Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

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Extended results for Brazil
Based on data through August 24, 2020
Outline of Slides

• Basic data from Johns Hopkins CSSE (raw and smoothed)
• Brief summary of the model
• Baseline results (δ = 1.0%, γ = 0.2, θ = 0.1)
• Simulation of re-opening – possibilities for raising $R_0$
• Results with alternative parameter values:
  o Lower mortality rate, δ = 0.8%
  o Higher mortality rate, δ = 1.2%
  o Infections last longer, γ = 0.15
  o Cases resolve more quickly, θ = 0.2
  o Cases resolve more slowly, θ = 0.07
• Data underlying estimates of $R_0(t)$
Underlying data from Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
Brazil: Daily Deaths per Million People (Smoothed)
**Brief Summary of Model**

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
Brazil: Estimates of $R_0(t)$

Brazil

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$
Brazil: Percent Currently Infectious

Brazil

Peak I/N = 0.25%  Final I/N = 0.23%  \( \delta = 0.010 \)  \( \theta = 0.10 \)  \( \gamma = 0.20 \)
Brazil: Growth Rate of Daily Deaths over Past Week (percent)

Brazil

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  
  o Black = current
  o **Red** = oldest, **Orange** = second oldest, **Yellow** = third oldest...
  o **Violet** (purple) = one day earlier

• For robustness graphs, same idea
  
  o Black = baseline (e.g. $\delta = 1.0\%$)
  o **Red** = lowest parameter value (e.g. $\delta = 0.8\%$)
  o **Green** = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it.
- For future, two approaches:
  1. Alternatively, we fit this equation:

\[
\log R_0(t) = a_0 - \alpha (\text{Daily Deaths})
\]

\[\Rightarrow \alpha \approx .05\]

$R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline.

- Robustness: Assume $R_0(t) = \text{final empirical value}$. Constant in future, so no $\alpha$ adjustment $\Rightarrow \alpha = 0$.
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = .05$ (see robustness section for $\alpha = 0$)
Brazil (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

Brazil

$R_0 = 1.3/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 6/7/11$

DATA THROUGH 24-AUG-2020
Brazil (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

Brazil

$R_0=1.3/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%Infect=6/7/11$

DATA THROUGH 24-AUG-2020
Brazil (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0.05$)

Brazil

$R_0=1.3/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 6/7/11

Cumulative deaths per million people
Robustness to Mortality Rate, $\delta$
Brazil: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

Brazil

$R_0 = 1.3/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect$= 6/7/11$

DATA THROUGH 24-AUG-2020
Brazil: Daily Deaths per Million People ($\delta = 0.01/0.08/0.12$)

Brazil

$R_0=1.3/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  $\%\text{Infect}=6/7/11$

DATA THROUGH 24-AUG-2020
Brazil: Cumulative Deaths per Million ($\delta = 0.01/.008/.012$)

Brazil

$R_0=1.3/1.1/1.1$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 6/7/11

DATA THROUGH 24-AUG-2020
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
Brazil: Re-Opening ($\alpha = .05$)

Brazil

$R_0(t)=1.1$, $R_0(suppress)=1.1$, $R_0(25/50)=1.3/1.5$, $\delta = 0.010$, $\alpha=0.05$
Brazil: Re-Opening ($\alpha = 0$)

Brazil

$R_0(t) = 1.1$, $R_0(\text{suppress}) = 1.1$, $R_0(25/50) = 1.3/1.5$, $\delta = 0.010$, $\alpha = 0.00$
Results for alternative parameter values
Brazil (7 days): Daily Deaths per Million People ($\alpha = 0$)

Brazil

$R_0 = 1.3/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 6/7/10

DATA THROUGH 24-AUG-2020
Brazil

\[ R_0 = 1.3/1.1/1.1 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect} = 6/7/10 \]

DATA THROUGH 24-AUG-2020

Brazil (7 days): Cumulative Deaths per Million (Future, \( \alpha = 0 \))
Brazil (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

Brazil

$R_0 = 1.3/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  $\% \text{Infect} = 6/7/10$

New York City

Italy

Cumulative deaths per million people
Brazil: Daily Deaths per Million People ($\delta = 0.8\%$)

Brazil

$R_0 = 1.3/1.1/1.1$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 7/9/14
Brazil: Cumulative Deaths per Million ($\delta = 0.8\%$)

Brazil

$R_0=1.3/1.1/1.1$  $\delta = 0.008$  $\theta=0.1$  $\gamma=0.2$  $%\text{Infect}=7/9/14$
Brazil: Daily Deaths per Million People ($\delta = 1.2\%$)

Brazil

$R_0 = 1.3/1.0/1.1 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 5/6/9$
Brazil: Cumulative Deaths per Million ($\delta = 1.2\%$)

Brazil

$R_0=1.3/1.0/1.1$  $\delta = 0.012$  $\theta=0.1$  $\gamma=0.2$  $\%\text{Infect}=5/6/9$
Brazil: Daily Deaths per Million People ($\gamma = .2/1.15$)

Brazil

$R_0 = 1.3/1.1/1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%$Infect = 6/7/11

DATA THROUGH 24-AUG-2020
Brazil: Cumulative Deaths per Million $\gamma = .2/ .15$)

Brazil

$R_0 = 1.3/1.1/1.1$, $\delta = 0.010$, $\alpha = 0.05$, $\theta = 0.1$, %Infect = 6/7/11

DATA THROUGH 24-AUG-2020

Cumulative deaths per million people
Brazil: Daily Deaths per Million People ($\theta = .1/ .07/ .2$)

Brazil

$R_0 = 1.3/1.1/1.1$  \( \delta = 0.010 \)  \( \alpha = 0.05 \)  \( \theta = 0.1 \)  \%Infect = 6/7/11

DATA THROUGH 24-AUG-2020
Brazil: Cumulative Deaths per Million People ($\theta = .1 / .07 / .2$)

DATA THROUGH 24-AUG-2020

Brazil

$R_0 = 1.3 / 1.1 / 1.1$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  $\%\text{Infect} = 6/7/11$

$\theta \equiv 0.7^7$
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
Brazil: Daily Deaths, Actual and Smoothed

\[ d = 0.010 \quad \theta=0.10 \quad \gamma=0.20 \]
Brazil: Change in Smoothed Daily Deaths

Brazil: Delta d
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Brazil: Change in (Change in Smoothed Daily Deaths)

Brazil: Delta (Delta d)

$\delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20$