Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Boston+Middlesex
Based on data through September 11, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results ($\delta = 1.0\%, \gamma = 0.2, \theta = 0.1$)
- Simulation of re-opening – possibilities for raising $R_0$
- Results with alternative parameter values:
  - Lower mortality rate, $\delta = 0.8\%$
  - Higher mortality rate, $\delta = 1.2\%$
  - Infections last longer, $\gamma = 0.15$
  - Cases resolve more quickly, $\theta = 0.2$
  - Cases resolve more slowly, $\theta = 0.07$
- Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Boston+Middlesex: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Boston+Middlesex: Estimates of $R_0(t)$

Boston+Middlesex

$\delta = 0.010 \; \theta=0.10 \; \gamma=0.20$
Boston+Middlesex: Percent Currently Infectious

Boston+Middlesex
Peak I/N = 1.77% Final I/N = 0.07% \( \delta = 0.010 \) \( \theta = 0.10 \) \( \gamma = 0.20 \)

Graph showing the percent currently infectious from March to September 2020 with peak in May and final percentage in September.
Boston+Middlesex: Growth Rate of Daily Deaths over Past Week (percent)
Notes on Intepreting Results
Guide to Graphs

• **Warning:** Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  ○ Black = current
  ○ Red = oldest, Orange = second oldest, Yellow = third oldest...
  ○ Violet (purple) = one day earlier

• For robustness graphs, same idea
  ○ Black = baseline (e.g. $\delta = 1.0\%$)
  ○ Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  ○ Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

     \[
     \log R_0(t) = a_0 - \alpha (\text{Daily Deaths})
     \]

     \[\Rightarrow \alpha \approx .05\]

     $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = 0.05$ (see robustness section for $\alpha = 0$)
Boston+Middlesex (7 days): Daily Deaths per Million People ($\alpha = .05$)

Boston+Middlesex

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect=13/14/16

DATA THROUGH 11 SEP 2020
Boston+Middlesex (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.05$)

$$R_0 = 2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 13/14/16$$

DATA THROUGH 11-SEP-2020
Boston+Middlesex (7 days): Cumulative Deaths per Million, Log Scale

Cumulative deaths per million people

Boston+Middlesex

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 13/14/16

New York City

Italy

Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan 2020
Robustness to Mortality Rate, $\delta$
Boston+Middlesex: Cumulative Deaths per Million ($\delta = .01/0.008/0.012$)

Boston+Middlesex

$R_0=2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=13/14/16$

DATA THROUGH 11-SEP-2020
Boston+Middlesex: Daily Deaths per Million People ($\delta = 0.01/0.008/0.012$)

DATA THROUGH 11 SEP 2020

Boston+Middlesex

$R_0 = 2.1/1.2/1.2$ \hspace{1em} \delta = 0.010 \hspace{1em} \alpha = 0.05 \hspace{1em} \theta = 0.1 \hspace{1em} \%Infect = 13/14/16
Boston+Middlesex: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Boston+Middlesex

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $%\text{Infect} = 13/14/16$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people

Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  2020
Reopening and Herd Immunity

– Black: assumes $R_0(\text{today})$ remains in place forever
– Red: assumes $R_0(\text{suppress}) = 1/s(\text{today})$
– Green: we move 25% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$
– Purple: we move 50% of the way from $R_0(\text{today})$ back to initial $R_0 = \text{“normal”}$

NOTE: Lines often cover each other up
Boston+Middlesex: Re-Opening ($\alpha = .05$)

Boston+Middlesex

$R_0(t)=1.2$, $R_0(\text{suppress})=1.2$, $R_0(25/50)=1.4/1.7$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
Boston+Middlesex: Re-Opening ($\alpha = 0$)

Boston+Middlesex

$R_0(t)=1.2$, $R_0$(suppress)$=1.2$, $R_0$(25/50)$=1.4/1.7$, $\delta = 0.010$, $\alpha = 0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
Boston+Middlesex (7 days): Daily Deaths per Million People ($\alpha = 0$)

Boston+Middlesex

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\%$Infect = 13/14/17

DATA THROUGH 11 SEP 2020
Boston+Middlesex (7 days): Cumulative Deaths per Million (Future, $\alpha = 0.1$)

**Boston+Middlesex**

$R_0=2.1/1.2/1.2$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  $\%$Infect=13/14/17

DATA THROUGH 11-SEP-2020
Boston+Middlesex (7 days): Cumulative Deaths per Million, Log Scale

Boston+Middlesex

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect=13/14/17

Cumulative deaths per million people

New York City

Italy
Boston+Middlesex: Daily Deaths per Million People ($\delta = 0.8\%$)

Boston+Middlesex

$R_0=2.1/1.3/1.2$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $\%Infect=17/17/20$
Boston+Middlesex: Cumulative Deaths per Million ($\delta = 0.8\%$)

Boston+Middlesex

$R_0 = 2.1/1.3/1.2 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 17/17/20$
Boston+Middlesex: Daily Deaths per Million People ($\delta = 1.2\%$)

Boston+Middlesex

$R_0 = 2.1/1.2/1.1 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 11/12/13$
Boston+Middlesex: Cumulative Deaths per Million ($\delta = 1.2\%$)

Boston+Middlesex
$R_0=2.1/1.2/1.1$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $%\text{Infect}=11/12/13$
Boston+Middlesex: Daily Deaths per Million People ($\gamma = .2/.15$)

Boston+Middlesex

$R_0=2.1/1.2/1.2 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}=13/14/16$

DATA THROUGH 11 SEP 2020

Daily deaths per million people

Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec  2020
Boston+Middlesex: Cumulative Deaths per Million $\gamma = 0.2/0.15$)

Boston+Middlesex

$R_0 = 2.1/1.2/1.2$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect $= 13/14/16$

DATA THROUGH 11-SEP-2020
Boston+Middlesex: Daily Deaths per Million People ($\theta = .1/\ .07/\ .2$)

Boston+Middlesex

$R_0=2.1/1.2/1.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}=13/14/16$

DATA THROUGH 11 SEP 2020
Boston+Middlesex: Cumulative Deaths per Million People ($\theta = 0.1/0.07/0.2$)

Boston+Middlesex

$R_0=2.1/1.2/1.2$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%$Infect$=13/14/16$

DATA THROUGH 11-SEP-2020

Cumulative deaths per million people
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Boston+Middlesex: Daily Deaths, Actual and Smoothed

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Boston+Middlesex: Change in Smoothed Daily Deaths

Delta d
\( \delta = 0.010 \) \( \theta=0.10 \) \( \gamma=0.20 \)
Boston+Middlesex: Change in (Change in Smoothed Daily Deaths)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]