Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Bosnia and Herzegovina
Based on data through September 11, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, $\delta = 0.8\%$
  o Higher mortality rate, $\delta = 1.2\%$
  o Infections last longer, $\gamma = 0.15$
  o Cases resolve more quickly, $\theta = 0.2$
  o Cases resolve more slowly, $\theta = 0.07$
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

– Raw data
– Smoothed = 7 day centered moving average
– No “excess deaths” correction (change as of Aug 6 run)
Bosnia and Herzegovina: Daily Deaths per Million People
Bosnia and Herzegovina: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Bosnia and Herzegovina: Estimates of $R_0(t)$

Bosnia and Herzegovina

$\delta = 0.010$  $\theta=0.10$  $\gamma=0.20$
Bosnia and Herzegovina: Percent Currently Infectious

Bosnia and Herzegovina

Peak I/N = 0.16%  Final I/N = 0.10%  $\delta=0.010$  $\theta=0.10$  $\gamma=0.20$
Bosnia and Herzegovina: Growth Rate of Daily Deaths over Past Week

Bosnia and Herzegovina
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• **7 days of forecasts:** Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  - **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  - **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it.
- For future, two approaches:
  
  1. Alternatively, we fit this equation:

     $$\log R_0(t) = a_0 - \alpha(Daily Deaths)$$

     $$\Rightarrow \alpha \approx 0.05$$

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) = final empirical value$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Bosnia and Herzegovina (7 days): Daily Deaths per Million People (\( \alpha = \))

\[ R_0 = 0.8/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 2/3/3 \]

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina (7 days): Cumulative Deaths per Million (Future)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 2/3/3$

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina (7 days): Cumulative Deaths per Million, Log Scale

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 2/3/3$

New York City

Italy
Robustness to Mortality Rate, $\delta$
Bosnia and Herzegovina: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 2/3/3

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina: Daily Deaths per Million People ($\delta = 0.01/0.008$)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$ Infect = 2/3/3

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina: Cumulative Deaths per Million ($\delta = 0.01, 0.008, 0.012$).

DATA THROUGH 11-SEP-2020

$R_0 = 0.8/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 2/3/3$
Reopening and Herd Immunity

- **Black**: assumes \( R_0(\text{today}) \) remains in place forever
- **Red**: assumes \( R_0(\text{suppress}) = \frac{1}{s(\text{today})} \)
- **Green**: we move 25% of the way from \( R_0(\text{today}) \) back to initial \( R_0 = \text{“normal”} \)
- **Purple**: we move 50% of the way from \( R_0(\text{today}) \) back to initial \( R_0 = \text{“normal”} \)

NOTE: Lines often cover each other up
Bosnia and Herzegovina: Re-Opening ($\alpha = .05$)

Bosnia and Herzegovina

$R_0(t)=0.9$,  $R_0^{(\text{suppress})}=1.0$,  $R_0^{(25/50)}=1.2/1.4$,  $\delta = 0.010$,  $\alpha=0.05$

(Light bars = New York City, for comparison)
Bosnia and Herzegovina: Re-Opening ($\alpha = 0$)

Bosnia and Herzegovina

$R_0(t)=0.9, \quad R_0(\text{suppress})=1.0, \quad R_0(25/50)=1.1/1.4, \quad \delta = 0.010, \quad \alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Bosnia and Herzegovina (7 days): Daily Deaths per Million People ($\alpha =$)

Bosnia and Herzegovina

$R_0=0.8/0.9/0.9 \quad \delta = 0.010 \quad \alpha=0.00 \quad \theta=0.1 \quad \%\text{Infect}=2/3/3$

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina (7 days): Cumulative Deaths per Million (Future)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/0.9$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%\text{Infect} = 2/3/3$

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina (7 days): Cumulative Deaths per Million, Log Scale

Bosnia and Herzegovina

$R_0 = 0.8/0.9/0.9 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \% \text{Infect} = 2/3/3$
Bosnia and Herzegovina: Daily Deaths per Million People ($\delta = 0.8\%$)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  $\%\text{Infect} = 3/3/4$

0 0.5 1 1.5 2 2.5 3 3.5

Daily deaths per million people

Bosnia and Herzegovina: Cumulative Deaths per Million ($\delta = 0.8\%$)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 3/3/4$
Bosnia and Herzegovina: Daily Deaths per Million People ($\delta = 1.2\%$)

Bosnia and Herzegovina

$R_0=0.8/0.9/1.0$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect} = 2/2/2$
Bosnia and Herzegovina: Cumulative Deaths per Million ($\delta = 1.2\%$)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0$ \hspace{1cm} $\delta = 0.012$ \hspace{1cm} $\theta = 0.1$ \hspace{1cm} $\gamma = 0.2$ \hspace{1cm} $\%$Infect $= 2/2/2$
Bosnia and Herzegovina: Daily Deaths per Million People ($\gamma = .2/1.5$)

Bosnia and Herzegovina

$R_0=0.8/0.9/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$ Infect= 2/3/3

Data through 11-Sep-2020
Bosnia and Herzegovina: Cumulative Deaths per Million $\gamma = .2/.15$)

Bosnia and Herzegovina

$R_0=0.8/0.9/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect= 2/3/3$

$\gamma = 0.2$

$\gamma = 0.15$

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina: Daily Deaths per Million People ($\theta = .1/.07/.2$)

Bosnia and Herzegovina

$R_0=0.8/0.9/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect = 2/3/3

DATA THROUGH 11-SEP-2020
Bosnia and Herzegovina: Cumulative Deaths per Million People ($\theta = 0.1$)

Bosnia and Herzegovina

$R_0 = 0.8/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 2/3/3$

DATA THROUGH 11-SEP-2020

$\theta = 0.2$

$\theta = 0.1$

$\theta = 0.07$
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Bosnia and Herzegovina: Daily Deaths, Actual and Smoothed

\[
\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20
\]
Bosnia and Herzegovina: Change in Smoothed Daily Deaths

Bosnia and Herzegovina: Delta $d$

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Bosnia and Herzegovina: Change in (Change in Smoothed Daily Deaths)

Bosnia and Herzegovina: Delta (Δd)

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$