Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities

Jesús Fernández-Villaverde and Chad Jones

Extended results for California
Based on data through September 11, 2020
Outline of Slides

- Basic data from Johns Hopkins CSSE (raw and smoothed)
- Brief summary of the model
- Baseline results \( \delta = 1.0\%, \gamma = 0.2, \theta = 0.1 \)
- Simulation of re-opening – possibilities for raising \( R_0 \)
- Results with alternative parameter values:
  - Lower mortality rate, \( \delta = 0.8\% \)
  - Higher mortality rate, \( \delta = 1.2\% \)
  - Infections last longer, \( \gamma = 0.15 \)
  - Cases resolve more quickly, \( \theta = 0.2 \)
  - Cases resolve more slowly, \( \theta = 0.07 \)
- Data underlying estimates of \( R_0(t) \)
Underlying data from
Johns Hopkins CSSE

- Raw data
- Smoothed = 7 day centered moving average
- No “excess deaths” correction (change as of Aug 6 run)
California: Daily Deaths per Million People

California
California: Daily Deaths per Million People (Smoothed)
### Brief Summary of Model

- See the paper for a full exposition
- A 5-state SIRDC model with a time-varying $R_0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>1.0%</td>
<td>Mortality rate from infections (IFR)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2</td>
<td>Rate at which people stop being infectious</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>Rate at which cases (post-infection) resolve</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.05</td>
<td>Rate at which $R_0(t)$ decays with daily deaths</td>
</tr>
<tr>
<td>$R_0$</td>
<td>...</td>
<td>Initial base reproduction rate</td>
</tr>
<tr>
<td>$R_0(t)$</td>
<td>...</td>
<td>Base reproduction rate at date $t$ ($\beta_t/\gamma$)</td>
</tr>
</tbody>
</table>
Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
(see end of slide deck for this data)
California: Estimates of $R_0(t)$

California
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
California: Percent Currently Infectious

California
Peak I/N = 0.18%  Final I/N = 0.11%  δ = 0.010  θ = 0.10  γ = 0.20
California: Growth Rate of Daily Deaths over Past Week (percent)

California

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
Notes on Interpreting Results
Guide to Graphs

• **Warning**: Results are often very uncertain; this can be seen by comparing across multiple graphs. See the original paper.

• 7 days of forecasts: Rainbow color order!
  ROY-G-BIV (old to new, low to high)
  - Black = current
  - Red = oldest, Orange = second oldest, Yellow = third oldest...
  - Violet (purple) = one day earlier

• For robustness graphs, same idea
  - Black = baseline (e.g. $\delta = 1.0\%$)
  - Red = lowest parameter value (e.g. $\delta = 0.8\%$)
  - Green = highest parameter value (e.g. $\delta = 1.2\%$)
How does $R_0$ change over time?

- Inferred from death data when we have it
- For future, two approaches:
  1. Alternatively, we fit this equation:

   \[
   \log R_0(t) = a_0 - \alpha(Daily \ Deaths) \]

   \[\Rightarrow \alpha \approx .05\]

   $R_0$ declines by 5 percent for each new daily death, or rises by 5 percent when daily deaths decline

- Robustness: Assume $R_0(t) =$ final empirical value. Constant in future, so no $\alpha$ adjustment $\rightarrow \alpha = 0$
Repeated “Forecasts” from the past 7 days of data

– After peak, forecasts settle down.
– Before that, very noisy!
– If the region has not peaked, do not trust
– With $\alpha = 0.05$ (see robustness section for $\alpha = 0$)
California (7 days): Daily Deaths per Million People ($\alpha = 0.05$)

California

$R_0 = 1.4/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect}= 4/4/4$

DATA THROUGH 11-SEP-2020
California (7 days): Cumulative Deaths per Million (Future, $\alpha = .05$)

California

$R_0 = 1.4/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4$

DATA THROUGH 11-SEP-2020
California (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = .05$)

California

$R_0=1.4/0.9/1.0$  \( \delta = 0.010 \)  \( \alpha=0.05 \)  \( \theta=0.1 \)  \%Infect= 4/ 4/ 4

New York City

Italy
Robustness to Mortality Rate, $\delta$
California: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

California

$R_0=1.4/0.9/1.0$  $\delta = 0.010$  $\alpha=0.05$  $\theta=0.1$  %Infect= 4/ 4/ 4

DATA THROUGH 11-SEP-2020
California: Daily Deaths per Million People ($\delta = .01/.008/.012$)

California

$R_0 = 1.4/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4$

DATA THROUGH 11-SEP-2020
California: Cumulative Deaths per Million ($\delta = .01/.008/.012$)

