Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

Jesús Fernández-Villaverde and Chad Jones

Extended results for Belgium
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results \((\delta = 1.0\%, \gamma = 0.2, \theta = 0.1)\)
• Simulation of re-opening – possibilities for raising \(R_0\)
• Results with alternative parameter values:
  - Lower mortality rate, \(\delta = 0.8\%\)
  - Higher mortality rate, \(\delta = 1.2\%\)
  - Infections last longer, \(\gamma = 0.15\)
  - Cases resolve more quickly, \(\theta = 0.2\)
  - Cases resolve more slowly, \(\theta = 0.07\)
• Data underlying estimates of \(R_0(t)\)
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Belgium: Daily Deaths per Million People

[Graph showing daily deaths per million people in Belgium from April to September 2020.]
Belgium: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t / \gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Belgium: Estimates of $R_0(t)$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Belgium: Percent Currently Infectious

Belgium
Peak I/N = 1.51%   Final I/N = 0.02%   δ = 0.010   θ = 0.10   γ = 0.20
Belgium: Growth Rate of Daily Deaths over Past Week (percent)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

- **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order!
  - ROY-G-BIV (old to new, low to high)
    - Black = current
    - Red = oldest, Orange = second oldest, Yellow = third oldest...
    - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it

- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     \[ \log R_0(t) = a_0 - \alpha (\text{Daily Deaths}) \]

     \[ \Rightarrow \alpha \approx .05 \]

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Belgium (7 days): Daily Deaths per Million People ($\alpha = .05$)

Belgium

$R_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/9$

DATA THROUGH 11-SEP-2020
Belgium (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Belgium

$R_0=2.1/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 9/9/9

DATA THROUGH 11-SEP-2020
Belgium (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Belgium

$R_0=2.1/0.2/0.2\ \ \delta = 0.010\ \ \alpha=0.05\ \ \theta=0.1\ %Infect= 9/\ 9/\ 9$
Robustness to Mortality Rate, $\delta$
Belgium: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

DATA THROUGH 11-SEP-2020

Belgium

$R_0=2.1/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=9/9/9$
Belgium: Daily Deaths per Million People ($\delta = .01/.008/.012$)

Belgium

$R_0 = 2.1/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect $= 9/9/9$

DATA THROUGH 11-SEP-2020
Belgium: Cumulative Deaths per Million \( (\delta = 0.01/0.008/0.012) \)

\[
R_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infected} = 9/9/9
\]

\( \delta = 0.008 \)

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people
Reopening and Herd Immunity

- **Black**: assumes $R_0(today)$ remains in place forever
- **Red**: assumes $R_0(suppress) = 1/s(today)$
- **Green**: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
- **Purple**: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

**NOTE**: Lines often cover each other up
Belgium: Re-Opening ($\alpha = 0.05$)

Belgium

$R_0(t)=0.2, \ R_0(\text{suppress})=1.1, \ R_0(25/50)=0.7/1.2, \ \delta = 0.010, \ \alpha=0.05$

(Light bars = New York City, for comparison)
Belgium: Re-Opening ($\alpha = 0$)

Belgium

$R_0(t)=0.2, \ R_0(\text{suppress})=1.1, \ R_0(25/50)=0.7/1.2, \ \delta = 0.010, \ \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Belgium (7 days): Daily Deaths per Million People ($\alpha = 0$)

Belgium

$R_0 = 2.1/0.2/0.2 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 9/9/9$

DATA THROUGH 11-SEP-2020
Belgium (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Belgium

$R_0 = 2.1/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 9/9/9

DATA THROUGH 11-SEP-2020
Belgium (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Belgium

$R_0=2.1/0.2/0.2$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  %Infect= 9/ 9/ 9

Cumulative deaths per million people

New York City

Italy

Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  2020

1  2  4  8  16  32  64  128  256  512  1024  2048  4096
Belgium: Daily Deaths per Million People ($\delta = 0.8\%$)

Belgium

$R_0=2.1/0.2/0.2 \quad \delta = 0.008 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}=11/11/11$

SOME ERRORS IN ESTIMATION...
Belgium: Cumulative Deaths per Million ($\delta = 0.8\%$)

Belgium

$R_0 = 2.1/0.2/0.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  %Infect=11/11/11

SOME ERRORS IN ESTIMATION...
Belgium: Daily Deaths per Million People ($\delta = 1.2\%$)

Belgium

$R_0 = 2.1/0.2/0.2$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  $\%$Infect = 7/7/7

SOME ERRORS IN ESTIMATION...
Belgium: Cumulative Deaths per Million ($\delta = 1.2\%$)

Belgium

$R_0=2.1/0.2/0.2 \quad \delta = 0.012 \quad \theta=0.1 \quad \gamma=0.2 \quad \%\text{Infect}=7/7/7$

SOME ERRORS IN ESTIMATION...
Belgium: Daily Deaths per Million People ($\gamma = .2/.15$)

Belgium

$R_0 = 2.1/0.2/0.2$   $\delta = 0.010$   $\alpha = 0.05$   $\theta = 0.1$   $\%\text{Infect} = 9/9/9$

DATA THROUGH 11-SEP-2020
Belgium: Cumulative Deaths per Million $\gamma = .2/.15$)

Belgium

$R_0=2.1/0.2/0.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect= 9/9/9$

$\gamma = 0.25$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  Jan  2020
Belgium: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Belgium

$R_0=2.1/0.2/0.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 9/ 9/ 9

DATA THROUGH 11-SEP-2020
Belgium: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Belgium

$R_0=2.1/0.2/0.2\ \ \delta = 0.010\ \ \alpha=0.05\ \ \theta=0.1\ \ \%\text{Infect}= 9/9/9$

DATA THROUGH 11-SEP-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Belgium: Daily Deaths, Actual and Smoothed

Belgium: Daily deaths, d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Belgium: Change in Smoothed Daily Deaths

Belgium: Delta d

\( \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \)
Belgium: Change in (Change in Smoothed Daily Deaths)

Belgium: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]