25/50)=0.8/1.2$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Arizona (7 days): Daily Deaths per Million People ($\alpha = 0$)

Arizona

$R_0 = 1.2/0.4/0.4$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 8/8/8

DATA THROUGH 09-OCT-2020
Arizona (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Arizona

$R_0 = 1.2/0.4/0.4 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8$

DATA THROUGH 09-OCT-2020
Arizona (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Arizona

$R_0$ = 1.2/0.4/0.4  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 8/8/8
Arizona: Daily Deaths per Million People ($\delta = 0.8\%$)

Arizona

$R_0=1.2/0.4/0.4$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect}=10/10/10$

SOME ERRORS IN ESTIMATION...
Arizona: Cumulative Deaths per Million ($\delta = 0.8\%$)

Arizona

$R_0 = 1.2/0.4/0.4$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 10/10/10

SOME ERRORS IN ESTIMATION...
Arizona: Daily Deaths per Million People ($\delta = 1.2\%$)

Arizona

$R_0=1.2/0.4/0.4$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $%\text{Infect}=7/7/7$

SOME ERRORS IN ESTIMATION...
Arizona: Cumulative Deaths per Million ($\delta = 1.2\%$)

Arizona

$R_0$=1.2/0.4/0.4  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  %Infect= 7/7/7

SOME ERRORS IN ESTIMATION...
Arizona: Daily Deaths per Million People ($\gamma = .2/15$)

Data through 09-Oct-2020

\[ R_0 = 1.2/0.4/0.4 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8 \]
Arizona: Cumulative Deaths per Million $\gamma = .2/.15$)

Arizona

$R_0 = 1.2/0.4/0.4 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8$

$\gamma = 0.25$

DATA THROUGH 09-OCT-2020
Arizona: Daily Deaths per Million People ($\theta = 0.1/0.07/0.2$)

Arizona

$R_0 = 1.2/0.4/0.4$, $\delta = 0.010$, $\alpha = 0.05$, $\theta = 0.1$, $\% \text{Infect} = 8/8/8$

DATA THROUGH 09-OCT-2020
Arizona: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Arizona

$R_0 = 1.2/0.4/0.4 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 8/8/8$

DATA THROUGH 09-OCT-2020

Cumulative deaths per million people

Mar 2020 - Feb 2021
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Arizona: Daily Deaths, Actual and Smoothed

Arizona: Daily deaths, $d$

$\delta = 0.010$  $\theta = 0.10$  $\gamma = 0.20$
Arizona: Change in Smoothed Daily Deaths

Arizona: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Arizona: Change in (Change in Smoothed Daily Deaths)

Arizona: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]