California

$R_0=1.4/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \% \text{Infect} = 4/4/4$

$\delta = 0.008$

DATA THROUGH 11-SEP-2020
Reopening and Herd Immunity

– Black: assumes $R_0(today)$ remains in place forever
– Red: assumes $R_0(suppress) = 1/s(today)$
– Green: we move 25% of the way from $R_0(today)$ back to initial $R_0 = “normal”$
– Purple: we move 50% of the way from $R_0(today)$ back to initial $R_0 = “normal”$

NOTE: Lines often cover each other up
California: Re-Opening ($\alpha = .05$)

California

$R_0(t)=0.9$, $R_0$ (suppress)$=1.0$, $R_0(25/50)=1.1/1.4$, $\delta = 0.010$, $\alpha=0.05$

(Light bars = New York City, for comparison)
California: Re-Opening ($\alpha = 0$)

California

$R_0(t)=0.8$, $R_0\text{(suppress)}=1.0$, $R_0(25/50)=1.1/1.4$, $\delta = 0.010$, $\alpha=0.00$

(Light bars = New York City, for comparison)
Results for alternative parameter values
California (7 days): Daily Deaths per Million People ($\alpha = 0$)

California

$R_0 = 1.4/0.8/0.8 \quad \delta = 0.010 \quad \alpha = 0.00 \quad \theta = 0.1 \quad \%\text{Infect}= 4/4/4$

DATA THROUGH 11-SEP-2020
California (7 days): Cumulative Deaths per Million (Future, $\alpha = 0$)

California

$R_0=1.4/0.8/0.8$  $\delta = 0.010$  $\alpha=0.00$  $\theta=0.1$  %Infect= 4/4/4

DATA THROUGH 11-SEP-2020
California (7 days): Cumulative Deaths per Million, Log Scale ($\alpha = 0$)

California

$R_0 = 1.4/0.8/0.8$  $\delta = 0.010$  $\alpha = 0.00$  $\theta = 0.1$  %Infect = 4/4/4

New York City

Italy
California: Daily Deaths per Million People ($\delta = 0.8\%$)

California

$R_0 = 1.4/0.9/1.0$  $\delta = 0.008$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 5/5/5
California: Cumulative Deaths per Million ($\delta = 0.8\%$)

California

$R_0 = 1.4/0.9/1.0 \quad \delta = 0.008 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 5/5/5$
California: Daily Deaths per Million People ($\delta = 1.2\%$)

California

$R_0 = 1.4/0.9/1.0 \quad \delta = 0.012 \quad \theta = 0.1 \quad \gamma = 0.2 \quad \%\text{Infect} = 3/3/4$
California: Cumulative Deaths per Million ($\delta = 1.2\%$)

R$_0$ = 1.4/0.9/1.0  $\delta = 0.012$  $\theta = 0.1$  $\gamma = 0.2$  %Infect = 3/3/4
California: Daily Deaths per Million People ($\gamma = .2/.15$)

\[ R_0 = 1.4/0.9/1.0 \quad \delta = 0.010 \quad \alpha=0.05 \quad \theta=0.1 \quad \%\text{Infect}= 4/ 4/ 4 \]

DATA THROUGH 11-SEP-2020
California: Cumulative Deaths per Million \( \gamma = .2/.15 \)

California

\[ R_0 = 1.4/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4 \]

\( \gamma \equiv 0.15 \)

DATA THROUGH 11-SEP-2020
California: Daily Deaths per Million People ($\theta = .1/.07/.2$)

\[ R_0 = 1.4/0.9/1.0 \quad \delta = 0.010 \quad \alpha = 0.05 \quad \theta = 0.1 \quad \%\text{Infect} = 4/4/4 \]

DATA THROUGH 11-SEP-2020
California: Cumulative Deaths per Million People ($\theta = .1/ .07/ .2$)

California

$R_0 = 1.4/0.9/1.0$  $\delta = 0.010$  $\alpha = 0.05$  $\theta = 0.1$  %Infect = 4/4/4

DATA THROUGH 11-SEP-2020
Data Underlying Estimates of Time-Varying $R_0$

– Inferred from daily deaths, and
– the change in daily deaths, and
– the change in (the change in daily deaths)
California: Daily Deaths, Actual and Smoothed

California: Daily deaths, \( d \)

\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]
California: Change in Smoothed Daily Deaths

δ = 0.010  θ=0.10  γ=0.20
California: Change in (Change in Smoothed Daily Deaths)

California: Delta (Delta d)
\[ \delta = 0.010 \quad \theta = 0.10 \quad \gamma = 0.20 \]