Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Bahrain
Based on data through August 24, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from
Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Bahrain: Daily Deaths per Million People

![Graph showing daily deaths per million people in Bahrain from April to August 2020. The graph displays a fluctuating line chart with the y-axis representing daily deaths per million people and the x-axis showing months. The data peaks in July and August.]
Bahrain: Daily Deaths per Million People (Smoothed)
Brief Summary of Model

• See the paper for a full exposition

• A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Bahrain: Estimates of $R_0(t)$

Bahrain
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Bahrain: Percent Currently Infectious

Peak I/N = 0.10%  Final I/N = 0.05%  δ = 0.010  θ = 0.10  γ = 0.20
Bahrain: Growth Rate of Daily Deaths over Past Week (percent)

\[
\text{Bahrain} \\
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]
Notes on Interpreting Results
Guide to Graphs

- **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

- 7 days of forecasts: Rainbow color order! ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

- For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it.
- For future, two approaches:
  1. Alternatively, we fit this equation:

$$\log R_0(t) = a_0 - \alpha(Daily\ Deaths)$$

$$\Rightarrow \alpha \approx .05$$

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$. 
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Bahrain (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

DATA THROUGH 24-AUG-2020

Bahrain

$R_0 = 1.4/1.1/1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect $= 1/2/6$
Bahrain (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

$$R_0 = 1.4/1.1/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/2/6$$

DATA THROUGH 24-AUG-2020
Bahrain (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

Bahrain

$R_0 = 1.4/1.1/1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 1/2/6$
Robustness to Mortality Rate, $\delta$
Bahrain: Cumulative Deaths per Million ($\delta = 0.01/0.008/0.012$)

DATA THROUGH 24-AUG-2020

Bahrain

$R_0=1.4/1.1/1.0 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 1/2/6$
Bahrain: Daily Deaths per Million People ($\delta = .01/0.008/0.012$)

Bahrain

$R_0 = 1.4/1.1/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 1/2/6$

DATA THROUGH 24-AUG-2020
Bahrain: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

**Bahrain**

$R_0=1.4/1.1/1.0 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%Infect= 1/ 2/ 6$

DATA THROUGH 24-AUG-2020
Reopening and Herd Immunity

– Black: assumes $R_0$(today) remains in place forever
– Red: assumes $R_0$(suppress) = $1/s(today)$
– Green: we move 25% of the way from $R_0$(today) back to initial $R_0$ = “normal”
– Purple: we move 50% of the way from $R_0$(today) back to initial $R_0$ = “normal”

NOTE: Lines often cover each other up
Bahrain: Re-Opening ($\alpha = .05$)

Bahrain

$R_0(t)=1.1$, $R_0$ (suppress) = 1.0, $R_0(25/50)$ = 1.3/1.6, $\delta = 0.010$, $\alpha=0.05$
Bahrain: Re-Opening ($\alpha = 0$)

Bahrain

$R_0(t)=1.1$, $R_0(\text{suppress})=1.0$, $R_0(25/50)=1.4/1.6$, $\delta = 0.010$, $\alpha=0.00$
Results for alternative parameter values
Bahrain (7 days): Daily Deaths per Million People ($\alpha = 0$)

R$_0$=1.4/1.1/1.1  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  %Infect= 1/ 2/16

DATA THROUGH 24-AUG-2020
Bahrain (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

Bahrain

$R_0 = 1.4/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect$= 1/2/16$

DATA THROUGH 24-AUG-2020
Bahrain (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Bahrain

$R_0 = 1.4/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \% \text{Infect} = 1/2/16$
Bahrain: Daily Deaths per Million People ($\delta = 0.8\%$)

Bahrain

$R_0 = 1.4/1.1/1.1 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 1/2/7$
Bahrain: Cumulative Deaths per Million ($\delta = 0.8\%$)

Bahrain

$R_0=1.4/1.1/1.1 \quad \delta = 0.008 \quad \theta=0.1 \quad \gamma=0.2 \quad \%Infect=1/2/7$
Bahrain: Daily Deaths per Million People ($\delta = 1.2\%$)

R$_0$=1.4/1.1/1.0  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  %Infect= 1/1/5
Bahrain: Cumulative Deaths per Million ($\delta = 1.2\%$)

Bahrain

$R_0 = 1.4/1.1/1.0$  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 1/1/5
Bahrain: Daily Deaths per Million People ($\gamma = .2 / .15$)

DATA THROUGH 24-AUG-2020

Bahrain

$R_0 = 1.4 / 1.1 / 1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 1 / 2 / 6
Bahrain: Cumulative Deaths per Million $\gamma = .2/.15$

Bahrain

$R_0=1.4/1.1/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect$= 1/2/6$

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people


$\gamma = 0.15$

$\gamma = 0.2$
Bahrain: Daily Deaths per Million People ($\theta = .1/.07/.2$)

DATA THROUGH 24-AUG-2020

Bahrain

$R_0=1.4/1.1/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 1/ 2/ 6
Bahrain: Cumulative Deaths per Million People ($\theta = .1/.07/.2$)

Bahrain

$R_0=1.4/1.1/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect= 1/2/6

DATA THROUGH 24-AUG-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Bahrain: Daily Deaths, Actual and Smoothed

Bahrain: Daily deaths, $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Bahrain: Change in Smoothed Daily Deaths

Bahrain: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Bahrain: Delta (Delta d)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]